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RESEARCH ARTICLE

Remote acoustic cloaks

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Due to the correspondence of the acoustic equations to Maxwell's equations of one polarization in two dimensions, we exploit theoretically the acoustic counterpart of the recently proposed remote invisibility cloak. The cloak consists of a circular cylindrical core with designed bulk moduli, and an "anti-object" embedded inside a shell with anisotropic mass densities. The material parameters of the cloaking shells are obtained by using the coordinate transformation method. The essence of the new design of cloaks relies on the ability that the cloaked object is no longer deafened by the cloaking shell, which is verified by both the far-field and near-field full-wave finite-element simulations in two dimensions.

Keywords cloak, acoustics, finite-element simulations

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1 Introduction

In recent years, invisibility cloaks have aroused great interest both theoretically and experimentally [1–6]. Tremendous success has been obtained in this field by combining transformation optics and complementary media, and using metamaterials with specified permittivities and permeabilities. At the same time, a trend in metamaterial-based studies is to generalize the electromagnetic (EM) devices to other fields, such as acoustics [7–15] and thermodynamics [16], in order to achieve new applications. Acoustic cloaks were proposed, numerically verified, and even experimentally realized in both the two-dimensional (2D) and three-dimensional cases. But one non-ignorable downside of the traditional cloak design is that the object hidden inside the cloaked domain has to be "blind" or "deafened", since no incident waves can penetrate the cloaking shell surrounding the cloaked object. Very recently, Lai *et al.* proposed a new recipe of invisibility cloak, the remote cloak, which can overcome this shortcoming [17]. In their remote cloak, the concepts of complementary media and transformation optics are used together to obtain the design of invisibility cloak which employs an "anti-object" embedded in a negative-index shell and makes another object that lies outside the cloaking shell invisible. With the successful precedent of the EM cloak proposed by them [17–19], a natural, but potentially important, question would be the feasibility of acoustic analogue of this design. If successful, they may shed new light on important applications like submarines' stealth. In this work, we design such a 2D acoustic cloak that can achieve the same functionalities, namely, cloaking an object outside the cloak, thus called *remote acoustic cloaks*. Their performance will be justified by both the far-field and near-field full-wave finite-element simulations by using model plane and cylindrical waves, respectively.

2 Theory

The theoretical feasibility of 2D acoustic cloaks owns to the invariance property of the 2D acoustic equation in the time harmonic form:

$$\nabla \cdot \left[\frac{1}{\rho(x,y)} \nabla p(x,y) \right] + \frac{\omega^2}{\lambda(x,y)} p(x,y) = 0 \qquad (1)$$

where $\rho(x,y)$ and $\lambda(x,y)$ are, respectively, the spatially varying mass density and bulk modulus, and $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$. Here, $p(x,y)$ and ω are the pressure field at position (x,y) and the angular frequency of an incident acoustic wave, respectively. This invariance property enables the functionality of the coordinate transformation method in the design of 2D acoustic cloaks, in a quite similar procedure as in transformation optics. Since the complementary media can be regarded as a special kind of transformation media, a full spectrum of complementary-media-based EM devices like superscatterers [20] and invisibility cloaks mentioned above can be generalized to the acoustic cases. Furthermore, Cummer and Schurig demonstrated that in a 2D geometry, the acoustic equations in a fluid are identical to the Maxwell equations of a particular polarization via a variable exchange that also preserves boundary conditions. In principle, any 2D transformation-media-based EM devices can be implemented in an acoustic version due to such identicalness. In the following, we shall demonstrate how to transform the remote cloak design by Lai *et al.* [17] into the corresponding acoustic one.

The key idea of our design is borrowed from the invisibility cloak design by Lai *et al.* [17] The cloaked object can have arbitrary shape, but the cloak should be correspondingly designed to ensure the cloaking effect. The procedure can be described in two steps. First, the object as well as the surrounding space is acoustically canceled by using a complementary media layer with an embedded complementary "image" of the object, namely, an "anti-object". Then, the correct acoustic path in the canceled space is restored by a core with a designed bulk modulus. As a result, the whole system is effectively equal to a piece of space, which is filled with the homogeneous environment media outside the system. In this sense, the desired cloaking effect is just realized.

Like the case of the permittivity and permeability in the invisibility cloak, the anisotropic mass density and the bulk modulus are the material parameters that we should manipulate to enable the remote cloaking effect. Cummer and Schurig have presented the general transformation equations for the mass density tensor ρ' and the bulk modulus λ' in the transformed space (x', y', z') under the 2D coordinate transformation, namely, transformation of a space into another space with different shape and size,

$$\rho_{i'j'} = \det(\Lambda)^{-1}(\epsilon_{i'k'z}\Lambda_k^{k'}\epsilon_{kiz})(\epsilon_{j'l'z}\Lambda_l^{l'}\epsilon_{ljz})\rho_{ij} \tag{2}$$

$$\lambda' = \det(\Lambda)(\Lambda_z^{z'})^{-2}\lambda \tag{3}$$

where ρ_{ij} and λ are, respectively, the anisotropic mass density and the bulk modulus in the original space (x, y, z), and Λ is the Jacobian transformation tensor with components $\Lambda_i^{i'} = \partial x^{i'}/\partial x^i$. Here, the subscript $i = x, y$, and z, and $i' = x', y'$, and z'. The transformation media exhibit two important properties which enable the design of a kind of complementary media using a special kind of coordinate transformation, i.e., folding a piece of space into another: first, the wave propagation path in the transformation media is identical as that in the original space; second, the transformation media are reflectionless if the coordinate transformation preserves the outer boundary condition. The original and the folded spaces give an exact canceling path when an acoustic wave travels across them.

Such a mechanism is illustrated in Fig. 1(a). We choose circular cylinders as our geometrical model for simplicity and clarity. In Fig. 1(a), a circular layer of acoustic complementary media with material parameters ρ' and λ' is presented. From the perspective of acoustic waves, this complementary media layer cancels an outer circular layer of homogeneous media whose material parameters are chosen as units throughout this work. The complementary media can be obtained by a coordinate transformation of folding the layer of homogeneous media ($R_2 < r < R_3$) into the layer of complementary media ($R_1 < r' < R_2$). Here, R_1, R_2 and R_3 are the core radius, the outer radius of complementary layer, and the outer radius of the canceled layer of homogeneous media, respectively; see Fig. 1(a). Let us consider a simple linear coordinate transformation

$$r' = -\frac{R_2 - R_1}{R_3 - R_2}r + \frac{R_3 - R_1}{R_3 - R_2}R_2 \tag{4}$$

$$\theta' = \theta \tag{5}$$

$$z' = z \tag{6}$$

By using Eqs. (2) and (3), straightforward calculations give the relative mass density tensor ρ' and the relative bulk modulus λ' of the complementary layer

$$\rho_{x'x'} = -\frac{(R_3 - R_2)^2 r'^2 \cos^2\theta' + \Delta^2 \sin^2\theta'}{(R_3 - R_2)r'\Delta} \tag{7}$$

$$\rho_{x'y'} = \rho_{y'x'} = -\frac{R_2(R_3 - R_1)\sin\theta'\cos\theta'[r'(R_3 - R_2) - \Delta]}{(R_3 - R_2)r'\Delta} \tag{8}$$

$$\rho_{y'y'} = -\frac{(R_3 - R_2)^2 r'^2 \sin^2\theta' + \Delta^2 \cos^2\theta'}{(R_3 - R_2)r'\Delta} \tag{9}$$

$$\lambda' = -\frac{(R_2 - R_1)^2 r'}{(R_3 - R_2)\Delta} \tag{10}$$

where

$$\Delta = R_2(R_3 - R_1) - (R_3 - R_2)r' \tag{11}$$

If we enforce the interface between the core and the complementary layer satisfying the scattering boundary condition, like sound-soft boundary ($p = 0$), the system becomes a new device that behaves as a scatterer bigger than its physical size and with an effective scattering boundary at radius R_3. We name it as "acoustic superscatterers" after its EM counterparts [20]. Then, in order to restore the original wave traveling path, we need to compress a large circle of homogeneous media with radius R_3 into the core region with radius R_1. Correspondingly, the relative bulk modulus of the core materials would be R_1^2/R_3^2 while the mass density is kept unchanged, which are deduced by coordinate transformation of $r' = R_1 r/R_3$. With such a choice of the material parameters of the core region, the acoustic wave experiences the same path as that in an circle of environmental homogeneous media with radius R_3. In other words, the whole system, including the outer homogeneous media layer, the complementary media layer and the core region, is equal to a circle of homogeneous media with radius R_3, from the perspective of acoustic waves. Thus, the acoustic cloak manifests its effect regardless of any form of external sound sources.

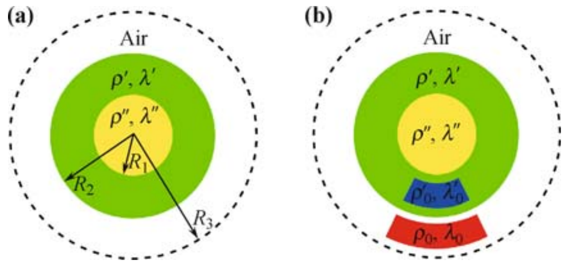

Fig. 1 **(a)** A system composed of a circular layer of air ($R_2 < r < R_3$), a circular layer of complementary media of ρ' and $\lambda'(R_1 < r' < R_2)$ and a core material of ρ'' and $\lambda''(r'' < R_1)$, which is acoustically equal to a large circle of air ($r < R_3$). **(b)** A scheme of cloaking an object of ρ_0 and λ_0 by placing a complementary "image" object ("anti-object") of ρ'_0 and λ'_0 in the complementary media layer of ($R_1 < r' < R_2$).

Now, we move to the next step. Suppose an object with relative mass density ρ_0 and relative bulk modulus λ_0 is placed in the outer circular layer of homogeneous media [Fig. 1(b)], replacing the otherwise homogeneous media occupying its space, and our purpose is to render it undetectable to any sound sources. Then, as the transformation media theory indicates, if we introduce a complementary "image" object ("anti-object") with material parameters

$$\rho_{0i'j'} = \det(\Lambda)^{-1}(\epsilon_{i'k'z}\Lambda_k^{k'}\epsilon_{kiz})(\epsilon_{j'l'z}\Lambda_l^{l'}\epsilon_{ljz})\rho_{0ij} \quad (12)$$

$$\lambda'_0 = \det(\Lambda)(\Lambda_z^{z'})^{-2}\lambda_0 \quad (13)$$

we can expect that the object of ρ_0 and λ_0 is canceled from the perspective of acoustic waves; see Fig. 1(b). The whole system then scatters no wave. This design of acoustic cloaks gives us a new approach to conceal an object from acoustic waves within a certain distance outside the cloak.

3 Simulations

Now, the performance of our design of acoustic cloaks is demonstrated by full wave simulation using the commercial finite-element simulation package COMSOL Multiphysics 3.5. Hereafter, we choose air as the environment media surrounding the devices (Fig. 1), and its material parameters as units for the mass density and the bulk modulus. All of our simulations use acoustic waves of frequency 4000 Hz without sacrificing the generality of our results. First, we give the simulation result of the scheme described in Fig. 1(a), for $R_1 = 0.1$ m, $R_2 = 0.2$ m, and $R_3 = 0.3$ m. Then we can write the transformation mapping Eq. (4) into $r' = 0.4 - r$, with material parameters of complementary media obtained by using Eqs. (7)–(10). In Fig. 2(a), the simulated distribution of pressure fields is presented. The absence of scattered waves justifies the cloaking function of our design. Next, we demonstrate the scheme shown in Fig. 1(b), the cloaking of an external object by complementary media. A dense curved sheet of thickness 0.06 m with relative parameters $\rho_0 = 5$ and $\lambda_0 = 1$ to be cloaked is placed between the circles of $r = 0.22$ m and $r = 0.28$ m. In Fig. 2(b), the scattering pattern of such a dense curved sheet is shown. In order to achieve the acoustic cloaking effect, we change the complementary media layer in Fig. 2(a) to include a complementary "image" object, i.e., the "anti-object", with parameters obtained by using Eqs. (7)–(10), positioned between the circles of $r' = 0.12$ m and $r' = 0.18$ m. The complete cloak consists of the modified complementary layer embedded with the "anti-object" and a core region. The simulated pressure-field distribution is shown in Fig. 2(c), which definitely demonstrates the "external" cloaking effect. The regions where the value of the amplitude of pressure fields exceeds the bounds of bars appear to be white. It should be noted that the cloaking effect does not exist in the space closely adjacent to the cloaked object since the cloaking effect originates from the embedded anti-object's canceling function. We stress that there are no geometrical or material parameter constraints on the object to be cloaked, as long as it can be placed into the region between $R_2 < r < R_3$. Next, we will demonstrate the cloaking effect under a cylindrical wave source located at (-0.4 m, 0 m) with amplitude 100 Pa. The configuration of this system appears to be more complicated, with two cloaked curved sheets with different material parameters included. The sheet on the downside has the identical configuration as that in Fig. 2(b) and (c). The sheet on the upper-right side bounded between 0.23 m$< r <$ 0.25 m has a bulk modulus of $\lambda_0 = 16$. The corresponding two "anti-objects" with material parameters obtained by using Eqs. (7)–(10) are embedded at the "image" positions in the complementary media shell. Again, perfect cloaking effect is demonstrated.

We will demonstrate the function of our design with another example. Consider a shell of $\rho_0 = 3$ and $\lambda_0 = 4$ bounded between 0.265 m$< r <$ 0.285 m, as shown in Fig. 3(a), which also shows its scattering pattern under an incident plane wave. In this case, the "anti-object" is an "image" shell positioned between 0.115 m$< r' <$ 0.135 m inside the complementary shell. In Fig. 3(b), we show the simulated pressure field distribution when the shell of 0.265 m$< r <$ 0.285 m is "cloaked" by a shell composed of a complementary media 0.1 m$< r' <$ 0.2 m with "anti-object" and a core material $r'' <$ 0.1 m. It should be again noted that the cloaked shell is outside the cloak. The figure clearly demonstrates the cloaking

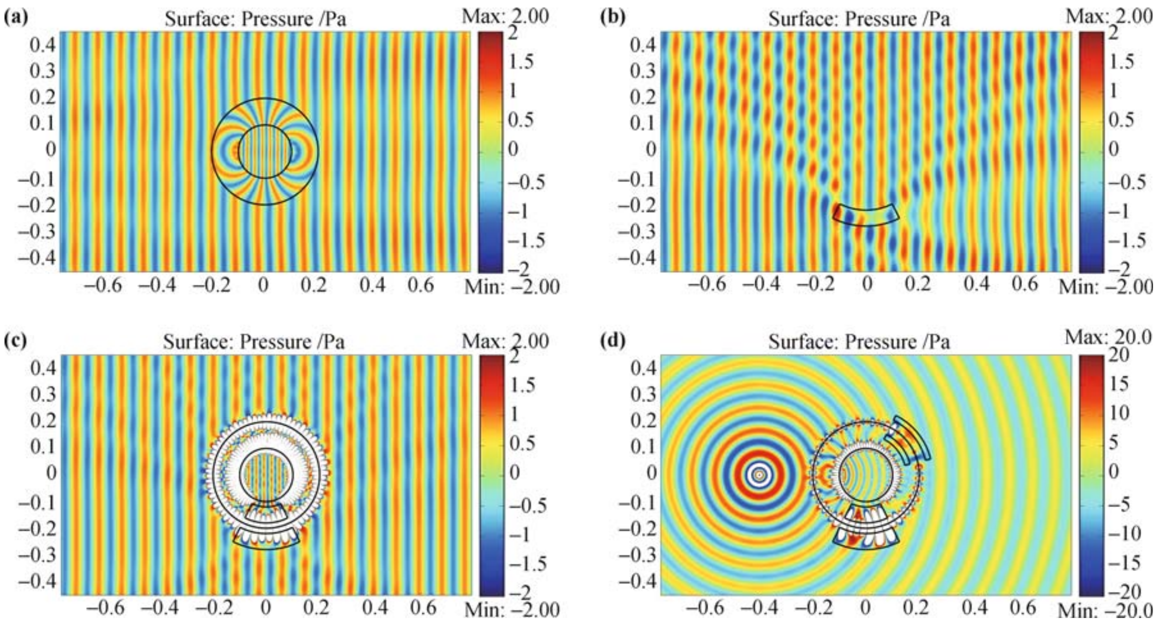

Fig. 2 Snapshots of the pressure fields under an incident acoustic (a)–(c) plane wave with amplitude 1 Pa and (d) cylindrical wave with amplitude 100 Pa, whose source is located at (−0.4 m, 0 m). **(a)** A circular complementary media layer of 0.1 m< $r' < 0.2$ m and a core material of $r'' < 0.1$ m. **(b)** A curved sheet of thickness 0.06 m, with relative mass density $\rho_0 = 5$ and relative bulk modulus $\lambda_0 = 1$. **(c)** The curved sheet in (b) is cloaked by an acoustic cloak composed of a circular layer of complementary media with an embedded "anti-object" and a core material. **(d)** A curved sheet on the downside with $\rho_0 = 5$ and $\lambda_0 = 1$, and another curved sheet on upper-right side with $\rho_0 = 1$ and $\lambda_0 = 16$, are both cloaked by an acoustic cloak with two corresponding "anti-objects" embedded in the complementary media layer.

effect. In Fig. 3(c) and (d), we demonstrate the case of identical geometric and material configuration as that of Fig. 3(a) and (b) under an incident cylindrical wave. The source is located at (−0.8 m, 0 m) with amplitude 100 Pa. Comparing the pattern of pressure fields with almost no scattered wave in Fig. 3(d) to that of Fig. 3(c), we are fully convinced about the cloaking effect of the whole system.

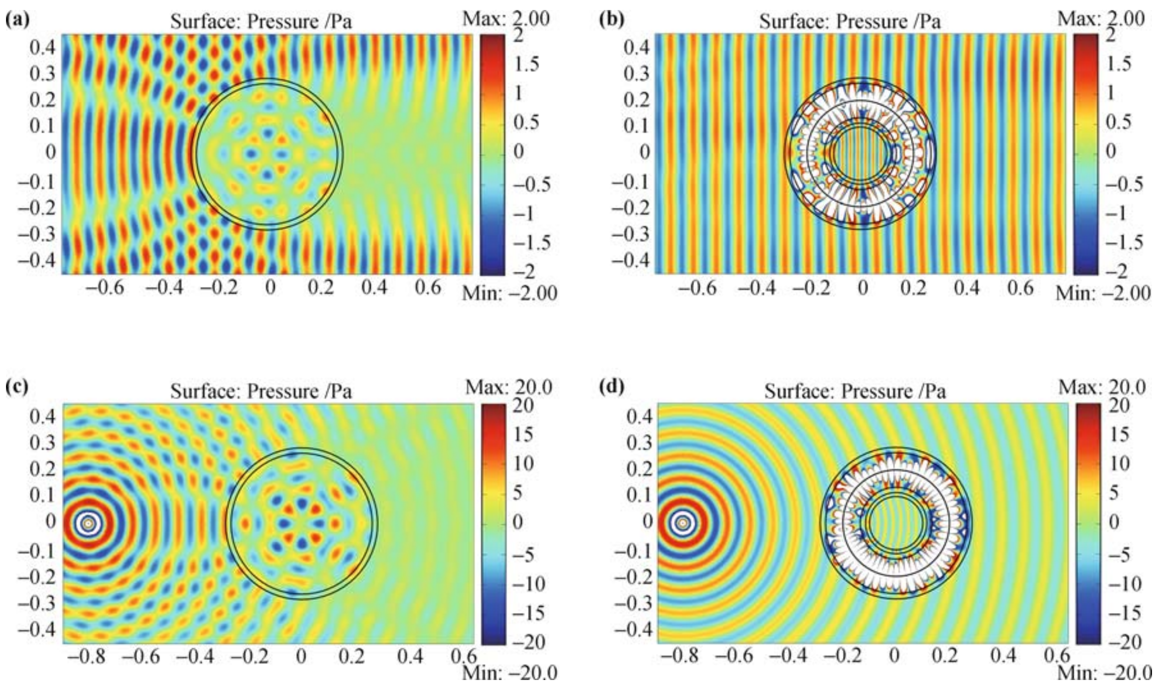

Fig. 3 Snapshots of the pressure fields under (a), (b) an incident plane acoustic wave with amplitude 1 Pa and (c), (d) a cylindrical wave with amplitude 100 Pa, whose source is located at (−0.8 m, 0 m). **(a)** A circular shell with relative mass density $\rho_0 = 3$ and relative bulk modulus $\lambda_0 = 4$. **(b)** The circular shell in (a) is cloaked by an acoustic cloak composed of a circular layer of complementary media with an embedded "anti-object" shell and a core material inside the complementary media layer. **(c)** and **(d)** are the same as (a) and (b), respectively, but for a cylindrical wave.

4 Discussion and conclusions

Here, some comments are in order. In the figures, the region with pressure exceeding the color bar is shown in white, and it comes to appear due to the complex metamaterial in use. The maximum value in this region depends on the mesh size adopted in the simulation. On the other hand, the scattering effect, if it is non-zero, e.g., in Fig. 2(c), is solely due to the numerical error arising in the simulations with specific meshes. This is because we have observed the definite convergence to the perfect cloaking effect when the finite-element mesh gets finer and finer. In other words, the present quality of the figures is the outcome of the compromise between simulation accuracy and computational time.

In principle, the results we have obtained can be valid for arbitrary acoustic frequencies as long as the metamaterial properties (which are frequency-dependent) satisfy Eqs. (7)–(10) and Eqs. (12) and (13).

To sum up, we have theoretically studied a circular acoustic cloak that can conceal an object outside its domain, in contrast to a rectangular counterpart [15]. This design conquers the traditional problem of former acoustic cloak in which the hidden object is deafened, since it lies inside the cloaking shell. Potential applications of our design include the stealth submarines in which the sonar detectors can be installed on its outer surface to receive the genuine incoming acoustic waves while keeping the whole submarine totally undetectable to any enemy's sonar devices. Realization of such rather unusual materials would be challenging but may be achievable by layered structures [10].

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