Front. Phys., 2012, 7(2): 239–243
DOI 10.1007/s11467-011-0158-3

RESEARCH ARTICLE

Diffraction of entangled photon pairs by ultrasonic waves

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Received November 3, 2010; accepted January 17, 2011

In this paper, we have presented and established a new theoretical formulation of photon optics based on photon path and Feynman path integral idea. We have used Feynman path integral approach to discuss Fraunhofer, Fresnel diffraction of single photon and entangled photon pairs by ultrasonic wave and obtained the following results: i) quantum state and probability distribution of single photon and entangled photon pairs by Fraunhofer and Fresnel ultrasonic diffraction, ii) oblique incidence Raman–Nath and Bragg diffraction conditions, iii) total correlation state and its probability distribution. Our calculation results are in agreement with the experiment results. Comparing one-photon and two-photon diffraction effects by ultrasonic waves, we have found that two-photon diffraction by ultrasonic waves is also a sub-wavelength diffraction.

Keywords atom optics, diffraction and interference, path integral

PACS numbers 42.25.Fx, 43.35.+d, 42.50.Ar

1 Introduction

In 1921, Brillouin predicted that light wave could be diffracted by sound wave [1] and Debye verified this experimentally (ultrasonic light diffraction) in 1932 [2]. After that, some authors had researched this phenomenon using classic theory (the differential equations method [3−8], integral equations method [9, 10]) and quantum theory [11]. Though from these theories they can obtain diffraction maximum relation between diffraction angle and wave length, the expression of diffraction fringe intensity is approximate, not compact [12, 13]. Till now, nobody has discussed ultrasonic diffraction of entangled photon pairs.

From Feynman path integral conception [14], we can obtain probability distribution expression of ultrasonic diffraction fringes not only for photon, but also for entangled photon pairs. Our theory is a quantum-mechanical theory, which is applying Feynman atom path integral diffraction theory to ultrasonic diffraction of photon [15, 16].

The diffraction of entangled photon pairs by ultrasonic waves can be applied to acoustic light modulators, scanners, deflectors, acoustic light waveguide modulators, distributed feedback lasers, and distributed Bragg-reflector lasers. Using diffraction effects of entangled photon pairs by ultrasonic waves, these acousto–optic devices can be modulated more accurately than current devices.

2 Feynman path integral expression of photon wave function

Although photon has not concentrated mass as particle, we can still use the concept of similar particles to describe its propagation and interaction with other matters, e.g., photoelectric effect, Compton effect, interaction between light and chemical molecules in photographic plate etc. Therefore, using the idea of Feynman path integral propagator, we can express a propagator by the action of photon from point S to P as [14]

$$U(P,S)=\sum_{\substack{\text{over all paths}\\ \text{from } S \text{ to } P}} \exp\left(\frac{\mathrm{i}}{\hbar}S(P,S)\right) \tag{1}$$

It shows that the propagator of a photon is probability amplitude superposition of all possible paths from point S to P. From the initial state to the end, it can be expressed as a propagator to

$$\Psi(\boldsymbol{r},T)=\int \mathrm{d}\boldsymbol{r}_c \int \mathrm{d}\boldsymbol{r}_S \exp\left\{\frac{\mathrm{i}}{\hbar}[S(P,C) + S(C,S)]\right\}\Psi(\boldsymbol{r}_S,0) \tag{2}$$

where $S(C,S)$and $S(P,C)$ are the actions of photon from point S to C and C to P, respectively. The possible paths are shown in Fig. 1. For single photons, S_1 coincides with S_2, that is, $S_1 = S_2 = S$ along x-axis.

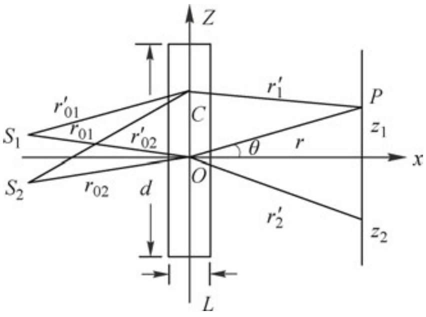

Fig. 1 Diffraction of entangled photon pairs by ultrasonic waves. S_1 and S_2 stand for two separated centers of wave packets, respectively. C is a point in ultrasonic wave propagating region along z axis. θ is the diffraction angle. P is a point on the detection plane.

For photons of frequency ω and wave vector k, its action is $S = \hbar(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)$. Formula (2) with the action is not a form of plane wave superposition, because the plane wave superposition is that with different wave vectors; it is not the superposition of secondary spherical wavelets at the wave-front in Kirchhoff diffraction theory, because these secondary spherical wavelets are from each element of other wave-front as the center of a secondary disturbance. Our superposition is that of probability amplitude for all possible paths of photon from the initial point to the end point. Here, the so-called photon path is indicated as a possible path in photon propagation, hence it is not the conception of classical light line.

3 Quantum state and probability distribution of photon by Fresnel ultrasonic diffraction

For a single photon, suppose that the initial state is

$$\Psi(\boldsymbol{r}_S, 0) = \delta(\boldsymbol{r}_S - \boldsymbol{r}_0) \tag{3}$$

Using Fresnel diffraction approximation,

$$r' = f + \frac{(y-y_c)^2 + (z-z_c)^2}{2f} \tag{4}$$

where f is a distance of the point 0 from the screen. When $\boldsymbol{r}_S$ is a distance far from the point 0, Eq. (2) is changed to

$$\begin{aligned}\Psi(\boldsymbol{r}, T) &= \int \mathrm{d}\boldsymbol{r}_c \exp[\mathrm{i}(\boldsymbol{k}'\cdot\boldsymbol{r}' + \boldsymbol{k}''\cdot\boldsymbol{L} - \boldsymbol{k}_0\cdot\boldsymbol{r}_0 - \omega T)] \\ &= \exp[\mathrm{i}(kf - \boldsymbol{k}_0\cdot\boldsymbol{r}_0 + kLn_0 - \omega T)] \\ &\quad \cdot \frac{(1+\mathrm{i})\lambda}{2} \sum_{m=-\infty}^{\infty} \mathrm{i}^m J_m(kL\Delta n)\cdot \exp\Big[\mathrm{i}\Big(\frac{m\kappa z}{\pi} \\ &\quad - \frac{\lambda f}{4}\frac{m^2\kappa^2}{\pi}\Big)\Big][F(\mu_2) - F(\mu_1)]\end{aligned} \tag{5}$$

where L is the width of the ultrasonic wave region. $\boldsymbol{k}'' \approx \boldsymbol{k}n$ is the wave vector of photon in a medium. $\boldsymbol{k}_0\cdot\boldsymbol{r}_c = 0$ indicates that the direction of wave vector $\boldsymbol{k}_0$ is vertical to $\boldsymbol{r}_c$. n is the refractive index of the medium. For ultrasonic wave propagation in the medium, the refractive index is changed to [12]

$$n = n_0 + \Delta n \cos(\kappa z_c) \tag{6}$$

where Δn is the maximum change of refractive index, κ is the wave vector of ultrasonic wave, subscript c is a point where a photon passes through ultrasonic wave region, z_c is the coordinate of the point.

$$\mu_1 = \sqrt{\frac{2}{\lambda f}\Big(z - \frac{d}{2}\Big)} - \sqrt{\frac{\lambda f}{2}}\frac{m\kappa}{\pi} \tag{7}$$

$$\mu_2 = \sqrt{\frac{2}{\lambda f}\Big(z + \frac{d}{2}\Big)} - \sqrt{\frac{\lambda f}{2}}\frac{m\kappa}{\pi} \tag{8}$$

$$F(\omega) = \int_0^{\omega} \mathrm{d}t \exp\Big(\frac{\mathrm{i}\pi t^2}{2}\Big) \tag{9}$$

The probability distribution of single photons by Fresnel ultrasonic diffraction is

$$\begin{aligned}|\Psi(\boldsymbol{r}, T)|^2 = \lambda^2\Big| &\sum_{m=-\infty}^{\infty} \mathrm{i}^m J_m(kL\Delta n) \exp\Big[\mathrm{i}\Big(\frac{m\kappa z}{\pi} \\ &- \frac{\lambda f}{4}\frac{m^2\kappa^2}{\pi}\Big)\Big][F(\mu_2) - F(\mu_1)]\Big|^2\end{aligned} \tag{10}$$

For the entangled photon pairs, Eq. (2) is changed to

$$\begin{aligned}\Psi(\boldsymbol{r}, T) = \int \mathrm{d}\boldsymbol{r}_c \int &\mathrm{d}\boldsymbol{r}_{S_1}\mathrm{d}\boldsymbol{r}_{S_2} \exp\Big\{\frac{\mathrm{i}}{\hbar}[S_1(P, C) \\ &+ S_2(P, C) + S_1(C, S_1) + S_2(C, S_2)]\Big\} \\ &\cdot \Psi(\boldsymbol{r}_{S_1}, \boldsymbol{r}_{S_2}, 0)\end{aligned} \tag{11}$$

Suppose the initial state is the entangled photon pairs with maximum entanglement

$$\begin{aligned}\Psi(\boldsymbol{r}_{S_1}, \boldsymbol{r}_{S_2}, 0) = \frac{1}{\sqrt{2}}[&\Psi_1(\boldsymbol{r}_{S_1}, 0)\,\Psi_2(\boldsymbol{r}_{S_2}, 0) \\ &+ \Psi_1(\boldsymbol{r}_{S_2}, 0)\,\Psi_2(\boldsymbol{r}_{S_1}, 0)]\end{aligned} \tag{12}$$

where $\Psi_1(\boldsymbol{r}_{S_1}, 0)$, $\Psi_2(\boldsymbol{r}_{S_2}, 0)$, $\Psi_1(\boldsymbol{r}_{S_2}, 0)$, and $\Psi_2(\boldsymbol{r}_{S_1}, 0)$ are wave functions of the initial time for photons 1 and 2, respectively. For the entangled photon pairs with separated centers of two wave packets [17], the initial state can be supposed as

$$\begin{aligned}\Psi(\boldsymbol{r}_{S_1}, \boldsymbol{r}_{S_2}, 0) = \frac{1}{\sqrt{2}}[&\delta_1(\boldsymbol{r}_{S_1} - \boldsymbol{r}_{0_1})\delta_2(\boldsymbol{r}_{S_2} - \boldsymbol{r}_{0_2}) \\ &+ \delta_1(\boldsymbol{r}_{S_2} - \boldsymbol{r}_{0_2})\delta_2(\boldsymbol{r}_{S_1} - \boldsymbol{r}_{0_1})]\end{aligned} \tag{13}$$

where $\delta_1(\boldsymbol{r}_{S_1} - \boldsymbol{r}_{0_1})$ represents the position $\boldsymbol{r}_{0_1}$ of photon 1 at $t = 0$, $\delta_2(\boldsymbol{r}_{S_2} - \boldsymbol{r}_{0_2})$ represents the position $\boldsymbol{r}_{0_2}$ of photon 2 at $t = 0$.

When $\boldsymbol{r}_{S_1}$ and $\boldsymbol{r}_{S_2}$ are distances far from the point 0, and for Fresnel diffraction, with radial probability into account, Eq. (11) is changed to

$$\begin{aligned}\Psi(\boldsymbol{r},T)=&\frac{2}{\sqrt{2}}\exp\{\mathrm{i}[(k_1+k_2)f-\boldsymbol{k}_{01}\cdot\boldsymbol{r}_{01}\\&-\boldsymbol{k}_{02}\cdot\boldsymbol{r}_{02}+(k_1+k_2)Ln_0-2\omega T]\}\frac{(1+\mathrm{i})\lambda_0}{2}\\&\cdot\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m[(k_1+k_2)L\Delta n]\\&\cdot\exp\left[\mathrm{i}\left(\frac{m\kappa z}{\pi}-\frac{\lambda_0 f}{4}\frac{m^2\kappa^2}{\pi}\right)\right][F(\nu_2)-F(\nu_1)]\end{aligned}\tag{14}$$

where $\lambda_0=\lambda/2$,

$$\nu_1=\sqrt{\frac{2}{\lambda_0 f}}(z-\frac{d}{2})-\sqrt{\frac{\lambda_0 f}{2}}\frac{m\kappa}{\pi}\tag{15}$$

$$\nu_2=\sqrt{\frac{2}{\lambda_0 f}}(z+\frac{d}{2})-\sqrt{\frac{\lambda_0 f}{2}}\frac{m\kappa}{\pi}\tag{16}$$

The probability distribution of entangled photon pairs by Fresnel ultrasonic diffraction is

$$\begin{aligned}|\Psi(\boldsymbol{r},T)|^2=2\lambda_0^2\Big|&\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m[(k_1+k_2)L\Delta n]\\&\cdot\exp\left[\mathrm{i}\left(\frac{m\kappa z}{\pi}-\frac{\lambda_0 f}{4}\frac{m^2\kappa^2}{\pi}\right)\right][F(\nu_2)-F(\nu_1)]\Big|^2\end{aligned}\tag{17}$$

4 Quantum state and probability distribution of photon by Fraunhofer ultrasonic diffraction

For a single photon, using Fraunhofer diffraction approximations,

$$r'=f-z_c\sin\theta\tag{18}$$

where θ is the diffraction angle.

When $\boldsymbol{r}_S$ is a distance far from the point 0 and the screen is also far from it, the first equation of Eq. (5) is changed to

$$\begin{aligned}\Psi(\boldsymbol{r},T)=&d\exp\{\mathrm{i}[\boldsymbol{k}\cdot\boldsymbol{r}-\boldsymbol{k}_0\cdot\boldsymbol{r}_0-\omega T+kn_0L]\}\\&\cdot\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m(\boldsymbol{k}\cdot\boldsymbol{L}\Delta n)\frac{\sin\left[(k\sin\theta-m\kappa)\dfrac{d}{2}\right]}{(k\sin\theta-m\kappa)\dfrac{d}{2}}\end{aligned}\tag{19}$$

Its probability distribution is

$$\begin{aligned}|\Psi(\boldsymbol{r},T)|^2=d^2\Big|&\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m(\frac{2\pi L\Delta n}{\lambda})\\&\cdot\frac{\sin\left[\left(\dfrac{\Lambda}{\lambda}\sin\theta-m\right)\dfrac{\pi d}{\Lambda}\right]}{\left(\dfrac{\Lambda}{\lambda}\sin\theta-m\right)\dfrac{\pi d}{\Lambda}}\Big|^2\end{aligned}\tag{20}$$

For the entangled photon pairs, when $\boldsymbol{r}_{S_1}$ and $\boldsymbol{r}_{S_2}$ are distances far from the point 0, and the screen is also far from it, we may take $\boldsymbol{k}'_1\approx\boldsymbol{k}_1$, $\boldsymbol{k}''_1\approx\boldsymbol{k}_1n$, $\boldsymbol{k}'_{01}\approx\boldsymbol{k}_{01}$, $\boldsymbol{k}'_{01}\cdot\boldsymbol{r}_c\approx\boldsymbol{k}_{01}\cdot\boldsymbol{r}_c\approx 0$, $\boldsymbol{k}'_2\approx\boldsymbol{k}_2$, $\boldsymbol{k}''_2\approx\boldsymbol{k}_2n$, $\boldsymbol{k}'_{02}\approx\boldsymbol{k}_{02}$, $\boldsymbol{k}'_{02}\cdot\boldsymbol{r}_c\approx\boldsymbol{k}_{02}\cdot\boldsymbol{r}_c\approx 0$, where $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are wave vectors of the first and second photon in the air, respectively. $\boldsymbol{k}''_1$ and $\boldsymbol{k}''_2$ are wave vectors of the first and the second photon in the medium, respectively. $\boldsymbol{k}_{01}\cdot\boldsymbol{r}_c\approx\boldsymbol{k}_{02}\cdot\boldsymbol{r}_c\approx 0$ indicates that the direction of wave vectors is vertical to propagation direction. Eq. (11) is changed to

$$\begin{aligned}\Psi(\boldsymbol{r},T)=&\frac{2}{\sqrt{2}}\exp\{\mathrm{i}[(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{r}-\boldsymbol{k}_{01}\cdot\boldsymbol{r}_{01}\\&-\boldsymbol{k}_{02}\cdot\boldsymbol{r}_{02}-2\omega T+(k_1+k_2)n_0L]\}\\&\cdot\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m[(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{L}\Delta n]\\&\cdot\int_{-d/2}^{d/2}\mathrm{d}z_c\exp\{-\mathrm{i}[(k_1+k_2)\sin\theta-m\kappa]z_c\}\end{aligned}\tag{21}$$

When $m\neq(k_1+k_2)\sin\theta/\kappa$, the ultrasonic diffraction state of entangled photon pairs is

$$\begin{aligned}\Psi(\boldsymbol{r},T)=&\frac{2d}{\sqrt{2}}\exp\{\mathrm{i}[(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{r}-\boldsymbol{k}_{01}\cdot\boldsymbol{r}_{01}\\&-\boldsymbol{k}_{02}\cdot\boldsymbol{r}_{02}-2\omega T+(k_1+k_2)n_0L]\}\\&\cdot\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m[(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{L}\Delta n]\\&\cdot\frac{\sin\left\{[(k_1+k_2)\sin\theta-m\kappa]\dfrac{d}{2}\right\}}{[(k_1+k_2)\sin\theta-m\kappa]\dfrac{d}{2}}\end{aligned}\tag{22}$$

For degenerate entangled photon pairs, $k_1=k_2=k$, $k=\frac{2\pi}{\lambda}$, $\kappa=\frac{2\pi}{\Lambda}$, where λ is the wave length of the light wave, Λ is the wave length of the ultrasonic wave, so the probability distribution of degenerate entangled photon pairs by ultrasonic diffraction is

$$\begin{aligned}|\Psi(\boldsymbol{r},T)|^2=2d^2\Big|&\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m(\frac{4\pi L\Delta n}{\lambda})\\&\cdot\frac{\sin\left[\left(\dfrac{2\Lambda}{\lambda}\sin\theta-m\right)\dfrac{\pi d}{\Lambda}\right]}{\left(\dfrac{2\Lambda}{\lambda}\sin\theta-m\right)\dfrac{\pi d}{\Lambda}}\Big|^2\end{aligned}\tag{23}$$

Comparing the probability distribution of degenerate entangled photon pairs with that of single photon, that is, Eqs. (20) and (23), we can see the diffraction wave length of degenerate entangled photon pairs is half of that of a single photon. It is the sub-wavelength diffraction.

Our calculation result of the probability distribution for single photon ultrasonic diffraction state is in agreement with the experimental results [18]. As shown in Fig. 2, the dashed line indicates the experimental result and the black solid line indicates the calculation result, $\alpha=\frac{2\pi}{\lambda}L\Delta n$, $\beta=\frac{d}{\Lambda}$, $x=\frac{\Lambda}{\lambda}\sin\theta$.

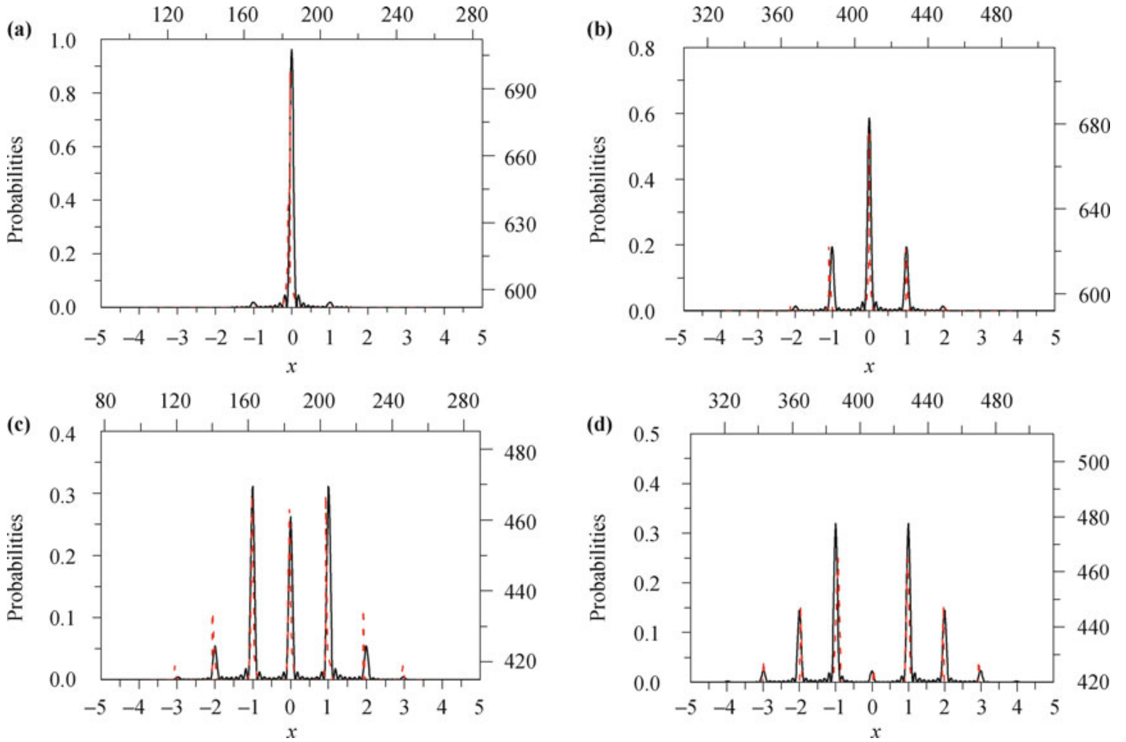

Fig. 2 The probability distribution for single photon ultrasonic diffraction corresponding to the following data: **(a)** $\alpha = 0.28, \beta = 8$; **(b)** $\alpha = 1.0, \beta = 8$; **(c)** $\alpha = 1.5, \beta = 8$; **(d)** $\alpha = 2.13, \beta = 8$. The upper and right scales correspond to the dashed experimental curves. The downword and left scales correspond to the solid theoretical curves.

5 Raman–Nath and Bragg diffraction conditions

For the oblique angle of incidence ϕ, $\sin\theta \to \sin\theta - \sin\phi$ in Eq. (21), Eq. (21) is changed to

$$|\Psi(\boldsymbol{r},T)|^2 = 2d^2\Big|\sum_{m=-\infty}^{\infty} \mathrm{i}^m J_m\Big(\frac{4\pi L\Delta n}{\lambda}\Big) \cdot \frac{\sin\left\{\left[\dfrac{2\Lambda}{\lambda}(\sin\theta-\sin\phi)-m\right]\dfrac{\pi d}{\Lambda}\right\}}{\left[\dfrac{2\Lambda}{\lambda}(\sin\theta-\sin\phi)-m\right]\dfrac{\pi d}{\Lambda}}\Big|^2 \tag{24}$$

The condition of the oblique incidence diffraction maximum for entangled photon pairs, namely, Raman-Nath diffraction condition, is

$$\Lambda(\sin\theta - \sin\phi) = m\frac{\lambda}{2}, \quad m = 0, \pm1, \pm2, \cdots \tag{25}$$

Therefore, we can get Bragg diffraction condition:

$$\Lambda\sin\theta = \frac{\lambda}{4} \tag{26}$$

The probability distribution of ultrasonic diffraction state for the oblique incidence single photon is

$$|\Psi(\boldsymbol{r},T)|^2 = d^2\Big|\sum_{m=-\infty}^{\infty} \mathrm{i}^m J_m\Big(\frac{2\pi L\Delta n}{\lambda}\Big) \cdot \frac{\sin\left\{\left[\dfrac{\Lambda}{\lambda}(\sin-\sin\phi)-m\right]\dfrac{\pi d}{\Lambda}\right\}}{\left[\dfrac{\Lambda}{\lambda}(\sin-\sin\phi)-m\right]\dfrac{\pi d}{\Lambda}}\Big|^2 \tag{27}$$

Raman–Nath diffraction condition for the oblique incidence single photon is

$$\Lambda(\sin\theta - \sin\phi) = m\lambda \tag{28}$$

Therefore, the corresponding Bragg diffraction condition is

$$\Lambda\sin\theta = \frac{\lambda}{2} \tag{29}$$

Eqs. (28) and (29) are consistent with the results in classical wave optics [12, 13].

6 Correlation state of entangled photon pairs

For Fraunhofer diffraction of the entangled photon pairs, from Fig. 1 and initial condition Eq. (13), the first photon starts from S_1, goes through point C and ends at z_1; the second photon starts from S_2, goes through point C and ends at z_2; the first photon starts from S_1, goes through point C and ends at z_2; the second photon starts from S_2, goes through point C and ends at z_1. The correlation state of them is

$$\Psi(z_1, z_2, T) = \frac{2}{\sqrt{2}}\exp\{\mathrm{i}[\boldsymbol{k}_1\cdot\boldsymbol{r}_1 + \boldsymbol{k}_2\cdot\boldsymbol{r}_2$$

$$-\boldsymbol{k}_{01}\cdot\boldsymbol{r}_{01}-\boldsymbol{k}_{02}\cdot\boldsymbol{r}_{02}+(k_1+k_2)n_0L-2\omega T]\}$$
$$\cdot\int_{-d/2}^{d/2}\mathrm{d}z_c\exp\{-\mathrm{i}[(k_1\sin\theta_1+k_2\sin\theta_2)z_c$$
$$-(k_1+k_2)L\Delta n\cos(\kappa z_c)]\}\tag{30}$$

that is, the correlation state of entangled photon pairs is

$$\begin{aligned}\Psi(z_1,z_2,T)=&\frac{2}{\sqrt{2}}\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}_1+\boldsymbol{k}_2\cdot\boldsymbol{r}_2\\&-\boldsymbol{k}_{01}\cdot\boldsymbol{r}_{01}-\boldsymbol{k}_{02}\cdot\boldsymbol{r}_{02}+2kn_0L-2\omega T)]\\&\cdot\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m(2kL\Delta n)\\&\cdot\frac{d\sin\left\{\left[\dfrac{\varLambda(z_1+z_2)}{\lambda f}-m\right]\dfrac{\pi d}{\varLambda}\right\}}{\left[\dfrac{\varLambda(z_1+z_2)}{\lambda f}-m\right]\dfrac{\pi d}{\varLambda}}\end{aligned}\tag{31}$$

The probability distribution of the correlation state for the entangled photon pairs is

$$\begin{aligned}|\Psi(z_1,z_2,T)|^2=&2d^2\Bigg|\sum_{m=-\infty}^{\infty}\mathrm{i}^mJ_m(2kL\Delta n)\\&\cdot\frac{d\sin\left\{\left[\dfrac{\varLambda(z_1+z_2)}{\lambda f}-m\right]\dfrac{\pi d}{\varLambda}\right\}}{\left[\dfrac{\varLambda(z_1+z_2)}{\lambda f}-m\right]\dfrac{\pi d}{\varLambda}}\Bigg|^2\end{aligned}\tag{32}$$

When $z_1=z_2=z$, Eq. (32) becomes Eq. (23), which is the probability distribution of degenerate entangled photon pairs by the ultrasonic diffraction. When $z_1\neq z_2$, Eq. (32) is the probability distribution of the correlation state for entangled photon pairs, so-called “ghost interference”.

7 Conclusions

In this paper, we have used Feynman path integral idea to discuss Fraunhofer, Fresnel ultrasonic diffraction of entangled photon pairs and have obtained the following results: i) quantum state and probability distribution of entangled photon pairs by the Fraunhofer and Fresnel ultrasonic diffraction, ii) the oblique incidence Raman-Nath and Bragg diffraction conditions, and iii) the total correlation state and its probability distribution.

We have obtained the quantum state and probability distribution of single photon ultrasonic diffraction. Our calculation results are in agreement with the experiment results.

We have presented and established a new theoretical formulation of photon optics based on the photon path and Feynman path integral idea, and have given explicitly the wave function expression of the quantum state, correlation state and its probability distribution for single photon and entangled photon pairs passing through different configurations.

Acknowledgements The author was grateful to anonymous referees for helpful suggestions with regards to the theory statement.

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