Front. Phys., 2012, 7(3): 259–260
DOI 10.1007/s11467-011-0242-8

NEWS & VIEWS

Entanglement-assisted entropic uncertainty principle

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Received November 2, 2011; accepted November 30, 2011

In quantum mechanics, the ability to simultaneously predict the precise outcomes of two conjugate observables, such as the position and momentum, for a particle is restricted by the uncertainty principle [1]. For example, the more precisely the location of the particle is determined, the less accurate the momentum determination will be. Originally given by Heisenberg, the uncertainty principle is best known as the Heisenberg–Robertson [2] commutation $\Delta R \Delta S \geqslant \frac{1}{2}|\langle [R,S] \rangle|$, with $\Delta R(\Delta S)$ representing the standard deviation of the corresponding variable R (S).

Obviously, the bound on the right-hand side of above equation is state-dependent and can vanish even when R and S are non-commuting. To avoid this defect, the uncertainty relation is further extended to the entropic form to precisely reflect its physical meanings. The entropic uncertainty relation for any two general observables was first given by Deutsch [3]. Soon afterwards, an improved version was proposed by Kraus [4] and then proved by Maassen and Uffink [5]. The improved relation reads as $H(R) + H(S) \geqslant \log_2 \frac{1}{c}$, where H is the Shannon entropy and c represents the overlap between observables R and S.

The uncertainty principle is the essential characteristic of quantum mechanics. However, the possibility of violating Heisenberg's uncertainty relation has been considered early. In 1935, Einstein, Podolsky, and Rosen published the famous paper in which they considered using two particles entangled in position and momentum freedoms to violate Heisenberg's uncertainty relation and to challenge the correctness of quantum mechanics (EPR paradox) [6]. Popper also proposed an experimental scheme using entangled systems to demonstrate the violation of the Heisenberg's uncertainty relation [7]. After a long debate and many experimental work, it is known that these violations do not contradict the quantum theory and they are now implemented as a signature of entanglement which is the fundamental feature of quantum mechanics and important resource of quantum information processing.

One way to think about uncertainty relations is through the uncertainty game [8] between two players, Alice and Bob. Before the game commences, Alice and Bob agree on two measurements, R and S. The game proceeds as follows (Fig. 1). Bob prepares a particle in a chosen quantum state and sends it to Alice. Alice then carries out one of the two measurements and announces her choice to Bob. Bob's task is to minimize his uncertainty about Alice's measurement outcome.

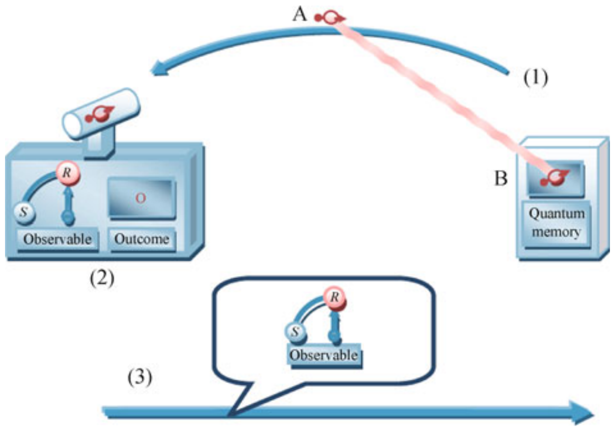

Fig. 1 Illustration of the uncertainty game. Reproduced from Ref. [8], Copyright © 2010 Nature Publishing Group.

Recently, a stronger entropic uncertainty relation, which uses previously determined quantum information, was proved by Berta *et al.* [8], whose equivalent form was previously conjectured by Renes and Boileau [9].

By initially entangling the particle of interest (A) to another particle that acts as a quantum memory (B), the uncertainty associated with the outcomes of two conjugate observables can be reduced to zero. The entropic uncertainty relation is mathematically expressed as $H(R|\mathrm{B}) + H(S|\mathrm{B}) \geqslant \log_2 \frac{1}{c} + H(\mathrm{A}|\mathrm{B})$, where $H(R|\mathrm{B})(H(S|\mathrm{B}))$ is the conditional von Neumann entropy representing the uncertainty of the measurement outcomes of R (S) obtained using the information stored in B. $H(\mathrm{A}|\mathrm{B})$ represents the conditional von Neumann entropy between A and B. The lower bound of the

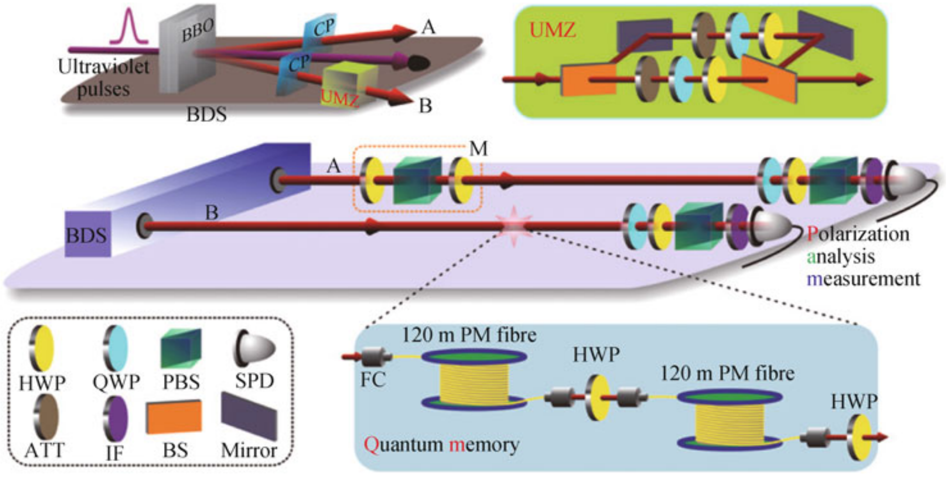

Fig. 2 Lattice and magnetic structure. Reproduced from Ref. [10], Copyright © 2011 Nature Publishing Group.

uncertainty is essentially dependent on the entanglement between the particle of interest and the quantum memory. This novel entropic uncertainty relation greatly extends the uncertainty principle.

In experiment [10] we prepare a special kind of entangled photon state, named the Bell diagonal state, in an entirely optical setup (Fig. 2). One of the photons is sent for measurement and the other one acts as an assisted particle carrying the quantum information of the one of interest. The assisted photon is stored in a spin-echo based quantum memory which consisted of two polarization maintaining fibers each of 120 m length and two half-wave plates. The storing time can reach as long as 1.2 μs. We then characterize the spin-echo based quantum memory using quantum process tomography. It is found that the operation of the memory is close to the identity, and the fidelity of the experimental result is about 98.3%. The lower bound of the uncertainty related to the outcomes of two conjugate observables is measured, which can be reduced to arbitrarily small values when the two particles share quasi-maximal entanglement. As a result, the entropic form of Heisenberg's uncertainty relation is violated and the novel one is confirmed. By measuring observables on both particles, the group further used the novel entropic uncertainty relation to witness entanglement and to compare the measurements with other entanglement measurements. The novel entanglement witness can be obtained by a few separate measurements on each of the entangled particles, which exhibits its ease of access.

The verified entropic uncertainty principle implies that the uncertainty principle is not only observable-dependent, but is also observer-dependent, providing a particularly intriguing perspective [11]. The method used to estimate uncertainties by directly performing measurements on both photons has a practical application in verifying the security of quantum key distribution [12]. This novel uncertainty relation would also find practical use in the area of quantum engineering.

The experimental investigation of the novel entropic uncertainty principle has caused great interests. Another relevant experimental work was performed independently by Prevedel and colleagues [13] and both papers are published in the same issue of Nature Physics.

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