

Quantum optical correlation through metamaterials

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Based on the quantization scheme of the radiation fields in the dispersive and absorptive magnetic media, the normally ordered correlation functions of the outgoing field through a metamaterial plate are obtained. Then the relative photon-number densities of the transmitted field, the reflected field and the absorbed field are gotten through the correlation functions. Furthermore, the contributions of the relative permittivity and permeability of the metamaterials to the transmission are analyzed. Our results show that the permittivity and permeability reinforce the transmission for frequencies that are big compared with the magnetic resonance frequency.

Keywords metamaterials, magnetic resonance, quantum optical correlation

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1 Introduction

Recently, the metamaterials have attracted a great deal of attention from both theoretical and experimental sides. The metamaterials have the artificial structure and the electromagnetic parameters are dependent on the resonant of the electric and the magnetic field [1]. In some frequency region the effective permittivity and the permeability of the materials can be simultaneously negative and the materials are also called the left-handed materials because the electric field, the magnetic field and the wave vector form a left-handed relation in the materials. These media exhibit a number of unusual electromagnetic properties such as negative refractive index [1], amplification of evanescent wave [3–7], subwavelength cavity resonator [8, 9], cloaking [10] and optical illusion [11, 12], etc. The adjustable effective permittivity and permeability play an important role in these novel electromagnetic properties that do not occur in the naturally existing materials. These unique features suggest fascinating applications and unusual phenomena of the metamaterials in the control of the electromagnetic wave [13–23]. It is found that both the negative permittivity and the negative permeability are necessary to the extraordinary property of the classical electromagnetic wave propagating in the metamaterials [19–21]. Recently theory and experiments have verified that the magnetic plasmon waves in a three-dimensional metamaterials own a quantum na-

ture and the metamaterials can steer the non-classical light waves [23]. Thus there should be some abnormal properties if the metamaterials are used to control the quantized radiation. In this work we derive the output field of the quantized radiation through the metamaterials and give support theoretically for the application of the metamaterials in the quantum information.

Based on the quantization scheme of the radiation fields and input–output relations in the dispersive and absorptive magnetic media [17], the quantum statistical properties of the outgoing radiation can be obtained from the incoming radiation and excitations associated with the media. In this work, the normally ordered correlation functions of the outgoing field through a slab of metamaterials are calculated. Then the relative photon-number densities of the transmitted field, the reflected field and the absorbed field are obtained through the correlation functions. The emphasis is put on the contributions of the permittivity and the permeability to the transmission.

2 Theory of quantum correlation functions

We consider the quantized radiation transmitting through a slab of metamaterial plate placed in the air along the x -direction shown in Fig. 1. The input–output relations of quantized radiation through multilayer structure with metamaterials have been given in Ref. [17].

Here we outline the main derivations to get the quantum optical correlation functions for our system.

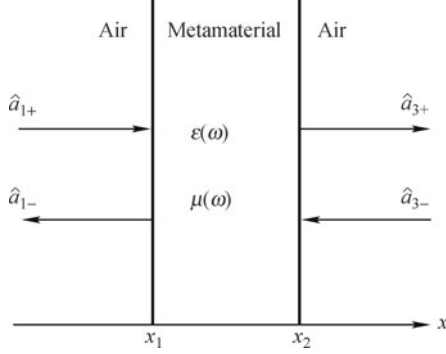


Fig. 1 Configuration of the quantized radiation propagating through a slab of metamaterial. The arrows, together with the amplitude operators, indicate incoming and outgoing fields.

The vector potential $\hat{A}(x)$ in the j th area ($x_{j-1} \leq x \leq x_j$, $j = 1, 2, 3$, $x_0 = -\infty$, $x_3 = \infty$) can be written as [17]

$$\hat{A}(x) = \int_0^\infty d\omega \sqrt{\frac{\hbar \zeta_j(\omega)}{4\pi\omega\epsilon_0 c \mathcal{A}}} \cdot \frac{\mu_j(\omega)}{n_j(\omega)} \times [e^{i\beta_j(\omega)\omega x/c} \hat{a}_{j+}(x, \omega) + e^{-i\beta_j(\omega)\omega x/c} \hat{a}_{j-}(x, \omega)] + H.c. \quad (1)$$

Here \mathcal{A} is the transverse normalized area and $\zeta_j(\omega) = \frac{\epsilon_j^I(\omega) - \kappa_j^I(\omega) |n_j(\omega)|^2}{2\gamma_j(\omega)}$ with $n_j(\omega) = \sqrt{\epsilon_j(\omega)} \times \sqrt{\mu_j(\omega)} = \beta_j(\omega) + i\gamma_j(\omega)$ being the complex refractive index of the medium in the j th area. $\epsilon_j(\omega)$ and $\mu_j(\omega)$ are the (relative) permittivity and the (relative) permeability of the media in the j th area respectively. $\kappa_j(\omega) = 1/\mu_j(\omega)$ is defined here and the superscript ‘‘I’’ represents the imaginary part of the corresponding physical quantity. The operators $\hat{a}_{j\pm}(x, \omega)$ which describe the amplitudes of the damped waves propagating to the right (subscript +) and left (subscript -) in the j th area can be represented as

$$\hat{a}_{j\pm}(x, \omega) = \hat{a}_{j\pm}(x', \omega) e^{\mp \gamma_j(\omega)\omega(x-x')/c} + \int_{x'}^x dy \hat{F}_{j\pm}(y, \omega) e^{\mp \gamma_j(\omega)\omega(x-y)/c} \quad (2)$$

with

$$\hat{F}_{j\pm}(x, \omega) = \pm i \sqrt{2\gamma_j(\omega) \frac{\omega}{c}} e^{\mp i\beta_j(\omega)\omega x/c} \times \frac{\sqrt{\epsilon_j^I(\omega)} \hat{f}_e(x, \omega) \mp i n_j(\omega) \sqrt{-\kappa_j^I(\omega)} \hat{f}_m(x, \omega)}{\sqrt{\epsilon_j^I(\omega) - \kappa_j^I(\omega) |n_j(\omega)|^2}} \quad (3)$$

In the air, the vector potential $\hat{A}(x)$ degenerates to the general form, which is consistent with the result in the Ref. [24],

$$\hat{A}(x) = \int_0^\infty d\omega \sqrt{\frac{\hbar \beta_j(\omega)}{4\pi\omega\epsilon_0 c \mathcal{A}}} \cdot \frac{1}{n_j(\omega)}$$

$$\times [e^{i\beta_j(\omega)\omega x/c} \hat{a}_{j+}(x, \omega) + e^{-i\beta_j(\omega)\omega x/c} \hat{a}_{j-}(x, \omega)] + H.c. \quad (4)$$

Based on Eq. (2) and the boundary continuity conditions of the vector potential $\hat{A}(x)$ at the interface $x = x_j$, the input-output relations for the amplitude operators through the one-layer structure shown in Fig. 1 can be obtained as [17, 24]

$$\begin{pmatrix} \hat{a}_{1-}(x_1, \omega) \\ \hat{a}_{3+}(x_2, \omega) \end{pmatrix} = \mathbf{T}(\omega) \begin{pmatrix} \hat{a}_{1+}(x_1, \omega) \\ \hat{a}_{3-}(x_2, \omega) \end{pmatrix} + \mathbf{A}(\omega) \begin{pmatrix} \hat{g}_+^{(1)}(\omega) \\ \hat{g}_-^{(1)}(\omega) \end{pmatrix} \quad (5)$$

Eq. (5) can be written in a compact form as

$$\hat{\mathbf{b}}(\omega) = \mathbf{T}(\omega) \hat{\mathbf{a}}(\omega) + \mathbf{A}(\omega) \hat{\mathbf{g}}(\omega) \quad (6)$$

where $\mathbf{T}(\omega)$ is the transformation matrix which is given in the appendix and $\mathbf{A}(\omega)$ is the absorption matrix. Here $\hat{\mathbf{b}}(\omega) = \begin{pmatrix} \hat{b}_1(\omega) \\ \hat{b}_2(\omega) \end{pmatrix} \equiv \begin{pmatrix} \hat{a}_{1-}(x_1, \omega) \\ \hat{a}_{3+}(x_2, \omega) \end{pmatrix}$ corresponds to the amplitude operators of the output fields, $\hat{\mathbf{a}}(\omega) = \begin{pmatrix} \hat{a}_1(\omega) \\ \hat{a}_2(\omega) \end{pmatrix} \equiv \begin{pmatrix} \hat{a}_{1+}(x_1, \omega) \\ \hat{a}_{3-}(x_2, \omega) \end{pmatrix}$ corresponds to the amplitude operators of the input fields from the two sides and $\hat{\mathbf{g}}(\omega) = \begin{pmatrix} \hat{g}_+^{(1)}(\omega) \\ \hat{g}_-^{(1)}(\omega) \end{pmatrix}$ represents the noise operators owing to the absorption.

The input-output relations can be used to obtain the quantum statistical properties of the outgoing radiation from the incoming radiation and the excitation associated with the plate. With regard to measurement, the quantum statistics of radiation is frequently in terms of normally ordered correlation functions of electric-field strength. The electric-field strength of the outgoing radiation reads as

$$\hat{E}'_i(x) = \hat{E}'_i^{(+)}(x) + \hat{E}'_i^{(-)}(x) \quad (7)$$

$$\hat{E}'_i^{(+)}(x) = i \int_0^\infty d\omega \sqrt{\frac{\hbar\omega}{4\pi c \epsilon_0 \mathcal{A}}} e^{i\omega \eta_i x/c} \hat{b}_i(\omega) \quad (8)$$

$$\hat{E}'_i^{(-)}(x) = [\hat{E}'_i^{(+)}(x)]^\dagger \quad (9)$$

where $i = 1$ represents the left side ($x \leq x_1$), $i = 2$ represents the right side ($x \geq x_2$) and $\eta_1 = 1$, $\eta_2 = -1$. Since the output photon operators can be related to the input photon operators according to Eq. (5), the normally ordered correlations of the outgoing radiation can be expressed as [24]

$$C'_{\{i_\mu\}}^{(m,n)}(\{x_\mu, t_\mu\}) = \left\langle \left[\prod_{\mu=1}^m \hat{E}'_{i_\mu}{}^{(-)}(x_\mu, t_\mu) \right] \times \left[\prod_{\mu=m+1}^{m+n} \hat{E}'_{i_\mu}{}^{(+)}(x_\mu, t_\mu) \right] \right\rangle \quad (10)$$

Using Eqs. (7)–(9) and the harmonic time evolution of photon destruction operators in the Heisenberg picture, we obtain

$$C'_{\{i_\mu\}}{}^{(m,n)}(\{x_\mu, t_\mu\}) = i^{n-m} \left(\frac{\hbar}{4\pi c \varepsilon_0 \mathcal{A}} \right)^{(n+m)/2} \times \int_0^\infty d\omega_1 \sqrt{\omega_1} e^{i\omega_1 \tau_{i1}} \dots \int_0^\infty d\omega_{m+n} \sqrt{\omega_{m+n}} e^{i\omega_{m+n} \tau_{im+n}} C'_{\{i_\mu\}}{}^{(m,n)}(\{\omega_\mu\}) \quad (11)$$

($\tau_{i_\mu} = t_\mu - \eta_{i_\mu} x_\mu / c$), where

$$C'_{\{i_\mu\}}{}^{(m,n)}(\{\omega_\mu\}) = \left\langle \left[\prod_{\mu=1}^m \hat{b}_{i_\mu}^\dagger(\omega_\mu) \right] \times \left[\prod_{\mu=m+1}^{m+n} \hat{b}_{i_\mu}(\omega_\mu) \right] \right\rangle = \left\langle \left\{ \prod_{\mu=1}^m \sum_{k_\mu=1}^2 [T_{i_\mu k_\mu}^*(\omega_\mu) \hat{a}_{k_\mu}^\dagger(\omega_\mu) + A_{i_\mu k_\mu}^*(\omega_\mu) \hat{g}_{k_\mu}^\dagger(\omega_\mu)] \right\} \left\{ \prod_{\mu=m+1}^{m+n} \sum_{k_\mu=1}^2 [T_{i_\mu k_\mu}(\omega_\mu) \hat{a}_{k_\mu}(\omega_\mu) + A_{i_\mu k_\mu}(\omega_\mu) \hat{g}_{k_\mu}(\omega_\mu)] \right\} \right\rangle \quad (12)$$

When a zero-temperature metamaterial plate is irradiated from the left side, the photon-number density of the outgoing field can be written as

$$N'_{phi i}(\omega) = \langle b_i^\dagger(\omega) b_i(\omega) \rangle = C'_{ii}{}^{(1,1)}(\omega, \omega) = |T_{i1}(\omega)|^2 N_{phi 1}(\omega), \quad i = 1, 2 \quad (13)$$

$N_{phi 1}(\omega)$ is the photon-number densities of the input field from the left side. Then the relative photon-number densities of the reflected outgoing field is

$$N_1(\omega) = N'_{phi 1}(\omega) / N_{phi 1}(\omega) = |T_{11}(\omega)|^2 \quad (14)$$

and the relative photon-number densities of the transmitted outgoing field can be obtained as

$$N_2(\omega) = N'_{phi 2}(\omega) / N_{phi 1}(\omega) = |T_{21}(\omega)|^2 \quad (15)$$

The relative photon-number densities of the absorbed field is

$$N_a(\omega) = 1 - N_1(\omega) - N_2(\omega) \quad (16)$$

3 Numerical results and discussion

Let us consider the radiation propagating through a single metamaterial plate with dispersive and absorbing (relative) permittivity and permeability given by [19]

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \quad (17)$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\omega\Gamma} \quad (18)$$

Here ω_p is the electronic plasma frequency and ω_0 is the magnetic resonance frequency. γ and Γ represent the electric and magnetic loss terms respectively. This metamaterial can be made by the SRR (split-ring resonator) structure [1]. Without loss of generality, the values $\omega_0 = 0.4\omega_p$, $\gamma = 0.03\omega_p$ and $\Gamma = 0.03\omega_0$, $F = 0.8$ are used in the numerical calculations. The (relative) permittivity and permeability of the surroundings (air) are taken as $\varepsilon(\omega) = 1$ and $\mu(\omega) = 1$. The real part of the permittivity and the real part of the permeability of the

metamaterials are plotted in Fig. 2(a) as a function of the normalized frequency ω/ω_0 . The results show that the metamaterials possess the magnetic resonance near the frequency $\omega = \omega_0$. The real part and the imaginary part of the refractive index ($n(\omega) = \sqrt{\varepsilon(\omega)} \times \sqrt{\mu(\omega)}$) are given in Fig. 2(b). We can find that a large imaginary part of the refractive index appears near the magnetic resonance frequency $\omega = \omega_0$. These properties play an important role in the transmission of the quantized radiation.

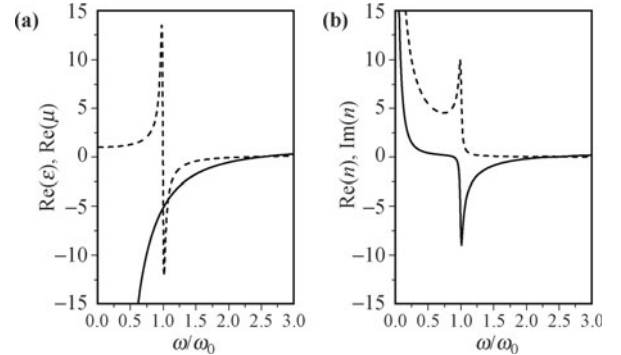


Fig. 2 (a) The real part of the permittivity (solid line) and the real part of the permeability (dashed line) of the metamaterials as a function of the normalized frequency ω/ω_0 . (b) The real part of the refractive index (solid line) and the imaginary part of the refractive index (dashed line) of the metamaterials as a function of the normalized frequency ω/ω_0 .

In Figs. 3 and 4 the relative photon-number densities of the reflected outgoing field ($N_1(\omega)$) and the transmitted outgoing field ($N_2(\omega)$) are shown as a function of the normalized frequency and the normalized thickness ($\omega_0 l / c$) of the metamaterial plate. The relative photon-number density of the absorbed field ($N_a(\omega)$) in the metamaterials is shown in Fig. 5. For frequencies that are small compared with the magnetic resonance frequency ω_0 , almost all the photons incident on the metamaterials with certain thickness are reflected. The metamaterials behave like a lossy mirror due to the large value of the real part and the imaginary part of the refractive index. The reflectivity decreased rapidly when the frequency is increasing toward the magnetic resonance frequency. The relative photon-number densities of the absorbed

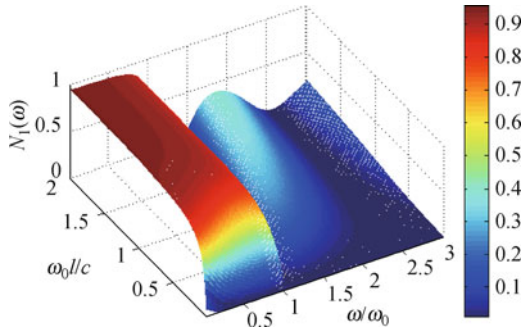


Fig. 3 The relative photon-number densities of the reflected outgoing field through a metamaterial plate.

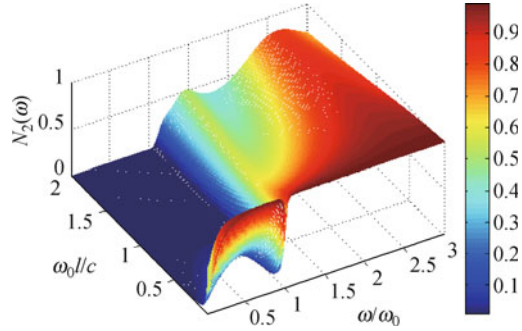


Fig. 4 The relative photon-number densities of the transmitted outgoing field through a metamaterial plate.

field are approaching the unit near the magnetic resonance frequency, which means most of the photons can be absorbed by the plate. The peak in Fig. 5 corresponds to the peak of the imaginary part of the refractive index in Fig. 2(b) located at the magnetic resonance frequency. The absorptivity is in proportion to the thickness of the metamaterial plate. The results show that the large imaginary part of the refractive index causes the large absorption. With a further increase of the frequency that is associated with a relative small absolute value of the refractive index, both the reflectivity and the absorptivity are reduced. The transmission is enhanced in oscillating way due to the interference effect of the interface. The metamaterials become nearly transparent to the photons for large frequencies.

To show the independent contributions of the permittivity and permeability to the transmitted outgoing field,

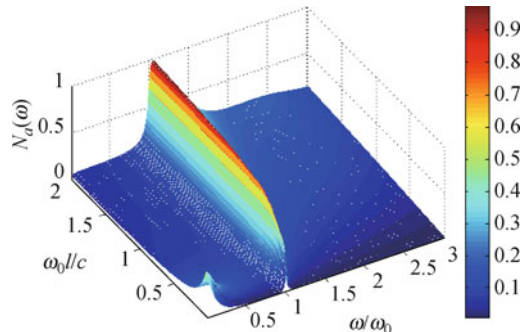


Fig. 5 The relative photon-number densities of the absorbed field in a metamaterial plate.

in Fig. 6 we show the relative photon-number densities of the transmitted outgoing field for three cases: i) the metamaterial plate with the permittivity and permeability given by Eq. (17) and Eq. (18); ii) the plate with the permittivity given by Eq. (17) and $\mu(\omega) = 1$; iii) the plate with $\varepsilon(\omega) = 1$ and the permeability given by Eq. (18). Figures 6(a) and (b) correspond to the plate with the normalized thickness $\omega_0 l/c = 0.5$ and $\omega_0 l/c = 1.0$ respectively. For frequencies that are small compared with the magnetic resonance frequency ω_0 , the relative photon-number densities through the metamaterials are small, which is consistent with the case ii): the plate with permittivity given by Eq. (17) and $\mu(\omega) = 1$. It means that the quantum feature is mainly dependent on the permittivity in low frequencies. When the frequency is near the magnetic resonance frequency ω_0 , the relative photon-number densities of the transmitted field are low because of the big absorption. For frequencies that are big compared with the magnetic resonance frequency ω_0 , the relative photon-number densities of the transmitted field are large. The results show that the permittivity and permeability reinforce the transmission and this feature is more obvious for the thick plate.

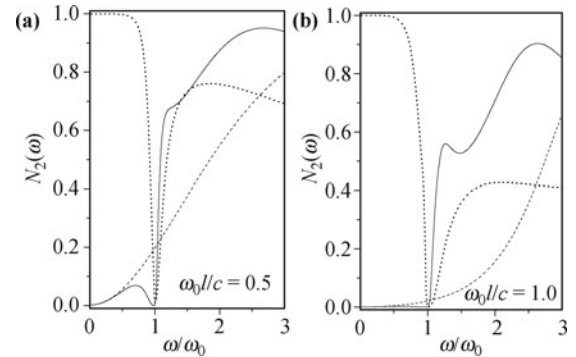


Fig. 6 The relative photon-number densities of the transmitted outgoing field: solid lines correspond to the metamaterial plate with permittivity and permeability given by Eq. (17) and Eq. (18); dashed lines and dotted lines correspond to the plate with the permittivity given by Eq. (17) but $\mu(\omega) = 1$ and $\varepsilon(\omega) = 1$ but the permeability given by Eq. (18). (a) and (b) correspond to different thickness of the plate ($\omega_0 l/c = 0.5$ and $\omega_0 l/c = 1.0$).

4 Summary

Based on the quantization scheme of the radiation fields and input-output relations in the dispersive and absorptive magnetic media, we calculate the quantum optical correlation through a metamaterial plate. Then the relative photon-number densities of the transmitted field, the reflected field and the absorbed field are obtained through the correlation functions. For frequencies that are small compared with the magnetic resonance frequency ω_0 , almost all the photons incident on the metamaterials with certain thickness are reflected. Near the magnetic resonance frequency ω_0 , most of the photons

which can enter the metamaterials are absorbed due to the large imaginary part of the refractive index. The transmission is enhanced with a further increase of the frequency that is associated with a relative small absolute value of the refractive index. Furthermore, the contributions of the permittivity and the permeability of the metamaterials to the transmission are analyzed. Our results show that both the permittivity and permeability are necessary to the high transmission for frequencies that are big compared with the magnetic resonance frequency ω_0 . These conclusions are similar to those in the classical situations. This indicates that the unusual properties of metamaterials might be used in the control of the quantized radiation.

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Appendix: Elements of transformation matrix

The elements of the matrix $\mathbf{T}(\omega)$ can be obtained by [24]

$$T_{11}(\omega) = e^{-i\beta_1(\omega)\omega l/c} \times [r_{12}(\omega) + t_{12}(\omega)e^{2in_2(\omega)\omega l/c}r_{23}(\omega)v(\omega)t_{21}(\omega)] \quad (\text{A-1})$$

$$T_{12}(\omega) = \frac{n_1(\omega)}{n_3(\omega)} \sqrt{\frac{\beta_3(\omega)}{\beta_1(\omega)}} \times e^{-i[\beta_1(\omega)+\beta_3(\omega)]\omega l/(2c)} t_{32}(\omega) e^{in_2(\omega)\omega l/c} v(\omega) t_{21}(\omega) \quad (\text{A-2})$$

$$T_{21}(\omega) = \frac{n_3(\omega)}{n_1(\omega)} \sqrt{\frac{\beta_1(\omega)}{\beta_3(\omega)}} \times e^{-i[\beta_1(\omega)+\beta_3(\omega)]\omega l/(2c)} t_{12}(\omega) e^{in_2(\omega)\omega l/c} v(\omega) t_{23}(\omega) \quad (\text{A-3})$$

$$T_{22}(\omega) = e^{-i\beta_3(\omega)\omega l/c} \times [r_{32}(\omega) + t_{32}(\omega)e^{2in_2(\omega)\omega l/c}r_{21}(\omega)v(\omega)t_{23}(\omega)] \quad (\text{A-4})$$

where the interface reflection and transmission coefficients are defined by

$$r_{ij}(\omega) = -r_{ji}(\omega) = \frac{n_i(\omega)\mu_j(\omega) - n_j(\omega)\mu_i(\omega)}{n_i(\omega)\mu_j(\omega) + n_j(\omega)\mu_i(\omega)} \quad (\text{A-5})$$

$$t_{ij}(\omega) = \frac{2n_i(\omega)\mu_j(\omega)}{n_i(\omega)\mu_j(\omega) + n_j(\omega)\mu_i(\omega)} \quad (\text{A-6})$$

and the multiple reflections factor reads as

$$v(\omega) = [1 - r_{21}(\omega)r_{23}(\omega)e^{2in_2(\omega)\omega l/c}]^{-1} \quad (\text{A-7})$$

In the above expressions $l = x_2 - x_1$ is the thickness of the metamaterial plate.

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