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RESEARCH ARTICLE

A novel snowdrift game model with edge weighting mechanism on the square lattice

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We propose a novel snowdrift game model with edge weighting mechanism to explore the cooperative behaviors among the players on the square lattice. Based on the assumption of three types of weight distribution including uniform, exponential and power-law schemes, the cooperation level is largely boosted in contrast with the traditional snowdrift game on the unweighted square lattice. Extensive numerical simulations indicate that the fraction of cooperators greatly augments, especially for the intermediate range of cost-to-benefit ratio r. Furthermore, we investigate how the cooperative behaviors are affected by the undulation amplitude of weight distribution and noise strength of strategy selection, respectively. The simulation results will be conducive to further understanding and analyzing the emergence of cooperation, which is a ubiquitous phenomenon in social and biological science.

Keywords snowdrift game, edge weighting mechanism, cooperative dynamics

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1 Introduction

Although Darwin's evolutionism drastically drives the species development, the cooperative behaviors are often found among selfish individuals within many biological and social systems [1]. At present, the evolutionary game theory has become a theoretical tool to understand these behaviors [2]. For the traditional game theory, a rational individual adopts the defection strategy instead of mutual cooperation in order to make its personal payoff maximized. The players in the game always attempt to obtain the maximum benefit against the cost as little as possible, which yields the conflict between individual and overall optimal strategy, and this is the so-called social dilemma [3]. The evolutionary game theory provides a powerful framework to answer the question of how the cooperation among unrelated individuals emerges. In the past decades, various feasible mechanisms were presented to probe into the cooperative behaviors, which incorporate direct [4] and indirect [5] reciprocity, kin selection [6], group selection [7], reputation and punishment [8], and aspiration to the fittest [9, 10]. Especially, Nowak and May [5] have found that the spatial structure can greatly promote the cooperative behaviors among agents in the prisoner's dilemma game, and various spatial extension schemes which can facilitate the cooperation [11–13] have been come up with to promote the evolution of cooperation. However, the cooperation could not be generally enhanced due to the spatial structure and could even be eliminated when the cost-to-benefit ratio r is extremely high for the snowdrift game [14]. In Ref. [15], the authors put forward a spatial extended snowdrift game model with mixed strategy giving rise to a noteworthy promotion of cooperation, in which each player updates his strategy with a given probability in the event of discontentment with the current one. In the past few years, with more and more attention paid to the coevolutionary game dynamics in systems with complex and spatial topology, numerous related studies have been initiated [16–27].

At present, the heterogeneity of individual role or influence isn't taken into account in most previous works, although there exists proverbially the non-homogeneity of degree distribution in many natural, social and engineering systems [28]. Therefore, many real systems are modeled by complex networks in which nodes represent individuals and links denote the interactions between individuals. Currently, synchronization dynamics taking place upon networks have become one of the active topics in the physics community [29–31]. In addition, the highly heterogeneous topology usually leads to the absence of epidemic threshold [32] and has stimulated many related works about epidemic dynamics in complex networks [33–35]. Recently Liao *et al.* studied the self-sustained periodic oscillations in random networks consisting of excitable nodes and revealed the underlying dynamic structure by applying a dominant phase-advanced driving method [36]. Meanwhile, many real systems exhibit the weighted properties. Scientific collaboration networks [37] and the world-wide airport networks [38] are two representative examples of social and large infrastructure systems, respectively. These results provide a better description of the hierarchies and organizational principles at the basis of the architecture of weighted networks. As a matter of fact, the effect of complex topology on the cooperative behaviors is also paid much attention [39–41] in the recent years. Nevertheless, to the best of our knowledge, the influence of weighted properties on the cooperative behaviors is still absent. Based on the standard snowdrift game, we consider the role of vertex weight in cooperative dynamics and present a weighted snowdrift game model, and the results have shown that the vertex weight can effectively enhance the cooperation level [42]. Whether the edge weighting can promote the cooperative dynamics is a natural problem to be studied further. However, up to now, the edge weight has not been integrated into the snowdrift game model on the square lattice. For this reason, in this paper, we will fill this gap and construct a novel snowdrift game with edge weighting scheme to deeply understand the cooperative behaviors in the snowdrift game model.

The remainder of this paper is organized as follows. At the beginning, we bring forward the snowdrift game model with edge weighting mechanism in Section 2. Then, Section 3 describes the numerical simulation of this modified snowdrift game in a given lattice, and we analyze the simulation results in detail. At last, we give out some concluding remarks in Section 4.

2 The novel snowdrift game model

The original snowdrift game depicts that two cars are trapped by a snowdrift obstructing their way to home. By assuming that both the cars have shovels, each driver's benefit is affected by the strategy of the other one when they face the selection of shoveling the snow (Cooperation, C) or staying in the car (Defection, D). Taking cooperation, that is, getting out to shovel the snow, can gain the benefit b of getting home at the cost of c, on the condition of $b > c > 0$. In the case of choosing defection for either of them, what they can obtain is the zero benefit of $P = 0$, which results in being stuck outside and even suffering the cold. If they shovel the snow together, the workload for each driver is half, so they both can gain the total benefit of $R = b - c/2$. In the event of the opposite strategy, the cooperator's benefit is $S = b - c$, whereas the defector acquires the highest benefit $T = b$. The payoff matrix can be written as follows,

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} b-c/2 & b-c \\ b & 0 \end{pmatrix} \end{array} \tag{1}$$

where $T > R > S > P$. We conventionalize the above matrix into a normalized one,

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix} \end{array} \tag{2}$$

where $r = \frac{c/2}{b-c/2}$ denotes the cost-to-benefit ratio. And this payoff matrix implies that taking the opposite strategy of the opponent will maximize the individual' payoff. In this paper, based on the snowdrift game mentioned above, the edge weight is taken into account and the specific game rules are described as follows.

First, each cell on the regular lattice, which stands for an individual player or agent, is randomly initialized to be defector or cooperator with the equal probability. We define a specific edge weight between each player and his neighbors which include only the von Neuman neighborhood (that is, $n = 4$). Taking ξ as a random variable, the weight value conforms to $\omega_{ij} = 1.0 + \xi$. Here we can consider only three types of weight distributions, in the order of uniform, exponential and power-law distributed weight parameter, respectively.

$$\xi = A_1 (-2\chi + 1) \tag{3}$$

$$\xi = A_2 (-\ln\chi - 1) \tag{4}$$

$$\xi = A_3 \left(\chi^{-0.5} - 2\right) \tag{5}$$

In the above-mentioned three cases, χ is a uniformly distributed random number from the unit interval, and the parameter A_α ($0 \leqslant A_\alpha \leqslant 1$, $\alpha = 1$, 2 or 3) represents the undulation amplitude of ξ. Besides, on account of $\int_0^1 \xi(\chi)\,\mathrm{d}\chi = 0$, the mean value of ξ among all agents is zero so that the average of ω_{ij} among the total populations is equal to one. On one hand, $A_\alpha = 0$ means that $\omega_{ij} = 1$ and all links have the same interaction strength, where the system coincides with the traditional snowdrift game. On the other hand, $A_\alpha = 1$ is the maximum which makes sure that the weight $\omega_{ij} \geqslant 0$ is valid.

Second, through playing the game with its four nearest neighbors at each time step, player i obtains his initial payoff, and the final payoff P_i is calculated in the following function,

$$P_i = \sum_{j \in \mathcal{N}_i} \omega_{ij} \cdot P_{i \to j} \tag{6}$$

where $P_{i\to j}$ denotes the payoff of payer i after interacting with his neighbor j. ω_{ij} is the edge weight between player i and his neighbor j, which stands for the nonhomogeneous interaction or influence that player i inflicts to player j. Utilizing the similar method, we can also compute the payoff of all players on the square lattice.

Finally, in accordance with the synchronous update rule, at each time step player i adopts the strategy of one of his neighbors (e.g., player j, which is randomly selected from his four neighbors) with the probability related to the difference of their final payoff,

$$W_{(S_i \to S_j)} = \frac{1}{1 + \exp[(P_i - P_j)/K]} \tag{7}$$

i.e., the so-called Fermi rule [43]. Here, S_i and S_j are used to express the strategy of player i and j, respectively. P_i and P_j denote the payoff computed from Eq. (6). In addition, the environmental noise $0 < K < +\infty$ is introduced into the process of strategy adoption to stand for players' irrationality or error, and the environmental factors [3].

3 Numerical simulations

In this section, we will discuss the influence of different edge weight distributions on the cooperative behaviors on the square lattice through extensive numerical simulations. The simulation of this modified snowdrift game is performed on a square lattice with periodic boundary condition. Its size is set to be 200×200 (The different sizes $L = 100, 400$ are also performed and the result are similar and robust), and the size of neighborhood is set to be 4. In the initial setup, each player, occupying one of sites on the regular lattice, randomly adopts the cooperation or defection strategy with the identical probability. According to Eq. (3)–Eq. (5), a random edge weight between vertex i and its neighbor j is assigned. Through the calculation of each player's payoff during each step, the agents update the strategy synchronously with the relative probability according to Eq. (7).

First of all, the evolution of the cooperator's fraction $F_c(t)$ along with discrete time steps is investigated in Fig. 1, which is a typical realization of numerical simulations. It is obvious that $F_c(t)$ tends to be a steady state after a transitory period. In contrast with the traditional snowdrift game, the stationary density of cooperators $F_c \approx \lim_{t\to\infty} F_c(t)$ is largely increased as expected. Nevertheless, the results are slightly diverse for different cost-to-benefit ratios. When r is smaller or intermediate (e.g., $r = 0.3$ or $r = 0.45$), the order of promoting the cooperation from maximum to minimum is the power-law, exponential, uniform distribution of edge weight. For a higher r (e.g., $r = 0.625$), the cooperator's fraction is not strengthened for the uniform distribution of weight, and it is even lower than that in the traditional case since the cost of cooperation is too expensive. However, other two types of weight distribution still greatly improve the cooperation level.

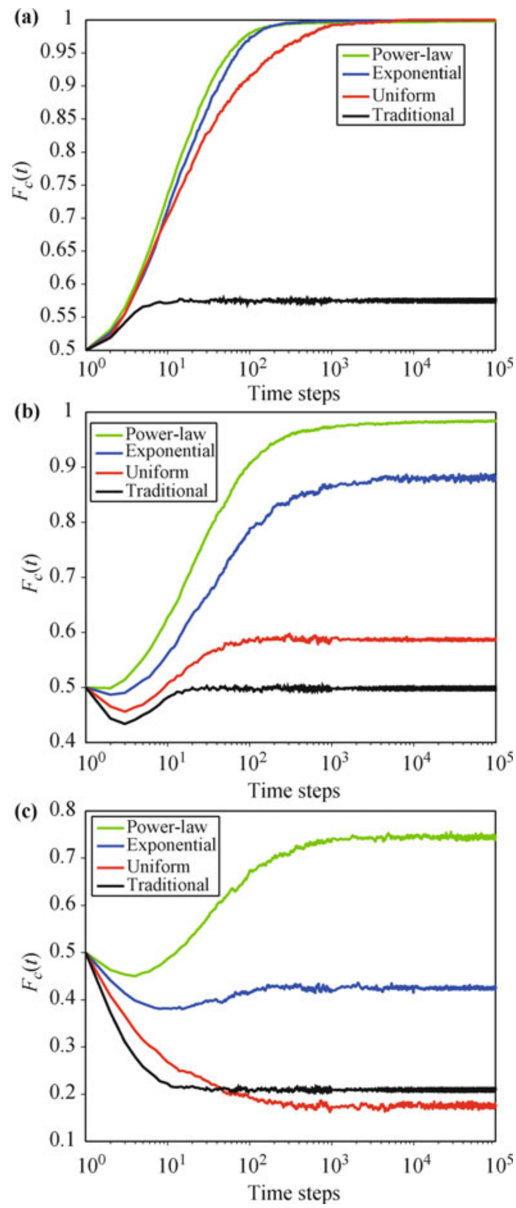

Fig. 1 The time evolution of the fraction of cooperators $F_c(t)$ for different ratios r. From subfigure **(a)** to **(c)**, the cost-to-benefit ratio r is 0.3, 0.45 and 0.625, respectively. Other parameters are set as $L = 200$, $A_\alpha = 1.0$ ($\alpha = 1$, 2 or 3) and $K = 0.1$ for three types of distributional schemes.

A quick look at Fig.1 can give us some hints, and it is the heterogeneity of the interaction strength among the agents that is responsible for the enhancement of cooperative behaviors among players, especially for the power-law edge distribution, because the heterogeneous interaction helps the players to form the boundary to resist the invasions of opposite strategies and induce the stable clusters of players.

At the basis of the above mentioned MC simulation, we take the average value of 10000 MCS after the evolution carries at a stationary state. Figure 2 shows the fraction of cooperators F_c of the steady state as a function of cost-to-benefit ratio r. For the range of $r < 0.3$, almost all the agents are apt to cooperate with each other except in the traditional case. When $r > 0.3$, the fraction of cooperators F_c starts to attenuate. However, in the intermediate range $0.3 < r < 0.6$, the heterogeneous edge weight distribution has often led to a distinct influence on the cooperation fraction and largely promotes the cooperative behaviors in the snowdrift game compared with the traditional case. When r is greater than 0.6, the cooperation for the uniform case is sharply reduced to a lower level than that in the traditional case, while for the other distributed edge weighting cases, it is still higher than that in the traditional case within a certain range, and ultimately no longer dominates over the game or dies out.

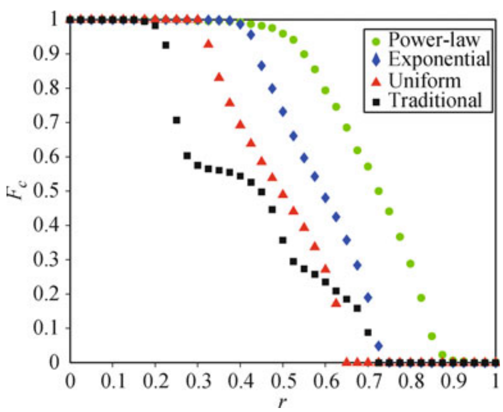

Fig. 2 The effect of cost-to-benefit r on the cooperation fraction F_c with the same parameters of $L = 200$, $A_\alpha = 1.0$ ($\alpha = 1$, 2 or 3) and $K = 0.1$ for distributional schemes.

In short, the lattice integrated with the edge weight promotes the cooperation among players for some certain ranges of r. To put it another way, the non-homogeneous properties of the relationship between two individuals (such as the interdependence) have a significant influence on the cooperative behaviors. Due to the strong interdependence between them, the cooperative clusters with the same strategy easily come into being, then counteract the impact of the clusters holding the opposite strategy. Therefore, the cooperation level in the edge weighting snowdrift game can be sustained, even higher than that in the traditional case in which the defection spreads quickly and the cooperation is extinct ultimately.

Next, let us see the weight distribution for three types of edge weighting schemes in order to interpret the reason for the promotion of cooperation in this novel game model. Figure 3 illustrates that the heterogeneity of weight distribution can be ranged as the power-law, exponential and uniform distribution by a decreasing order. Notably, the horizontal axis is a linear scale but the vertical axis is a logarithmic scale. What is more, the fraction of cooperators for $0.3 < r < 0.5$ in Fig. 2 is in the same order with the edge weighting distribution. Here, the uniform case with the lowest non-homogeneity cannot resist the temptation to defect and the cooperation level reduces quickly when $r > 0.3$. While the other two schemes with high heterogeneity can foster several clusters centered around some edges with higher weight, due to the stronger interaction between them, which plays a key role in restraining the propagation of defection strategy and forming the cooperative clusters. As a result, the cooperation level is higher than that in the uniform case.

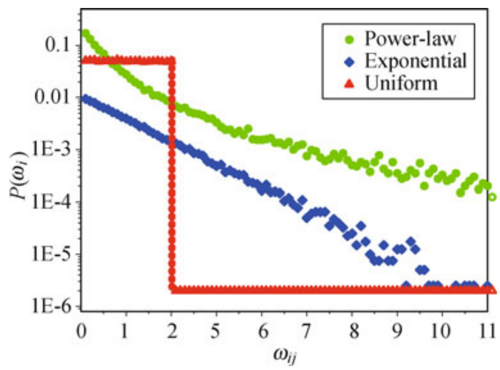

Fig. 3 A typical implementation of weight distribution between interactions among agents in which A_α (α=1, 2 or 3) is set to be 1.0 for three distributed schemes.

Figure 4 depicts the fraction of cooperators F_c as a function of the undulation amplitude of weight distribution A_α. The enhancement of cooperation exhibits a distinct phenomenon for different cost-to-benefit ratios. The cooperation level for the three types of weight distribution is advanced as the undulation amplitude A_α increases, and has a highest value for power-law distribution, which coincides with Fig. 3 when A_α is equal to 1. For a smaller ratio (e.g., $r = 0.3$), over half of the agents are inclined to cooperate and finally the cooperation dominates over the snowdrift game and saturates as A_α increases. For a middle ratio (e.g., $r = 0.45$), the fraction of cooperators decreases at first since the cooperation pays for a higher cost. As A_α continuously augments, the heterogeneity of weight distribution strengthens and the stronger interaction enhances the probability to cooperate. For the highest ratio (e.g., $r = 0.625$), the cooperation level for exponential and uniform distributions is not very high, and even slightly decreases especially for uniform case when A_α approaches the maximum value ($A_\alpha = 1$). In reality, increasing A_α means that

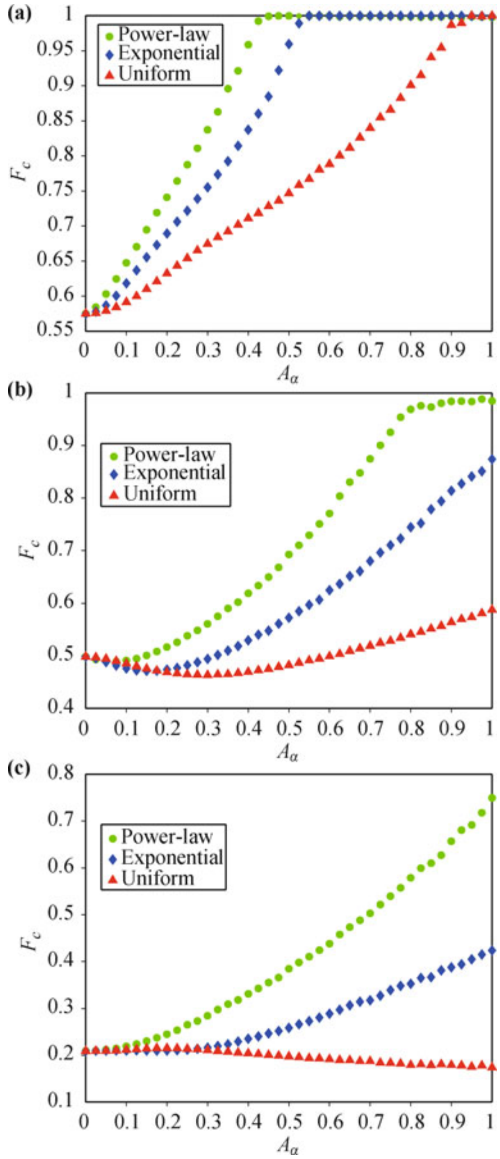

Fig. 4 The relationship between the undulation amplitude of weight distribution A_α and the fraction of cooperators F_c. From subfigure **(a)** to **(c)**, the cost-to-benefit r is 0.3, 0.45 and 0.625, respectively. And the noise strength K is set to be 0.1 and the lattice size is $L = 200$.

the undulation of edge weight can also be allowed to fluctuate acutely, and further result in the higher heterogeneity of weight distribution, which is beneficial to form the boundary of identical strategies to stop the exploitation of players. Therefore, the cooperation level is largely enhanced in the end as A_α increases for the more heterogeneous weight distribution, such as power-law and exponential distribution, which highlights the key role of much stronger links in the cooperation dynamics.

Finally, we investigate the influence of noise strength K on the fraction of cooperators. The results shown in Fig. 5 is completely diverse for the different cost-to-benefit ratios r. When the cost-to-benefit ratio is smaller (e.g., $r = 0.3$), almost all agents adopt the cooperative strategy, and the influence of K can be neglected. When the cost-to-benefit is in the middle range (e.g., $r = 0.45$), the cooperation is the massive behaviors among all agents for the power-law weight distribution, and the cooperation level advances as the noise strength K is aggrandized. But for the exponential and uniform edge weight distribution, the lowest cooperation level appears for a specific noise strength K_s, and the cooperation can be highly excited when K goes beyond K_s. In particular, when K is approximately greater than 70, nearly all the players cooperate with each other regardless of the types of weight distribution, hence the intermediate noise strength is enough for the cooperative behaviors among agents. On the contrary, the increase of noise strength K inhibits the cooperative behaviors among the players for the three types of weight distributions when the cooperation cost parameter r is larger (e.g., $r = 0.625$). Confronted with the severe noise, the cooperation is sharply reduced, and even dies out when K runs over 20. The numerical results reveal that too

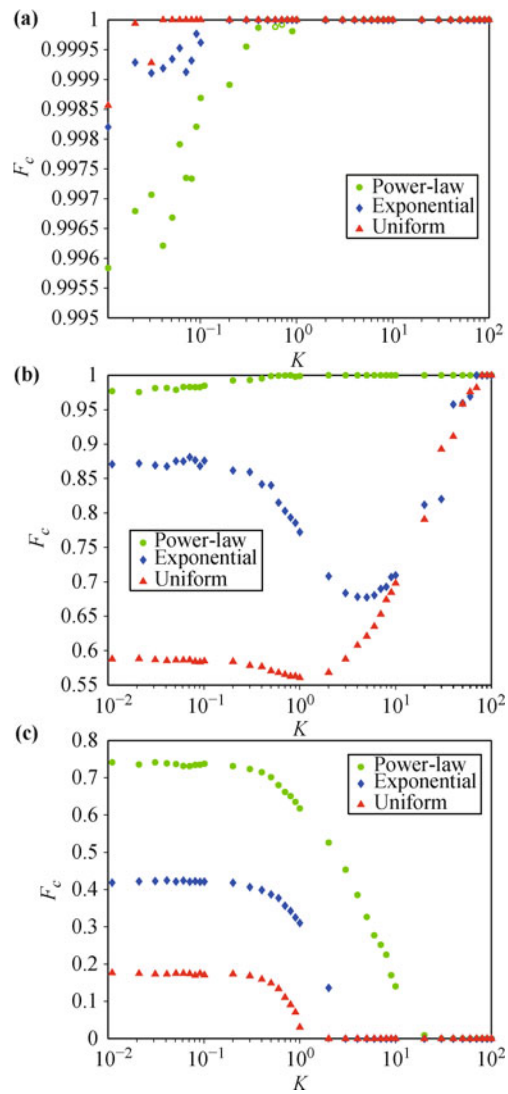

Fig. 5 The relationship between the fraction of cooperators F_c and the strength of noise K. From subfigure **(a)** to **(c)**, the cost-to-benefit r is 0.3, 0.45 and 0.625, respectively. And the amplitude of undulation A_α ($\alpha = 1$, 2 or 3) is 1.0 and the lattice size is $L = 200$.

much noise really goes against the collective behaviors of agents, to a great extent, and there exists an optimal noise strength to facilitate the cooperation among players on the weighted square lattice.

4 Conclusions and discussion

In summary, based on the traditional snowdrift game model on the square lattice, we introduce the edge weight distribution, which can characterize the heterogeneous and asymmetrical interaction or influence (such as the dependence relationship) between two agents, into the snowdrift game model. Generally speaking, the vertex weight indicates the status or role of individuals, while the edge weight portrays the strength of interaction or influence which one inflicts to another. In the numerical simulations, we assume that the interaction weight among players conforms to three typically distributed schemes including the uniform, exponential and power-law cases. Compared with the traditional snowdrift game, the non-homogeneous edge weight distribution greatly facilitates the cooperation for most ranges of cost-to-benefit ratio r. Furthermore, the relationship between the undulation amplitude of weight distribution A_α and the cooperator's fraction is also investigated, and the results show that the ultimate cooperation level advances as A_α varies, no matter r is smaller or larger, while in the uniform case it slightly decreases when r is larger. At last, we discuss how the noise strength K influences the cooperative behaviors among agents. There exists a nontrivial phenomenon that the cooperation is strengthened for a smaller r but sharply reduced for a larger r. What we have got at present will be conducive to further understanding and analyzing the emergence of cooperation which is prevalent in many social and biological systems. For example, how the cooperation evolves in the weighted complex networks is an open question to be resolved.

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