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RESEARCH ARTICLE

New application of non-Hermitian Hamiltonian operator in solving master equation for laser process

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We find that laser process can be equivalently described by a symplectic evolution in the context of thermo field dynamics, and the corresponding coherent state evolution for the corresponding master equation is recognized. More interestingly, this embodies a new application of non-Hermitian Hamiltonian operator which can well expose the entanglement between the system and its environment.

Keywords non-Hermitian, master equation, laser process

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1 Introduction

In recent years, a class of non-Hermitian Hamiltonians, which were considered unacceptable in physics before, have been studied in quantum theory. Despite their non-Hermitian nature, these Hamiltonians possess real positive spectra, therefore proper quantum theory can be developed based on them [1]. The corresponding quasi-Hermitian quantum theories has been widely applied in problems such as Schwinger–Dyson Equations [2] and Lee Model [3].

In this paper we shall report another application of non-Hermitian Hamiltonians which is embodied in laser process. It has been known that a laser process includes both gain and damping, so it involves typical thermodynamic mechanism. One also knows that the ordinary Schrödinger equation for Hermitian Hamiltonian cannot describe dissipation mechanism, so it is expected that a Schrödinger-like equation for the laser process might be described by some non-Hermitian Hamiltonian in some approach. In fact, as one can see shortly later that by adopting the thermo entangled state representation the master equation for laser process can be really converted into a Schrödinger-like equation governed by a non-Hermitian Hamiltonian, and this equation can be exactly solved which cannot only well expose the entanglement between the system and its environment, but also exhibit coherent state evolution for the process, this also implies that the laser process corresponds to a symplectic transformation in classical dynamics.

In quantum optics theory the time evolution of laser in the lowest-order approximation can be described by the following master equation of density operator [4–10]

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} &= g[2a^\dagger\rho(t)a - aa^\dagger\rho(t) - \rho(t)aa^\dagger] \\ &\quad +\kappa[2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a]\end{aligned} \tag{1}$$

where g and κ are the cavity gain and loss, respectively, $a^\dagger$, a are photon creation and annihilation operator, respectively. It is also known that the evolution due to the interaction between a system and its environment can be ascribed to an evolution from the initial density operator ρ_0 to $\rho(t)$ [11, 12]

$$\rho(t) = \sum_{n=0}^{\infty} M_n\rho_0M_n^\dagger \tag{2}$$

Such an expression is named an operator-sum (Kraus) representation, and M_n is named Kraus operator. In this paper we shall show that the laser process cannot only be depicted by Kraus representation, but also be equivalently described by a symplectic evolution of initial density operator in the context of thermo-field dynamics. That is, a unitary time evolution in the n-mode coherent state representation is responsible for such a process. Remarkably, this also implies a new application of non-Hermitian Hamiltonian operator.

We begin with introducing the two-mode entangled state

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^* \tilde{a}^\dagger + a^\dagger \tilde{a}^\dagger\right)|0\tilde{0}\rangle \tag{3}$$

where $\tilde{a}^\dagger$ is a fictitious mode independent of the real mode $a^\dagger$, and $|\tilde{0}\rangle$ is annihilated by $\tilde{a}$, $[\tilde{a}, \tilde{a}^\dagger] = 1$. The state $|\eta = 0\rangle$ possesses the properties

$$\begin{aligned} &a|\eta = 0\rangle = \tilde{a}^\dagger|\eta = 0\rangle, \quad a^\dagger|\eta = 0\rangle = \tilde{a}|\eta = 0\rangle \\ &(a^\dagger a)^n|\eta = 0\rangle = (\tilde{a}^\dagger \tilde{a})^n|\eta = 0\rangle \end{aligned} \tag{4}$$

Operating the both sides of Eq. (1) on the state $|\eta = 0\rangle \equiv |I\rangle$, and denoting $|\rho\rangle = \rho|I\rangle$, and using Eq. (4) we have the time-evolution equation for $|\rho(t)\rangle$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}|\rho(t)\rangle = \{&g[2a^\dagger\tilde{a}^\dagger - aa^\dagger - \tilde{a}\tilde{a}^\dagger] \\ &+\kappa(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a})\}|\rho(t)\rangle \end{aligned} \tag{5}$$

where $|\rho_0\rangle \equiv \rho_0|I\rangle$, and ρ_0 is the initial density operator.

By comparing (5) with the usual Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H|\psi(t)\rangle \tag{6}$$

and identifying $|\rho(t)\rangle$ as $|\psi(t)\rangle$ we see that Eq. (5) involves the corresponding Hamiltonian H

$$H = \mathrm{i}g(2a^\dagger\tilde{a}^\dagger - aa^\dagger - \tilde{a}\tilde{a}^\dagger) + \mathrm{i}\kappa(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a}) \tag{7}$$

which is a non-Hermitian Hamiltonian. Thus, in solving the master equation in the entangled states representation, there naturally appears the non-Hermitian Hamiltonian in the Schrödinger equation. This is remarkable. As one can see shortly later, though H is not Hermitian, the evolution of $\rho(t)$ is still unitary.

The formal solution of Eq. (5) is

$$|\rho(t)\rangle = U(t)|\rho_0\rangle, \quad |\rho_0\rangle = \rho_0|I\rangle \tag{8}$$

and

$$\begin{aligned} U(t) = \exp\{&gt(2a^\dagger\widetilde{a}^\dagger - aa^\dagger - \widetilde{a}\widetilde{a}^\dagger) \\ &+\kappa t(2a\widetilde{a} - a^\dagger a - \widetilde{a}^\dagger\widetilde{a})\} \end{aligned} \tag{9}$$

It challenges us how to disentangle the exponential operator $U(t)$. We have been reminded of two theorems about the normally ordered expansion of multimode bosonic exponential operators and its coherent state representation which are helpful to disentangling $U(t)$ and sheds a new insight on the physical explanation of laser evolution.

2 Two theorems

The two new theorems are [13, 14]

Theorem 1 The multimode bosonic exponential operator $\exp\{\mathcal{H}\}$, $\mathcal{H} = \frac{1}{2}B\mathit{\Gamma}\widetilde{B}$, $\mathit{\Gamma}$ is a symmetric $2n \times 2n$ matrix, B is defined by

$$B \equiv (A^\dagger \; A) \equiv (a_1^\dagger \; a_2^\dagger \cdots \; a_n^\dagger \; a_1 \; a_2 \cdots \; a_n), \; \widetilde{B} = \begin{pmatrix} \widetilde{A}^\dagger \\ \widetilde{A} \end{pmatrix} \tag{10}$$

has its n-mode coherent state representation:

$$\begin{aligned} \exp\{\mathcal{H}\} = \sqrt{\det Q}\int\prod_{i=1}^{n}\frac{\mathrm{d}^2 Z_i}{\pi} \\ \cdot\left|\begin{pmatrix} Q & -L \\ -N & P \end{pmatrix}\begin{pmatrix} \widetilde{Z} \\ \widetilde{Z}^* \end{pmatrix}\right\rangle\left\langle\begin{pmatrix} \widetilde{Z} \\ \widetilde{Z}^* \end{pmatrix}\right| \end{aligned} \tag{11}$$

where the n-mode coherent state is

$$\left|\begin{pmatrix} \widetilde{Z} \\ \widetilde{Z}^* \end{pmatrix}\right\rangle \equiv |Z\rangle = D(Z)|\vec{0}\rangle, \quad D(Z) \equiv \exp\{A^\dagger\widetilde{Z} - A\widetilde{Z}^*\} \tag{12}$$

and Q, L, N, P are all $n \times n$ complex matrices and

$$\begin{pmatrix} Q & L \\ N & P \end{pmatrix} = \exp\{\mathit{\Gamma}\mathit{\Pi}\} \equiv M, \; \mathit{\Pi} = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix} \tag{13}$$

I_n is the $n \times n$ unit matrix.

The ket in (11) is

$$\begin{aligned} &\left|\begin{pmatrix} Q & -L \\ -N & P \end{pmatrix}\begin{pmatrix} \widetilde{Z} \\ \widetilde{Z}^* \end{pmatrix}\right\rangle \\ &= \exp\{A^\dagger(Q\widetilde{Z} - L\widetilde{Z}^*) - A(-N\widetilde{Z} + P\widetilde{Z}^*)\}|\vec{0}\rangle \\ &= \exp\{A^\dagger(Q\widetilde{Z} - L\widetilde{Z}^*) + \frac{1}{2}(Z\widetilde{N} - \widetilde{Z^*}P)(Q\widetilde{Z} - L\widetilde{Z}^*)\}|\vec{0}\rangle \end{aligned} \tag{14}$$

The fact that $\mathit{\Gamma}$ is symmetric guarantees that M is a symplectic matrix, obeying

$$M\mathit{\Pi}\tilde{M} = \mathit{\Pi}, \; \mathit{\Pi}\tilde{M}\mathit{\Pi} = -M^{-1} \tag{15}$$

or

$$Q\tilde{L} = L\tilde{Q}, \; N\tilde{P} = P\tilde{N}, \; Q\tilde{P} - L\tilde{N} = I, \; P\tilde{Q} - N\tilde{L} = I \tag{16}$$

In other words, the transformation defined by operator $\exp\{\mathcal{H}\}$ is symplectic.

Theorem 2 By performing the integration in Eq. (11) with the technique of integration within an ordered product of operators [10, 11], we have

$$\begin{aligned} \exp\{\mathcal{H}\} = \frac{1}{\sqrt{\det P}}\exp\left\{-\frac{1}{2}A^\dagger(LP^{-1})\widetilde{A}^\dagger\right\} \\ \times\exp\left\{A^\dagger(\ln\widetilde{P}^{-1})\widetilde{A}\right\}\exp\left\{\frac{1}{2}A(P^{-1}N)\widetilde{A}\right\} \end{aligned} \tag{17}$$

According to Theorem 1, putting $U(t)$ in (9) into the following symmetrized matrix form

$$U(t) = \mathrm{e}^{(\kappa-g)t}\exp\left\{\frac{1}{2}(\widetilde{a}^\dagger \quad a^\dagger \quad \widetilde{a} \quad a)\mathit{\Gamma}\begin{pmatrix} \widetilde{a}^\dagger \\ a^\dagger \\ \widetilde{a} \\ a \end{pmatrix}\right\} \tag{18}$$

with Γ being the symmetric matrix

$$\Gamma = t\begin{pmatrix} 0 & 2g & -g-\kappa & 0 \\ 2g & 0 & 0 & -g-\kappa \\ -g-\kappa & 0 & 0 & 2\kappa \\ 0 & -g-\kappa & 2\kappa & 0 \end{pmatrix}$$

$$= t\begin{pmatrix} 2gJ_2 & -(g+\kappa)I_2 \\ -(g+\kappa)I_2 & 2\kappa J_2 \end{pmatrix} \tag{19}$$

here

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$J_2^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 \tag{20}$$

we calculate $\exp(\Gamma\Pi)$ by first evaluating

$$\Gamma\Pi = t\begin{pmatrix} 2gJ_2 & -(g+\kappa)I_2 \\ -(g+\kappa)I_2 & 2\kappa J_2 \end{pmatrix}\begin{pmatrix} 0 & -I_2 \\ I_2 & 0 \end{pmatrix}$$

$$= t\begin{pmatrix} -(g+\kappa)I_2 & -2gJ_2 \\ 2\kappa J_2 & (g+\kappa)I_2 \end{pmatrix} \tag{21}$$

On observing that

$$\begin{pmatrix} -(g+\kappa)I_2 & -2gJ_2 \\ 2\kappa J_2 & (g+\kappa)I_2 \end{pmatrix}\begin{pmatrix} -gJ_2 & -gJ_2 \\ gI_2 & \kappa I_2 \end{pmatrix}$$

$$= \begin{pmatrix} g(\kappa-g)J_2 & g(g-\kappa)J_2 \\ g(g-\kappa)I_2 & \kappa(\kappa-g)I_2 \end{pmatrix}$$

$$= \begin{pmatrix} -gJ_2 & -gJ_2 \\ gI_2 & \kappa I_2 \end{pmatrix}\begin{pmatrix} (g-\kappa)I_2 & 0 \\ 0 & (\kappa-g)I_2 \end{pmatrix} \tag{22}$$

we can diagonalize $\Gamma\Pi$ as

$$t\begin{pmatrix} -(g+\kappa)I_2 & -2gJ_2 \\ 2\kappa J_2 & (g+\kappa)I_2 \end{pmatrix} = \begin{pmatrix} -gJ_2 & -gJ_2 \\ gI_2 & \kappa I_2 \end{pmatrix}\begin{pmatrix} (g-\kappa)tI_2 & 0 \\ 0 & (\kappa-g)tI_2 \end{pmatrix}\begin{pmatrix} -gJ_2 & -gJ_2 \\ gI_2 & \kappa I_2 \end{pmatrix}^{-1} \tag{23}$$

Therefore

$$\mathrm{e}^{\Gamma\Pi} = \exp\left\{\begin{pmatrix} -(g+\kappa)I_2 & -2gJ_2 \\ 2\kappa J_2 & (g+\kappa)I_2 \end{pmatrix} t\right\}$$

$$= \begin{pmatrix} -gJ_2 & -gJ_2 \\ gI_2 & \kappa I_2 \end{pmatrix}\begin{pmatrix} \mathrm{e}^{(g-\kappa)t}I_2 & 0 \\ 0 & \mathrm{e}^{(\kappa-g)t}I_2 \end{pmatrix}\begin{pmatrix} -gJ_2 & -gJ_2 \\ gI_2 & \kappa I_2 \end{pmatrix}^{-1}$$

$$= \frac{1}{g(g-\kappa)}\begin{pmatrix} -gJ_2 & -gJ_2 \\ gI_2 & \kappa I_2 \end{pmatrix}\begin{pmatrix} \mathrm{e}^{(g-\kappa)t}I_2 & 0 \\ 0 & \mathrm{e}^{(\kappa-g)t}I_2 \end{pmatrix}\begin{pmatrix} \kappa J_2 & gI_2 \\ -gJ_2 & -gI_2 \end{pmatrix}$$

$$= \frac{1}{g-\kappa}\begin{pmatrix} [g\mathrm{e}^{(\kappa-g)t} - \kappa\mathrm{e}^{(g-\kappa)t}]I_2 & g[\mathrm{e}^{(\kappa-g)t} - \mathrm{e}^{(g-\kappa)t}]J_2 \\ \kappa[\mathrm{e}^{(g-\kappa)t} - \mathrm{e}^{(\kappa-g)t}]J_2 & [g\mathrm{e}^{(g-\kappa)t} - \kappa\mathrm{e}^{(\kappa-g)t}]I_2 \end{pmatrix} \equiv \begin{pmatrix} Q & L \\ N & P \end{pmatrix} \tag{24}$$

with

$$Q \equiv \frac{g\mathrm{e}^{(\kappa-g)t} - \kappa\mathrm{e}^{(g-\kappa)t}}{g-\kappa}I_2, \quad L \equiv \frac{g[\mathrm{e}^{(\kappa-g)t} - \mathrm{e}^{(g-\kappa)t}]}{g-\kappa}J_2$$

$$N \equiv \frac{\kappa[\mathrm{e}^{(g-\kappa)t} - \mathrm{e}^{(\kappa-g)t}]}{g-\kappa}J_2, \quad P \equiv \frac{g\mathrm{e}^{(g-\kappa)t} - \kappa\mathrm{e}^{(\kappa-g)t}}{g-\kappa}I_2 \tag{25}$$

One can check that they obey Eq. (16). Thus using Eqs. (11), (17), and (24) we can reform Eq. (18) as

$$U(t) = \sqrt{\det Q}\int\prod_{i=1}^{n}\Pi\frac{\mathrm{d}^2 Z_i}{\Pi}$$

$$\cdot\left|\begin{pmatrix} Q & -L \\ -N & P \end{pmatrix}\begin{pmatrix} \widetilde{Z} \\ \widetilde{Z}^* \end{pmatrix}\right\rangle\left\langle\begin{pmatrix} \widetilde{Z} \\ \widetilde{Z}^* \end{pmatrix}\right|$$

$$= \mathrm{e}^{(\kappa-g)t}\frac{1}{\sqrt{\det P}}\exp\left[-\frac{1}{2}(\widetilde{a}^\dagger \quad a^\dagger)LP^{-1}\begin{pmatrix} \widetilde{a}^\dagger \\ a^\dagger \end{pmatrix}\right]$$

$$\times\exp\left[(\widetilde{a}^\dagger \quad a^\dagger)\ln P^{-1}\begin{pmatrix} \widetilde{a} \\ a \end{pmatrix}\right]$$

$$\times\exp\left[\frac{1}{2}(\widetilde{a} \quad a)P^{-1}N\begin{pmatrix} \widetilde{a} \\ a \end{pmatrix}\right]$$

$$= \frac{\kappa-g}{\kappa\mathrm{e}^{-2(g-\kappa)t} - g}\exp\left[\frac{g[1-\mathrm{e}^{-2(\kappa-g)t}]}{\kappa - g\mathrm{e}^{-2(\kappa-g)t}}\widetilde{a}^\dagger a^\dagger\right]$$

$$\times\exp\left[(\widetilde{a}^\dagger\widetilde{a} + a^\dagger a)\ln\frac{(\kappa-g)\mathrm{e}^{-(\kappa-g)t}}{\kappa - g\mathrm{e}^{-2(\kappa-g)t}}\right]$$

$$\times\exp\left[\frac{\kappa[1-\mathrm{e}^{-2(\kappa-g)t}]}{\kappa - g\mathrm{e}^{-2(\kappa-g)t}}a\widetilde{a}\right] \tag{26}$$

where we have used

$$LP^{-1} = \frac{g[1-\mathrm{e}^{-2(\kappa-g)t}]}{g\mathrm{e}^{-2(\kappa-g)t} - \kappa}J_2$$

$$P^{-1}N = \frac{\kappa[\mathrm{e}^{-2(\kappa-g)t} - 1]}{g\mathrm{e}^{-2(\kappa-g)t} - \kappa}J_2 \tag{17}$$

and

$$\sqrt{\det P} \equiv \frac{g\mathrm{e}^{(g-\kappa)t} - \kappa\mathrm{e}^{(\kappa-g)t}}{g-\kappa} \tag{28}$$

So far, we have seen that a laser process can be considered as a symplectic evolution in the context of thermo-field dynamics, which has, on the other hand, shown a new application of non-Hermitian Hamiltonian operator.

Further, let

$$T_1 = \frac{1-\mathrm{e}^{-2(\kappa-g)t}}{\kappa - g\mathrm{e}^{-2t(\kappa-g)}},\ T_2 = \frac{(\kappa-g)\mathrm{e}^{-(\kappa-g)t}}{\kappa - g\mathrm{e}^{-2t(\kappa-g)}}$$
$$T_3 = \frac{\kappa-g}{\kappa - g\mathrm{e}^{-2t(\kappa-g)}} = 1 - gT_1 \tag{29}$$

Using (26) we can express (8) as

$$\begin{aligned}|\rho(t)\rangle &= T_3 \exp[gT_1 a^\dagger \tilde{a}^\dagger]\colon \exp\{(T_2-1)(\tilde{a}^\dagger\tilde{a} + a^\dagger a)\}\colon \\ &\quad \times\ \exp[\kappa T_1 a\tilde{a}]|\rho_0\rangle \\ &= T_3 \exp[gT_1 a^\dagger \tilde{a}^\dagger] \exp[(\tilde{a}^\dagger\tilde{a} + a^\dagger a)\ln T_2] \\ &\quad \times\ \exp[\kappa T_1 a\tilde{a}]|\rho_0\rangle \\ &= \sum_{i,j=0}^{\infty} T_3 \frac{\kappa^i g^j T_1^{i+j}}{i!j!} a^{\dagger j} \exp[a^\dagger a \ln T_2] a^i \rho_0 a^{\dagger i} \\ &\quad \times\ \exp[a^\dagger a \ln T_2] a^j |\eta = 0\rangle \\ &= \sum_{i,j=0}^{\infty} T_3 \frac{\kappa^i g^j T_1^{i+j}}{i!j!} a^{\dagger j} \exp[a^\dagger a \ln T_2] a^i \rho_0 a^{\dagger i} \\ &\quad \times\ \exp[a^\dagger a \ln T_2] a^j |\eta = 0\rangle \\ &= \sum_{i,j=0}^{\infty} T_3 \frac{\kappa^i g^j T_1^{i+j}}{i!j!T_2^{2j}} \exp[a^\dagger a \ln T_2] a^{\dagger j} a^i \rho_0 a^{\dagger i} a^j \\ &\quad \times\ \exp[a^\dagger a \ln T_2]|\eta = 0\rangle\end{aligned} \tag{30}$$

Since $a^{\dagger i} a^j |\eta = 0\rangle = a^{\dagger i}\tilde{a}^{\dagger j}|\eta = 0\rangle$, if any one-mode operator $\sum_{i,j=0}^{\infty} C_{ij} a^{\dagger i} a^j$ satisfies $\sum_{i,j=0}^{\infty} C_{ij} a^{\dagger i} a^j |\eta = 0\rangle = \sum_{i,j=0}^{\infty} C_{ij} a^{\dagger i}\tilde{a}^{\dagger j}|\eta = 0\rangle = 0$, then $C_{ij} \equiv 0$, which indicates the Kraus representation for laser evolution

$$\rho(t) = \sum_{i,j=0}^{\infty} M_{ij}\rho_0 M_{ij}^\dagger \tag{31}$$

$$M_{ij} = \sqrt{\frac{\kappa^i g^j T_3 T_1^{i+j}}{i!j!T_2^{2j-1}}} \tag{32}$$

3 Normalization property of the Kraus Operator in Eq. (32)

We can derive the normalization property of the Kraus Operator in Eq. (32) straightforward through the following calculation:

$$\begin{aligned}&\sum_{i,j=0}^{\infty} M_{ij}^\dagger M_{ij} \\ &= \sum_{i,j=0}^{\infty} T_3 \frac{\kappa^i g^j T_1^{i+j}}{i!j!T_2^{2i-1}} a^{\dagger i} a^j a^{\dagger j} a^i \exp[2a^\dagger a \ln T_2] \\ &= T_3 \sum_{i,j=0}^{\infty} \frac{\kappa^i g^j T_1^{i+j}}{i!j!T_2^{2i}} a^{\dagger i} \int \frac{\mathrm{d}^2 z}{\pi} z^j z^{*j} |z\rangle\langle z| a^i \\ &\quad \times\ \exp(2a^\dagger a \ln T_2) \\ &= T_3 \sum_{i=0}^{\infty} \frac{\kappa^i T_1^i}{i!T_2^{2i}} a^{\dagger i} \int \frac{\mathrm{d}^2 z}{\pi} \exp[gT_1|z|^2] \\ &\quad \times : \exp[-|z|^2 + z^* a + z a^\dagger - a^\dagger a] : \\ &\quad \times\ a^i \exp[2a^\dagger a \ln T_2] \\ &= \frac{T_3}{1-gT_1} \sum_{i=0}^{\infty} \frac{\kappa^i T_1^i}{i!T_2^{2i}} a^{\dagger i} : \exp\left(\frac{gT_1}{1-gT_1} a^\dagger a\right) : \\ &\quad \times\ a^i \exp(2a^\dagger a \ln T_2) \\ &= \frac{T_3}{1-gT_1} : \exp\left(\frac{gT_1}{1-gT_1} a^\dagger a + \frac{\kappa T_1}{T_2^2} a^\dagger a\right) : \\ &\quad \times\ \exp(2a^\dagger a \ln T_2) \\ &= \frac{T_3}{1-gT_1} \exp\left[\ln\left(\frac{gT_1}{1-gT_1} + \frac{\kappa T_1}{T_2^2} + 1\right) a^\dagger a\right] \\ &\quad \times \exp(2a^\dagger a \ln T_2) \\ &= 1\end{aligned} \tag{33}$$

4 Conclusions

In summary, we found that laser process can be equivalently described by a symplectic evolution in the context of thermo-field dynamics, and the corresponding coherent state evolution for the corresponding master equation is recognized. More interestingly, in solving the master equation in the entangled states representation, there naturally appears the non-Hermitian Hamiltonian in the Schrödinger equation. Though it is not Hermitian, the evolution of density operator is still unitary. This embodies a new application of non-Hermitian Hamiltonian operator. For more applications of coherent states, we refer the readers to Ref. [15].

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