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RESEARCH ARTICLE

Andreev reflection and tunneling spectrum on metal–superconductor–metal junctions

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The tunneling spectrum of an electron and a hole in metal–superconductor–metal junctions is computed using the Blonder–Tinkham–Klapwijk method. The incident and the outgoing currents finally balance each other by an interface charge inside the superconductor and metal junction. The present computation shows a more abundant structure compared to that on a metal–superconductor junction, such as the resonance at bias voltages above the energy gap of the superconductor. The density of the interface charge shows a quantum-like oscillation.

Keywords Andreev reflection, superconductivity

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1 Introduction

The research on the Andreev reflection in metal–superconductor junctions has a long history and still attracts great interests of scientists recently [1–3]. The tunneling spectrum of electrons on a metal–superconductor (SC) junction is sensitive to the energy gap of the superconductor, thus becoming an important technique to measure the gaps and gap properties of superconductors. When an electron tunnels into a superconductor to form a Cooper pair there appears a hole reflection to conserve the charges, which is called the Andreev reflection. A commonly used theoretical method to investigate the Andreev reflection is the Blonder–Tinkham–Klapwijk (BTK) approach [1], which takes the interface in a metal–superconductor junction as a $\delta(x)$ potential barrier. This theory has been widely and successfully applied to systems like metal–superconductor junctions [4], ferromagnet–SC junctions [5], etc. Wei, Dong and Xing *et al.* studied the tunneling spectrum in metal–SC–metal (MSM) junctions using the BTK approach [6, 7]. They found that the incident and outgoing currents in both sides do not balance each other [6]. Thus they claimed at an earlier time that the BTK approach are not suitable for the Andreev reflection in MSM junctions and treated them in the Landauer–Büttiker formalism [8]. But later they balanced the current by adjusting the chemical potential of the superconductor inside the MSM junction.

The quasi-particle current and superconducting current are continuous inside the superconductor everywhere, hence, the charge density inside the superconductor does not change according to the continuity equation of charges. Therefore, the chemical potential thus the Fermi surface of the SC inside MSM junctions may not change. On the outgoing interface, however, the current does not balance on the two sides. Therefore, there must be a charge accumulation on this interface. This interface charge density changes the potential of the metal in the outgoing side and finally reaches a current balance. This is the main idea of this paper. Through detailed computation we demonstrate that this mechanism is reasonable. the charge accumulation should be a real effect in MSM junctions and can be measured in experiments. This dynamic process can be simulated by solving the time-dependent Bogoliubov–de Gennes (BdG) equations. In the present paper a simpler model is set up to describe the final balanced state of the MSM junctions.

2 Formalism

The MSM junctions are shown in Fig. 1, where two thin insulating layers exist inside the two junctions which become two potential barriers and are treated as a $\delta(x)$ potential in the BTK approach. An electron and a hole are incident into the junctions under a bias voltage on both sides. Due to the two barriers the electron and the hole are partially reflected and partially transmitted into the SC. In the SC a quasi-particle can only propagate above the energy gap Δ. When the bias voltage is smaller than Δ only Cooper pairs can propagate in the SC. In this case there will be a hole reflected on both sides, which are called Andreev reflection in literatures.

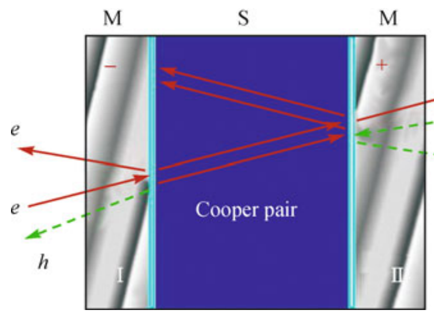

Fig. 1 MSM junctions with an incident electron and an incident hole from the left side and the right side respectively under a bias voltage. The metal and the SC are labeled by M and S, respectively. The interfaces are colored in cyan with a potential barrier.

The tunneling process of an electron and a hole in the MSM junctions is described by the following Bogoliubov–de Gennes (BdG) equation [9]

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\begin{pmatrix} u(\boldsymbol{r},t) \\ v(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} H_0 & \Delta \\ \Delta^* & -H_0 \end{pmatrix}\begin{pmatrix} u(\boldsymbol{r},t) \\ v(\boldsymbol{r},t) \end{pmatrix} \tag{1}$$

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - \mu \tag{2}$$

where $V(\boldsymbol{r}) = eV, -eV$ in the left and the right metals and 0 in the SC. On the interfaces $V(\boldsymbol{r}) = U\delta(x)$. A key idea of this paper is that the SC–metal interface at the right side will be charged due to the supercurrent in the SC. The charge accumulation is determined by the continuity equation of the BdG equation

$$\frac{\partial \rho_Q(\boldsymbol{r})}{\partial t} + \nabla\cdot(\boldsymbol{J}_Q(\boldsymbol{r}) + \boldsymbol{J}_S(\boldsymbol{r})) = 0 \tag{3}$$

where $\rho_Q(\boldsymbol{r})$ is the charge density and $\boldsymbol{J}_Q(\boldsymbol{r}) = \frac{e\hbar}{m}Im(u^*\nabla u + v^*\nabla v)$ the current density of quasi-particles. The supercurrent density is defined by

$$\nabla\cdot\boldsymbol{J}_S(\boldsymbol{r}) = -\frac{4e}{\hbar}Im(\Delta u^*(\boldsymbol{r})v(\boldsymbol{r})) \tag{4}$$

It can be proved that quasi-particle current is continuous along the whole MSM junctions, but the supercurrent is obviously discontinuous since no supercurrent exists in a metal. This charges the interface like the charging process on a capacitor with a bias voltage. The charging process can be seen by integrating the continuity equation over a slap volume across the interface, resulting in

$$\frac{\partial \sigma(t)}{\partial t} = J_S(x = L) \tag{5}$$

where $\sigma(t)$ is the surface charge density on the interface and $J_S(x = L)$ is the supercurrent at the interface. The interface charge distribution changes the potential of electrons and holes in metals II, and therefore the outgoing current is changed and finally balances with the incident current.

As seen from Fig. 2, when the bias voltage satisfies $eV > \Delta$ an incident particle in metals tunnels into the SC through an interface and propagate as a quasi-particle. It is then partially reflected by the following interface and partially tunnels as the following metal. The wave functions of the electron and hole inside the metals and the SC are written as

$$\psi_{\mathrm{I}} = \begin{pmatrix} \mathrm{e}^{\mathrm{i}k_+x} + b\mathrm{e}^{-\mathrm{i}k_+x} \\ a\mathrm{e}^{\mathrm{i}k_-x} \end{pmatrix} \tag{6}$$

$$\psi_{\mathrm{SC}} = \left[c\mathrm{e}^{\mathrm{i}q_+x}\begin{pmatrix} u_+ \\ v_+ \end{pmatrix} + d\mathrm{e}^{-\mathrm{i}q_-x}\begin{pmatrix} u_- \\ v_- \end{pmatrix} + e\mathrm{e}^{-\mathrm{i}q_+x}\begin{pmatrix} u_+ \\ v_+ \end{pmatrix} + f\mathrm{e}^{\mathrm{i}q_-x}\begin{pmatrix} u_- \\ v_- \end{pmatrix}\right] \tag{7}$$

$$\psi_{\mathrm{II}} = \begin{pmatrix} g\mathrm{e}^{\mathrm{i}k'_+x} \\ \mathrm{e}^{\mathrm{i}k'_-x} + h\mathrm{e}^{-\mathrm{i}k'_-x} \end{pmatrix} \tag{8}$$

where $\binom{u_+}{v_+}$ and $\binom{u_-}{v_-}$ are the quasi-electron and quasi-hole wave functions, respectively. They are given by

$$u_\pm = \sqrt{\frac{1}{2} \pm \frac{\varepsilon_{q\pm} - \mu}{2E}}, \quad v_\pm = \frac{|\Delta|}{\Delta}\sqrt{\frac{1}{2} \mp \frac{\varepsilon_{q\pm} - \mu}{2E}} \tag{9}$$

$$E = \sqrt{(\varepsilon_{q\pm} - \mu)^2 + |\Delta|^2}, \quad \varepsilon_{q\pm} = \frac{\hbar^2 q_\pm^2}{2m} \tag{10}$$

The wave vectors are determined by

$$E = eV = \frac{\hbar^2 k_+^2}{2m} - \mu = \mu - \frac{\hbar^2 k_-^2}{2m} \tag{11}$$

$$k_\pm/k_{\mathrm{F}} = \sqrt{1 \pm E/\mu} \tag{12}$$

$$q_\pm/k_{\mathrm{F}} = \sqrt{1 \pm \sqrt{E^2 - \Delta_q^2}/\mu} \tag{13}$$

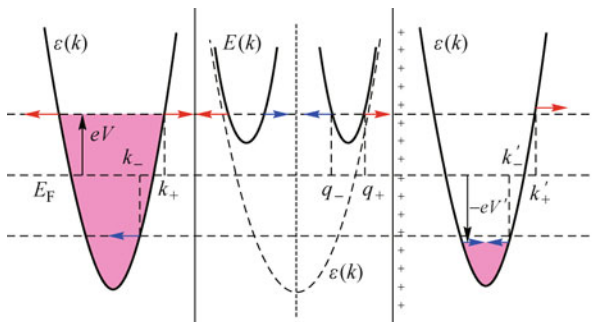

Fig. 2 The energy dispersion on the MSM junctions in the case of $eV > \Delta$. The arrows label the propagating directions of quasi-electrons (*red*) and quasi-holes (*blue*). The charges on the interface at the right side are labeled by a column of "+". $E(k)$ is the quasi-particle energy and $\varepsilon(k)$ is the free electron energy in the metals and SC. The energy of electrons and holes in the metal at the right side is increased a little by the interface charge density.

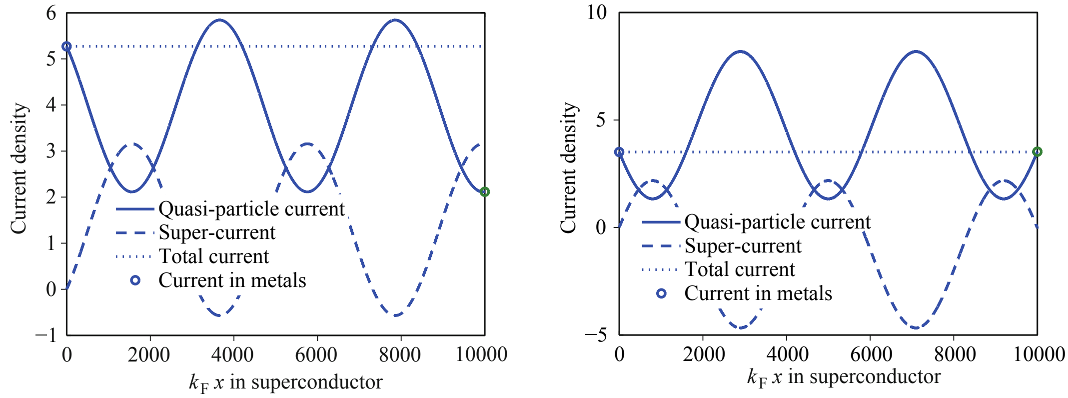

Fig. 3 Charge current densities inside the SC without (left, $\sigma = 0$) and with (right, $\sigma = 0.24 \times 10^{-4}e$) interface charging. The parameters are set to $\mu = 0.5, \Delta = 0.001\mu, Z = 1, k_F L = 10000, eV = 1.8\Delta$. Circles label the incident and outgoing currents in the metals.

$$-eV' = \frac{\hbar^2 k_-'^2}{2m} - \mu = \mu - \frac{\hbar^2 k_+'^2}{2m} \tag{14}$$

$$k'_\pm / k_F = \sqrt{1 \pm E'/\mu} \tag{15}$$

In the case of $eV < \Delta$, $q_\pm$ become complex numbers, so that the wave functions of electrons and holes in the SC become damping traveling waves and Cooper pairs appear.

The constants a, b, c, d, e, f, g, h in Eqs. (6) to (8) are obtained from the boundary conditions of wave functions on the interfaces, where the wave functions are continuous, but the first derivatives of them have jumps due to the $\delta(x)$ barriers, i.e.,

$$\psi_I(0^-) = \psi_{SC}(0^+) \tag{16}$$

$$\psi'_I(0^-) - \psi'_{SC}(0^+) + Zk_F\psi_I(0) = 0 \tag{17}$$

$$\psi_{SC}(L) = \psi_{II}(L) \tag{18}$$

$$\psi'_{SC}(L) - \psi'_{II}(L) + Zk_F\psi_{II}(L) = 0 \tag{19}$$

where $Z = \frac{2mU}{\hbar^2 k_F}$ and L is the thickness of the SC. Finally, the current density, the super-current and the differential electric conductance are computed with these constants, which are given by

$$J_Q(\mathrm{I}) = 2e[k_+(1 - |b(eV)|^2) + k_-|a(eV)|^2] \tag{20}$$

$$J_Q(\mathrm{II}) = 2e[k'_-(1 - |h(eV)|^2) + k'_+|g(eV)|^2] \tag{21}$$

$$\frac{\mathrm{d}I_1(eV)}{\mathrm{d}V} \sim 1 - |b(eV)|^2 + |a(eV)|^2 \tag{22}$$

$$\frac{\mathrm{d}I_2(eV)}{\mathrm{d}V} \sim 1 - |h(eV')|^2 + |g(eV')|^2 \tag{23}$$

3 Results of computation

To view the effect of the interface charges we first compute the current densities in the MSM junctions with and without interface charges. As shown in Fig. 3 (left), the incident and outgoing currents do not balance each other. The current has a jump at the interface in the right-hand side due to the supercurrent. It is natural to think that the interface at the right-hand side will be charged to saturation until both the currents balance each other. This is true as seen in Fig. 3 (right) where the two currents equal to each other after fully charged. It is interesting to note that the super-currents are always complementary to the charge currents in both cases so that the total currents are always constant in the SC. In addition, the quasi-particle current and the super-current with interface charges become symmetric about the center of the SC. This result convinces us that the interface charges have a true effect on the current in MSM junctions.

Further computation shows that the incident and outgoing currents actually oscillate on the interface charges. There exists a series of interface charge densities which balance the incident and outgoing currents, as shown by the matching points in Fig. 4. The physics of this periodicity is still an open question. One may find a quantum behavior of the interface charge densities.

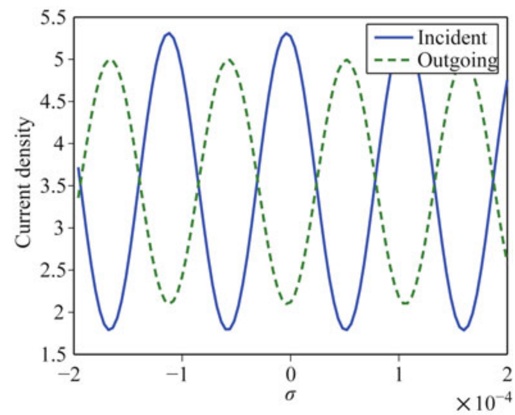

Fig. 4 The incident and outgoing current densities versus different interface charge densities at $eV = 1.8\Delta$. Other parameters see Fig. 3.

The tunneling spectrum, i.e., the normalized differential conductivities at different bias voltages without and with interface charges are shown in Fig. 5. Without interface charges the incident and outgoing currents match each other only under small bias voltages. After fully charged the two currents overlap in the whole bias region. Therefore, the conservation of currents in the MSM junctions is realized by means of the interface charging. In addition, the probability flux $|a|^2+|b|^2+|f|^2+|g|^2=2$ is always conserved for different bias voltages.

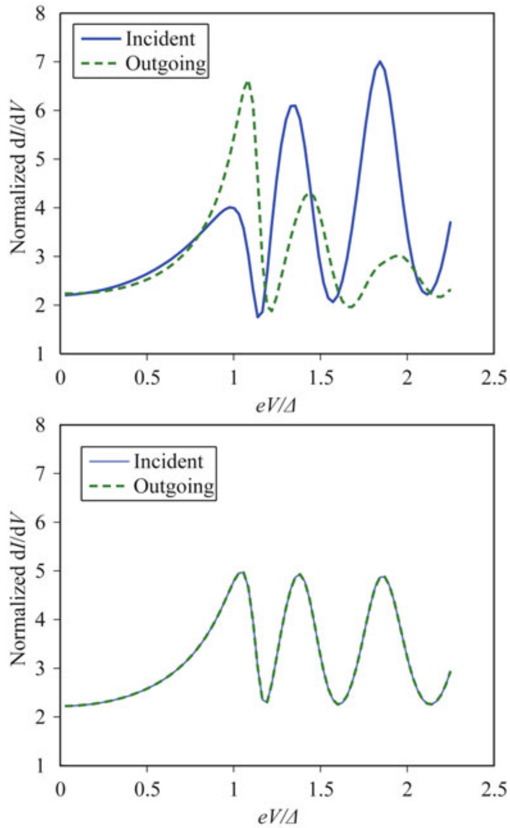

Fig. 5 The normalized differential conductance without (top panel) and with (bottom panel) interface charges versus bias potential. For parameters see Fig. 3.

Finally, the tunneling spectra for the differential electric conductance (DEC) across different interface potential barriers are computed, as shown in Fig. 6. Across a vanishing barrier the DEC approaches to a constant 4, which originates from the electron and hole injection. This is very different from that across a single metal–SC junction, where the DEC drops gradually from a constant value to about half at the bias voltage of the energy gap (Δ). The main difference between these two cases is that there is a hole injection from the right side into the MSM junctions. It is found that this hole injection is necessary for the Andreev reflection. Otherwise there would be no hole propagation at all in the MSM junctions.

As the interface barrier increases, the DEC shows a few sharper and sharper resonances with energies depending on the width of the SC layer. They are in fact the bound states of a quasi-particle in the SC well. This is similar to the cases observed by Tomasch [10] and by Rowell and McMillan [11]. Below the bias voltage Δ/e the DEC decreases rapidly, similar to the case across a metal–SC junction. This provides a measurement to the energy gap of the superconductor.

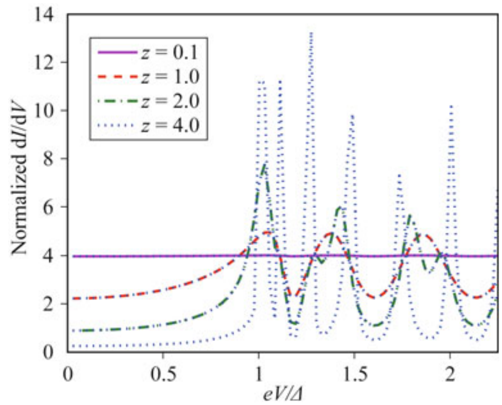

Fig. 6 The differential conductance versus the bias voltage for different interface barriers. For other parameters see Fig. 3.

4 Conclusion

In summary, the tunneling spectrum of an electron and a hole on MSM junctions is computed using the Bogoliubov–de Gennes (BdG) equation. It shows a more abundant structure compared to that on a metal–superconductor junction, such as the resonance above the gap voltage. An important effect occurring in the present device is that the incident current and the outgoing current balance each other only when there is an interface charge. The density of this interface charge shows a quantum-like oscillation, where a series of densities balance the currents.

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