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RESEARCH ARTICLE

Coherent manipulation of spin squeezing in atomic Bose–Einstein condensate via electromagnetically induced transparency

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We propose a scheme to coherently control spin squeezing of atomic Bose–Einstein condensate (BEC) via the technique of electromagnetically induced transparency (EIT). We study quantum dynamics of the mean spin vector and spin squeezing. It is shown that the mean spin vector and spin squeezing of the BEC can be controlled and manipulated by adjusting the external coupling fields or/and internal nonlinear interactions of the BEC. It is indicated that the spin squeezing can be generated rapidly in the dynamical process and maintained in a long time interval. It is found that a larger effective Rabi coupling between atoms and lasers can produce a stronger spin squeezing, and the squeezing can maintain a longer time interval.

Keywords spin squeezing, atomic Bose–Einstein condensates via electromagnetically induced transparency

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1 Introduction

Many-body quantum mechanics offers the possibility to overcome the single particle limit by the use of entanglement as a resource [1–3]. Spin squeezing [4, 5] is one example where entanglement provides a resource for quantum enhanced metrology [6]. For atomic two-level systems the concept of spin squeezed states and a mechanism to obtain them was introduced by Kitagawa and Ueda [7]. One year later Wineland *et al.* pointed out its potential usefulness for atom interferometry [8]. The basic idea is to use a quantum correlated spin state for Ramsey type interferometry in which the quantum fluctuations in the different spin directions are redistributed [9]. Atomic spin squeezing has already been experimentally demonstrated in vapor cell experiments [10–13], ion traps [14] and Bose–Einstein condensates (BECs) [15,16].

Recent studies indicate that there is a close relation between spin squeezing and quantum entanglement [17–21]. The advantage of spin squeezing over the well-known entanglement measures, such as concurrence and negativity is that spin squeezing can be used as a measurement of many-partite entanglement [22–24]. The interest in spin squeezing arises not only from the fact that it exhibits reduced fluctuations of the collection of atomic spins below the potential spin noise limit, but also from the possibility of interesting novel applications in high-precision spectroscopy [8] and atomic clocks [25].

Atomic BECs have become almost routinely available in the laboratory. They provide us with large coherent ensembles of ultra-cold atoms which can be used to generate spin squeezing and perform quantum state manipulation from an exquisitely well-controlled initial state. There have been several proposals of how to engineer spin squeezing in BECs [17, 26–29]. Sørensen *et al.* proposed a scheme for creating massive entangled spin squeezed states from a two-component condensate using the inherent atom–atom interactions [17]. Micheli *et al.* further investigated a scheme to dynamically create many-particle entangled states of a two-component BEC [26]. Zhang *et al.* considered a scheme that can create massively entangled states of BECs using an effective interaction between two atoms from coherent Raman processes through a (two-atom) molecular intermediate state [27]. Jin and Kim [28] proposed a simple scheme for storage of spin squeezing in a two-component Bose–Einstein condensate [30]. The purpose of this paper is to propose a scheme of controlling of spin squeezing in atomic BECs via electromagnetically induced transparency (EIT) technique [31–33]. We show that quantum dynamics of atomic spin squeezing in BECs can be coherently controlled and manipulated by appropriately adjusting the external laser fields or/and internal interatomic nonlinear interactions.

2 Physical model and solution

We consider a cloud of BEC atoms which have three internal states labeled by $|1\rangle$, $|2\rangle$, and $|3\rangle$ with energies E_1, E_2, and E_3, respectively. The two lower states $|1\rangle$ and $|3\rangle$ are Raman coupled to the upper state $|2\rangle$ via two classical laser fields of frequencies ω_1 and ω_2 in the Lambda configuration, respectively. The interaction scheme is shown in Fig. 1. The atoms in these internal states are subject to isotropic harmonic trapping potentials $V_i(\boldsymbol{r})$ for $i = 1, 2, 3$, respectively. Furthermore, the atoms in BEC interact with each other via elastic two-body collisions with the δ-function potentials $V_{ij}(\boldsymbol{r}-\boldsymbol{r}') = U_{ij}\delta(\boldsymbol{r}-\boldsymbol{r}')$,where $U_{ij} = 4\pi\hbar^2 a_{ij}^{sc}/m$ with m and a_{ij}^{sc} being the atomic mass and the s-wave scattering length between atoms in states i and j, respectively. A good experimental candidate of this system is the sodium atom condensate for which there exist appropriate atomic internal levels and external laser fields to form the Lambda configuration having been used to demonstrate ultraslow light propagation [34, 35], amplification of light and atoms [36] in atomic BECs, and generation of entangled squeezed states [37].

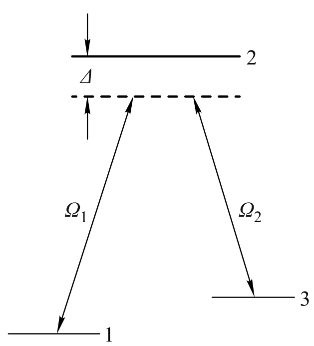

Fig. 1 Three-level Lambda-type atoms coupled to two classical laser fields with the two-photon detuning Δ.

The second quantized Hamiltonian to describe the system at zero temperature [38, 39] is given by

$$\hat{H} = \hat{H}_a + \hat{H}_{a-l} + \hat{H}_c \tag{1}$$

where $\hat{H}_a$ gives the free evolution of the atomic fields, $\hat{H}_{a-l}$ describes the dipole interactions between the atomic fields and laser fields, and $\hat{H}_c$ represents inter-atom two-body interactions.

The free atomic Hamiltonian is given by

$$\hat{H}_a = \sum_{i=1}^{3}\int \mathrm{d}\boldsymbol{x}\hat{\psi}_i^\dagger(\boldsymbol{x})\left[-\frac{\hbar^2}{2m}\nabla^2 + V_i(\boldsymbol{x}) + E_i\right]\hat{\psi}_i(\boldsymbol{x}) \tag{2}$$

where E_i are internal energies for the three internal states, $\hat{\psi}_i(\boldsymbol{x})$ and $\hat{\psi}_i^\dagger(\boldsymbol{x})$ are the boson annihilation and creation operators for the $|i\rangle$-state atoms at position $\boldsymbol{x}$, respectively, they satisfy the standard boson commutation relation $[\hat{\psi}_i(\boldsymbol{x}), \hat{\psi}_j^\dagger(\boldsymbol{x}')] = \delta_{ij}\delta(\boldsymbol{x}-\boldsymbol{x}')$ and $[\hat{\psi}_i(\boldsymbol{x}), \hat{\psi}_j(\boldsymbol{x}')] = 0 = [\hat{\psi}_i^\dagger(\boldsymbol{x}), \hat{\psi}_j^\dagger(\boldsymbol{x}')]$.

The atom-laser interactions in the dipole approximation can be described by the following Hamiltonian

$$\hat{H}_{a-l} = \frac{1}{2}\int \mathrm{d}\boldsymbol{x}[\Omega_1\hat{\psi}_2^\dagger(\boldsymbol{x})\hat{\psi}_1(\boldsymbol{x})\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{x}-\omega_1 t)} + \Omega_2\hat{\psi}_2^\dagger(\boldsymbol{x})\hat{\psi}_3(\boldsymbol{x})\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_2\cdot\boldsymbol{x}-\omega_2 t)} + H.c.] \tag{3}$$

where $\Omega_1 = -\mu_{21}\mathcal{E}_1/\hbar$ and $\Omega_2 = -\mu_{23}\mathcal{E}_2/\hbar$ are the Rabi frequencies of the two laser beams with μ_{ij} denoting a transition dipole-matrix element between states $|i\rangle$ and $|j\rangle$, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are wave vectors of correspondent laser fields.

The collision Hamiltonian is taken to be the following form

$$\hat{H}_c = \frac{2\pi\hbar^2}{m}\int \mathrm{d}\boldsymbol{x}\left[\sum_{i=1}^{3} a_i^{sc}\hat{\psi}_i^\dagger(\boldsymbol{x})\hat{\psi}_i^\dagger(\boldsymbol{x})\hat{\psi}_i(\boldsymbol{x})\hat{\psi}_i(\boldsymbol{x}) + \sum_{i\neq j} 2a_{ij}^{sc}\hat{\psi}_i^\dagger(\boldsymbol{x})\hat{\psi}_j^\dagger(\boldsymbol{x})\hat{\psi}_i(\boldsymbol{x})\hat{\psi}_j(\boldsymbol{x})\right] \tag{4}$$

where a_i^{sc} is s-wave scattering length of condensate in the internal state $|i\rangle$ and a_{ij}^{sc} between condensates in the internal states $|i\rangle$ and $|j\rangle$.

For a weakly interacting BEC at zero temperature there are no thermally excited atoms and the quantum depletion is negligible. The motion state is frozen to be approximately the ground state. One may neglect all modes except for the condensate mode and approximately factorize the atomic field operators as $\hat{\psi}_i(\boldsymbol{x}) \approx \hat{b}_i\phi_i(\boldsymbol{x})$ where $\phi_i(\boldsymbol{x})$ is a normalized wavefunction for the atoms in the BEC, which is given by the ground state of the following Schrödinger equation:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_i(\boldsymbol{x}) + E_i\right]\phi_i(\boldsymbol{x}) = \hbar\nu_i\phi_i(\boldsymbol{x}) \tag{5}$$

where $\hbar\nu_i$ is the energy of the mode i.

The valid conditions of the single-mode approximation were demonstrated in Refs. [40–43], which indicate that this approximation provides a reasonably accurate picture for weak many-body interactions. Substituting the single-mode expansions of the atomic field operators into Eqs. (2)–(4), we arrive at the following three-mode approximate Hamiltonian

$$\hat{H} = \hbar\sum_{i=1}^{3}\nu_i\hat{b}_i^\dagger\hat{b}_i - \hbar(g_1\hat{b}_2^\dagger\hat{b}_1\mathrm{e}^{-\mathrm{i}\omega_1 t} + g_2\hat{b}_2^\dagger\hat{b}_3\mathrm{e}^{-\mathrm{i}\omega_2 t} + H.c.) + \sum_{i=1}^{3}\lambda_i\hat{b}_i^{\dagger 2}\hat{b}_i^2 + \sum_{i\neq j}\lambda_{ij}\hat{b}_i^\dagger\hat{b}_i\hat{b}_j^\dagger\hat{b}_j \tag{6}$$

where the linear interaction between different BEC components is described by tunneling coupling constants defined by

$$g_i = \frac{1}{2}\Omega_i\int \mathrm{d}\boldsymbol{x}\phi_2^*(\boldsymbol{x})\phi_1(\boldsymbol{x})\mathrm{e}^{\mathrm{i}\boldsymbol{k}_i\cdot\boldsymbol{x}} \tag{7}$$

which is the Rabi frequency for the transition between the state $|2\rangle$ and state $|i\rangle$ $(i = 1, 2)$. And nonlinear interactions among atoms are described by the following coupling constants

$$\lambda_i = \frac{2\pi\hbar^2 a_i^{sc}}{m}\int \mathrm{d}\boldsymbol{x}|\phi_i(\boldsymbol{x})|^4 \tag{8}$$

$$\lambda_{ij} = \frac{4\pi\hbar^2 a_{ij}^{sc}}{m}\int \mathrm{d}\boldsymbol{x}|\phi_i(\boldsymbol{x})|^2|\phi_j(\boldsymbol{x})|^2, \quad i\neq j \tag{9}$$

which characterize the strength of the interatomic interaction in each condensate and between two different BEC components. They can be manipulated by Feshbach resonances. We note that an analogous three-mode Bose–Hubbard model has been discussed for BEC in a special optical lattice potential [44].

Going over to an interaction picture with respect to

$$H_0 = \hbar\nu_1\sum_{i=1}^{3}\hat{b}_i^\dagger\hat{b}_i + \hbar(\omega_1-\omega_2)\hat{b}_3^\dagger\hat{b}_3 + \hbar\omega_1\hat{b}_2^\dagger\hat{b}_2 \tag{10}$$

we can transfer the time-dependent Hamiltonian (6) to the following time-independent Hamiltonian

$$\hat{H}_I = \hbar\Delta\hat{b}_2^\dagger\hat{b}_2 - \hbar[g_1\hat{b}_2^\dagger\hat{b}_1 + g_2\hat{b}_2^\dagger\hat{b}_3 + H.c.] + \sum_{i=1}^{3}\lambda_i\hat{b}_i^{\dagger 2}\hat{b}_i^2 + \sum_{i\neq j}\lambda_{ij}\hat{b}_i^\dagger\hat{b}_i\hat{b}_j^\dagger\hat{b}_j \tag{11}$$

where $\Delta = \nu_2 - \nu_1 - \omega_1 = \nu_2 - \nu_3 - \omega_2$ is the two-photon detuning of the two laser beams.

We consider the situation of the ideal EIT which is attained only when the system is at the two-photon resonance with the two-photon detuning Δ. Initially the lasers are outside the BEC medium in which all atoms are in their ground state. The condensed atoms are generally in a superposition state of two lower states when they are in EIT. However, when the coupling laser is much stronger than the probe laser, atomic population at the intermediate level approaches zero while the upper level is unpopulation. Hence, under the condition of $(g_1/g_2)^2 \ll 1$, the terms that involve $\hat{b}_2^\dagger\hat{b}_2$ in Hamiltonian (11) may be neglected. After adiabatically eliminating the atomic field operators in the internal states $|2\rangle$, from Hamiltonian (11) we obtain $\hat{b}_2 = (g_1\hat{b}_1 + g_2\hat{b}_3)/\Delta$ and $\hat{b}_2^\dagger = (g_1^*\hat{b}_1^\dagger + g_2^*\hat{b}_3^\dagger)/\Delta$. Then, we can obtain the following effective Hamiltonian:

$$\hat{H}_{eff} = \omega_1'\hat{b}_1^\dagger\hat{b}_1 + \omega_3'\hat{b}_3^\dagger\hat{b}_3 + \lambda_1\hat{b}_1^{\dagger 2}\hat{b}_1^2 + \lambda_1\hat{b}_3^{\dagger 2}\hat{b}_3^2 + 2\lambda_{13}\hat{b}_1^\dagger\hat{b}_1\hat{b}_3^\dagger\hat{b}_3 + (g'\hat{b}_3^\dagger\hat{b}_1 + H.c.) \tag{12}$$

which contains only atomic field operators in internal states $|1\rangle$ and $|3\rangle$. Here we have set $\hbar = 1$ and introduced

$$\omega_1' = -\frac{|g_1|^2}{\Delta}, \quad \omega_3' = -\frac{|g_2|^2}{\Delta}, \quad g' = -\frac{g_1 g_2^*}{\Delta} \tag{13}$$

The analysis of the effective Hamiltonian can be greatly simplified by the introduction of the angular momentum operators:

$$\hat{J}_x = \frac{1}{2}(\hat{b}_3^\dagger\hat{b}_1 + \hat{b}_1^\dagger\hat{b}_3), \quad \hat{J}_y = \frac{i}{2}(\hat{b}_1^\dagger\hat{b}_3 - \hat{b}_3^\dagger\hat{b}_1)$$
$$\hat{J}_z = \frac{1}{2}(\hat{b}_3^\dagger\hat{b}_3 - \hat{b}_1^\dagger\hat{b}_1) \tag{14}$$

The Casimir invariant is $\hat{J}^2 = \hat{N}(\hat{N} + 1/2)$ with $\hat{N} = \hat{b}_3^\dagger\hat{b}_3 + \hat{b}_1^\dagger\hat{b}_1$ being the total number operator. In terms of angular momentum operators the effective Hamiltonian (12) may be rewritten as

$$\hat{H}_{eff} = \chi\hat{J}_z^2 + \delta\hat{J}_z + \Omega_x\hat{J}_x + \Omega_y\hat{J}_y \tag{15}$$

where $\Omega_x = \mathrm{Re}(2g')$ and $\Omega_y = \mathrm{Im}(2g')$ are the real Rabi couplings, and we have introduced the effective nonlinearity and effective detuning denoted by

$$\chi = \lambda_1 + \lambda_3 - \lambda_{13}$$
$$\delta = \omega_3' - \omega_1' + (\lambda_3 - \lambda_1)(N-1) \tag{16}$$

Note that the effective nonlinearity χ can be achieved by the proper engineering of the scattering lengths via Feshbach resonances. In the following, we choose the two Rabi frequencies to be real, and then the effective Hamiltonian can be reduced

$$\hat{H}_{eff}' = \chi\hat{J}_z^2 + \delta\hat{J}_z + \Omega_x\hat{J}_x \tag{17}$$

It is important for the following discussion to observe that the independent control of the Rabi pulses characterized by ω_3', ω_1' and Ω_x can be well carried out experimentally.

In general, it is difficult to treat the Hamiltonian (17) in an exact way because of the presence of nonlinear interactions between the atoms and the lasers. Here we use secular approximation that enables one to obtain approximate results in closed form. For convenience we define a new set of angular momentum operators obtained by rotation around the y axis [45]:

$$\begin{pmatrix} \hat{R}_x \\ \hat{R}_y \\ \hat{R}_z \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -\sin\eta & 0 & \cos\eta \\ \cos\eta & 0 & \sin\eta \end{pmatrix} \begin{pmatrix} \hat{J}_x \\ \hat{J}_y \\ \hat{J}_z \end{pmatrix} \tag{18}$$

where the auxiliary parameter η is defined by

$$\sin\eta = \frac{\delta}{\sqrt{\Omega_x^2+\delta^2}}, \quad \cos\eta = \frac{\Omega_x}{\sqrt{\Omega_x^2+\delta^2}} \tag{19}$$

Then, we have

$$\begin{aligned} \hat{R}_+ &= -\mathrm{i}\sin\eta\hat{J}_x + \mathrm{i}\cos\eta\hat{J}_z + \hat{J}_y \\ \hat{R}_- &= \mathrm{i}\sin\eta\hat{J}_x - \mathrm{i}\cos\eta\hat{J}_z + \hat{J}_y \end{aligned} \tag{20}$$

In terms of the $\hat{R}$ operators, the Hamiltonian (17) can be rewritten as

$$\hat{H}'_{eff} = \hat{H}_0 + \hat{H}_1 \tag{21}$$

where $\hat{H}_0$ and $\hat{H}_1$ are given by

$$\begin{aligned} \hat{H}_0 &= \Omega_x \sec\eta\hat{R}_z + \frac{\chi}{4}\cos^2\eta\{\hat{R}_+,\hat{R}_-\} + \chi\sin^2\eta\hat{R}_z^2 \\ \hat{H}_1 &= \chi\cos\eta\sin\eta\{\hat{R}_z,\hat{R}_y\} - \frac{\chi}{4}\cos^2\eta(\hat{R}_+^2+\hat{R}_-^2) \end{aligned} \tag{22}$$

where $\{\hat{R}_i,\hat{R}_j\}$ denotes the anti-commutator. Clearly $\hat{H}_0$ is diagonal in the representation consisting of the eigenstates $|\frac{N}{2},\frac{N}{2}-m\rangle$ of $\hat{R}_z$ and $\hat{R}^2 = \hat{R}_x^2+\hat{R}_y^2+\hat{R}_z^2$ with the eigenvalues given by

$$\begin{aligned} E(m) = {} & \chi\frac{N(N/2+1)}{4}\cos^2\eta + \Omega_x\left(\frac{N}{2}-m\right)\sec\eta \\ & +\chi\left(1-\frac{3}{2}\cos^2\eta\right)\left(\frac{N}{2}-m\right)^2 \end{aligned} \tag{23}$$

In the secular approximation we neglect the off-diagonal term $\hat{H}_1$ in the Hamiltonian (21). Hence the wave vector at arbitrary time $|\psi(t)\rangle$ under Hamiltonian evolution is formally written as

$$|\psi(t)\rangle \approx \mathrm{e}^{-\mathrm{i}\hat{H}_0 t/\hbar}|\psi(0)\rangle \tag{24}$$

3 Dynamics of spin squeezing

In this section, we study how to manipulate quantum dynamics of spin squeezing by EIT technique. We show that the spin squeezing of the BEC can be controlled by changing characteristic parameters of the condensed atoms and lasers in the EIT system.

Firstly, we investigate quantum dynamics of the mean spin vector. It is important to study the mean spin since we regard the spin as squeezed only if the variance of one spin component normal to the mean spin vector is smaller than the standard quantum limit. Suppose that the initial state of the BEC $|\psi(0)\rangle$ is a microscopic superposition state of N-atom wave function of the BEC

$$|\psi(0)\rangle = 2^{-N/2}\sum_{m=0}^{N}\sqrt{\frac{N!}{m!(N-m)!}}|\frac{N}{2},\frac{N}{2}-m\rangle \tag{25}$$

which can be prepared by applying a short $\pi/2$ pulse to a single-component BEC with all the atoms being in the internal ground state $|1\rangle$ [46, 47]. With the laser fields turned on at $t=0$, then at time $t>0$ the state of the system becomes

$$\begin{aligned} |\psi(t)\rangle = {} & 2^{-N/2}\mathrm{e}^{-\mathrm{i}\chi t\cos^2\eta N(N/2+1)/4} \\ & \times\sum_{m=0}^{N}\sqrt{C_m^N}\mathrm{e}^{-\mathrm{i}\alpha_m t}|\frac{N}{2},\frac{N}{2}-m\rangle \end{aligned} \tag{26}$$

where $C_m^N = N!/[m!(N-m)!]$, and the running frequency is given by

$$\alpha_m = \Omega_x\sec\eta\left(\frac{N}{2}-m\right) + \eta'\chi\left(\frac{N}{2}-m\right)^2 \tag{27}$$

where we have introduced the parameter $\eta' = 1 - \frac{3}{2}\cos^2\eta$.

According to the definition of spin squeezing, at first we need to know the expectation values $\langle\hat{J}_i\rangle$ for $i\in\{x,y,z\}$. After a short calculation, we can easily obtain the following results:

$$\begin{aligned} \langle\hat{J}_x\rangle &= \frac{N}{2}\cos^{N-1}(\eta'\chi t)\cos(\Omega_x\sec\eta t) \\ \langle\hat{J}_y\rangle &= \frac{N}{2}\cos^{N-1}(\eta'\chi t)\sin(\Omega_x\sec\eta t) \\ \langle\hat{J}_z\rangle &= 0 \end{aligned} \tag{28}$$

which means that the polar angle $\theta = \arccos\left(\frac{\langle\hat{J}_z\rangle}{R}\right) = \pi/2$, and the azimuth angle ϕ is

$$\phi = \begin{cases} \arccos\left(\frac{\langle\hat{J}_x\rangle}{R\sin\theta}\right) = \Omega_x t\sec\eta & \langle\hat{J}_y\rangle > 0 \\ 2\pi - \arccos\left(\frac{\langle\hat{J}_x\rangle}{R\sin\theta}\right) = 2\pi - \Omega_x t\sec\eta & \langle\hat{J}_y\rangle \leqslant 0 \end{cases} \tag{29}$$

where R is the length of mean spin defined by

$$\begin{aligned} R &= \sqrt{\langle\hat{J}_x\rangle^2+\langle\hat{J}_y\rangle^2+\langle\hat{J}_z\rangle^2} \\ &= \frac{N}{2}\left|\cos^{N-1}(\eta'\chi t)\right| \end{aligned} \tag{30}$$

If we denote the mean spin direction by $\boldsymbol{n}_3$ and the other two directions perpendicular to it are denoted by $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$, respectively, then the directions can be written in spherical coordinates as

$$\begin{pmatrix} \boldsymbol{n}_1 \\ \boldsymbol{n}_2 \\ \boldsymbol{n}_3 \end{pmatrix} = \begin{pmatrix} -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \\ \cos\phi & \sin\phi & 0 \end{pmatrix} \tag{31}$$

which indicates that the mean spin direction at time t can be controlled by the azimuth angle ϕ, i.e., by changing parameters of the BEC-laser system Ω_x and δ.

From Eq. (28) we can see that the mean spin vector moves in the x–y plane, and satisfies the following equation of motion

$$\langle \hat{J}_x \rangle^2 + \langle \hat{J}_y \rangle^2 = R^2 \tag{32}$$

From Eqs. (30) and (32) we can see that the mean spin length changes periodically with respect to the time evolution, and the maximal value of the mean spin length is $R_{\max} = N/2$, which is determined by the number of the condensed atoms. The evolution period can be manipulated by changing the internal parameter of the BEC χ and the coupling parameter of the external laser fields $\cos\eta$. In Fig. 2, we plot the mean spin length R for various Ω_x as a function of χt.

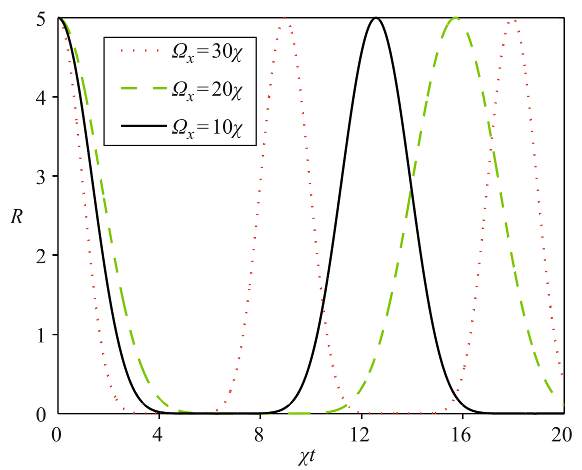

Fig. 2 The length of the mean spin R for various Ω_x as a function of χt with the total number of atoms $N = 10$.

In particular, from Eqs. (28)–(32), we can find that when the condition $\eta' = 0$, i.e., $\cos^2\eta = 2/3$ is satisfied, the evolution equation of the mean spin can be rewritten as the following form:

$$\frac{\mathrm{d}\langle \boldsymbol{J} \rangle}{\mathrm{d}t} = \boldsymbol{\Omega} \times \langle \boldsymbol{J} \rangle \tag{33}$$

where $\boldsymbol{\Omega} = \sqrt{\frac{3}{2}}\Omega_x \hat{\boldsymbol{n}}_z$ is the precessing frequency. Therefore, given the condition $\cos^2\eta = 2/3$, the evolution of the mean spin vector can be regarded as a precession around the z axis in the x–y plane. The value of the precessing frequency $\boldsymbol{\Omega}$ is only determined by the value of Ω_x, which can be controlled by adjusting the laser-atom interaction and the two-photon detuning. In Fig. 3 we plot the precession of the mean spin vector during the BEC evolution.

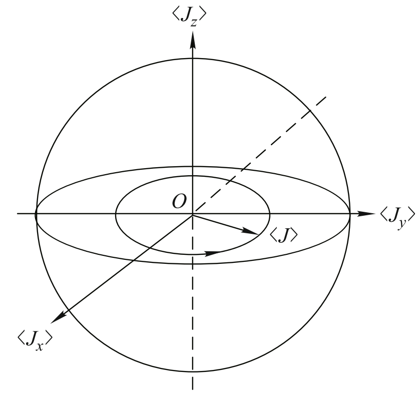

Fig. 3 Precession of the mean spin vector in the BEC time evolution when $\cos^2\eta = 2/3$.

Having studied the mean spin dynamics, we now consider the degree of spin squeezing which was first defined by Kitagawa and Ueda [7], and the spin squeezing parameter is quantified by

$$\xi^2 = \frac{4(\Delta \hat{J}_{\vec{n}_\perp})^2}{N} \tag{34}$$

where the subscript $\vec{n}_\perp$ refers to an axis perpendicular to the mean spin direction $\boldsymbol{n}_3$, in which the variance $(\Delta J)^2$ is minimal, and $J_{\vec{n}_\perp} = \boldsymbol{J} \cdot \vec{n}_\perp$. The inequality $\xi^2 < 1$ indicates that the system is spin squeezed.

In order to obtain the spin squeezing parameter ξ^2, we have to calculate the variance $(\Delta \hat{J}_{\vec{n}_\perp})^2$ which can be expressed as [48, 49]

$$(\Delta \hat{J}_{\vec{n}_\perp})^2 = \frac{1}{2}(C - \sqrt{A^2 + B^2}) \tag{35}$$

where the three parameters A, B and C are defined by

$$A = \langle \hat{J}^2_{\vec{n}_1} - \hat{J}^2_{\vec{n}_2} \rangle, B = \langle \{\hat{J}_{\vec{n}_1}, \hat{J}_{\vec{n}_2}\} \rangle$$
$$C = \langle \hat{J}^2_{\vec{n}_1} + \hat{J}^2_{\vec{n}_2} \rangle \tag{36}$$

where $\hat{J}_{\vec{n}_1} = -\hat{J}_x \sin\phi + \hat{J}_y \cos\phi$ and $\hat{J}_{\vec{n}_2} = \hat{J}_z$.

It is easy to see that the three parameters A, B and C can be expressed in terms of the three expectation values $\langle \hat{J}_z^2 \rangle$, $\langle \hat{J}_+^2 \rangle$ and $\langle \hat{J}_+(2\hat{J}_z + 1) \rangle$ as follows:

$$A = \frac{1}{2}\left\{\left[\frac{N}{2}\left(\frac{N}{2}+1\right) - 3\langle \hat{J}_z^2 \rangle\right] - \cos(2\phi)\mathrm{Re}\langle \hat{J}_+^2 \rangle\right\} - \frac{1}{2}\sin(2\phi)\mathrm{Im}\langle \hat{J}_+^2 \rangle$$
$$B = -\sin\phi\mathrm{Re}\langle \hat{J}_+(2\hat{J}_z+1) \rangle + \cos\phi\mathrm{Im}\langle \hat{J}_+(2\hat{J}_z+1) \rangle$$
$$C = \frac{1}{2}\left\{\left[\frac{N}{2}\left(\frac{N}{2}+1\right) + \langle \hat{J}_z^2 \rangle\right] - \cos(2\phi)\mathrm{Re}\langle \hat{J}_+^2 \rangle\right\} - \frac{1}{2}\sin(2\phi)\mathrm{Im}\langle \hat{J}_+^2 \rangle \tag{37}$$

For the wave function given by Eq. (26), it is straightforward to get the following mean values:

$$\langle \hat{J}_z^2 \rangle = \frac{N}{4}$$
$$\langle \hat{J}_+^2 \rangle = \frac{N(N-1)}{4} \mathrm{e}^{2\mathrm{i}\phi} \cos^{N-2}(2\eta' \chi t)$$
$$\langle \hat{J}_+(2\hat{J}_z+1) \rangle = \frac{N(N-1)}{2} \mathrm{e}^{\mathrm{i}\phi} \cos^{N-2}(\eta' \chi t) \times [\cos(\eta'\chi t) - \mathrm{e}^{-\mathrm{i}\eta'\chi t}] \tag{38}$$

Using these angular momentum relations [48] $\langle \hat{J}_x^2 + \hat{J}_y^2 \rangle = \frac{N}{2}\left(\frac{N}{2}+1\right) - \langle \hat{J}_z^2 \rangle$, $\langle \hat{J}_x^2 - \hat{J}_y^2 \rangle = \mathrm{Re}\langle \hat{J}_+^2 \rangle$, $\langle [\hat{J}_x, \hat{J}_y]_+ \rangle = \mathrm{Im}\langle \hat{J}_+^2 \rangle$, $\langle [\hat{J}_x, \hat{J}_z]_+ \rangle = \mathrm{Re}\langle \hat{J}_+(2\hat{J}_z+1) \rangle$ and $\langle [\hat{J}_y, \hat{J}_z]_+ \rangle = \mathrm{Im}\langle \hat{J}_+(2\hat{J}_z+1) \rangle$, after some tedious calculations, we further obtain

$$A = \frac{N(N-1)}{8}[1 - \cos^{N-2}(2\eta'\chi t)]$$
$$B = \frac{N(N-1)}{2}[\cos^{N-2}(\eta'\chi t)\sin(\eta'\chi t)]$$
$$C = A + \frac{N}{2} \tag{39}$$

In order to show that spin squeezing can be controlled and manipulated with the external coupling fields instead of the internal nonlinearity of the BEC, substituting these results into Eqs. (34) and (35), we can numerically calculate the spin squeezing parameter ξ^2 against the scaled time χt. In Fig. 4 we plot the squeezing parameter ξ^2 calculated numerically for the effective Rabi coupling Ω_x as a function of χt. From Fig. 4 we can see that the spin squeezing can be generated rapidly in the dynamical process and maintained in a long time interval. A larger effective Rabi coupling Ω_x can produce a stronger spin squeezing, and the squeezing can maintain a longer time interval. The optimal point of spin squeezing, i.e., the point of the minimal value of ξ^2), can be reached rapidly after beginning the dynamical evolution of the system.

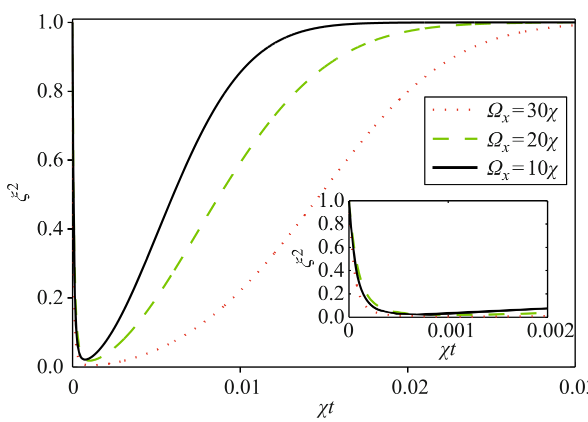

Fig. 4 ξ^2 as a function of χt for different Ω_x, where we have chosen the total number of atoms $N = 50000$.

The inset in Fig. 4 shows the magnification of the spin squeezing regime around the optimal point of the spin squeezing. We can see that the BEC can rapidly reach the optimal spin squeezing state in a short-time interval. In fact, in the short-time regime of $|\eta'\chi t| \ll 1$, we have $A \approx 0, B \approx N(N-1)|\eta'\chi|t/2$ and $C \approx N/2$. In this case, the spin squeezing parameter ξ^2 can be approximately written as the following simple form:

$$\xi^2 \approx 1 - (N-1)|\eta'\chi|t \tag{40}$$

which indicates that the spin squeezing at time t is determined by the number of condensed atoms N, and characteristic parameters of the BEC and laser systems η' and χ, and the optimal spin squeezing ($\xi \approx 0$) occurs at time $t_{\mathrm{optimal}} \approx 1/(N-1)|\eta'\chi|$.

It is interesting to note that from Eqs. (30) and (39) we can observe that for the mean spin vector and spin squeezing, the controllable parameter of the external laser fields η' plays the same role as that of the internal nonlinear interactions of the BEC χ. This means that the external-field-control way by EIT is equivalent to that of manipulating the internal nonlinear interactions from the point of view of control effect for the spin squeezing. The external-field parameter η' can be manipulated by varying the Rabi frequencies and two-photon detuning of the external fields while the internal nonlinear interactions of condensed atoms can be controlled through tuning the scattering lengths via Feshbach resonances. Therefore, we can conclude that the mean spin vector and spin squeezing of the BEC can be controlled and manipulated by adjusting the external coupling fields or/and internal nonlinear interactions of the BEC.

4 Conclusions

In conclusion, we have proposed a theoretical scheme to coherently control and manipulate spin squeezing of BEC via the EIT technique. We have studied quantum dynamics of the mean spin vector and spin squeezing. We have shown that the mean spin vector and spin squeezing of the BEC can be controlled and manipulated by adjusting the external coupling fields or/and internal interatomic nonlinear interactions of the BEC. We have found that the external-field-control way by EIT is equivalent to that of manipulating the internal nonlinear interactions from the point of view of control effect for the spin squeezing. We have indicated that spin squeezing can be generated rapidly in the dynamical process and maintained in a long time interval, and found that a larger effective Rabi coupling between atoms and lasers can produce a stronger spin squeezing, and the squeezing can maintain a longer time interval. Finally, it should be pointed out that in practice, the scheme presented in this work could be realized in the BEC for the ultra-slow light experiment [34].

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