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RESEARCH ARTICLE

Design of square-shaped heat flux cloaks and concentrators using method of coordinate transformation

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A square-shaped heat flux cloak and a square-shaped heat flux concentrator have been designed theoretically according to the invariance symmetry of steady state thermal conductive equation. The direction of heat flux in these devices can be modulated as desired. Using the method of coordinate transformation, the inhomogeneous and anisotropic thermal conductivity in the transformation region have been acquired. Two-dimensional finite element simulations were performed to confirm the theoretical results.

Keywords heat flux cloak, heat flux concentrator, transformation optics

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1 Introduction

Recently, much attention has been focused on the technique of transformation optics, which offers us an unconventional method to manipulate electromagnetic (EM) waves by applying a form-invariant coordinate transformation to Maxwell's equation [1–4]. Because the transformation optic opens up possibilities to control the electromagnetic fields, one can arbitrarily curve the optical space in a desired way and mold the flow of light. Therefore, many novel wave manipulation strategies have been proposed, such as cloaking [3, 4], concentrator [5], rotation coating [6], and splitting [7], etc. Because the coordinate transformation method is based on the invariance of Maxwell's equations, therefore, if the invariance symmetry of the wave equations in other fields is kept under the coordinate transformation, this method can be extended to other classical waves, such as acoustic cloak [8–10], matter wave cloak [11] and heat flux cloak [12, 13], etc.

Based on the coordinate transformation method, a spherical thermal cloak and an oblate spheroidal thermal cloak have been suggested by Fan *et al.* [13]. In these special cloaks, the heat flux goes around the inner domain and eventually returns to its original pathway, and the object inside the inner domain is protected from the invasion of external heat flux. In reality, the shielded object in the thermal cloak may have an arbitrary shape. Moreover, we have to consider whether the thermal cloak is easy to be placed or not in some applications. Therefore, the arbitrary shape thermal cloak may be considered. Besides the thermal cloaks, it is interesting to design a heat flux concentrator using the coordinate transformation method.

In this paper, we propose a square heat flux cloak and square heat flux concentrator using the method of the coordinate transformation, respectively. In classical physics, if the thermal radiation is ignored, the thermal conductive equation can be written as: $\rho C\frac{\partial T}{\partial t} + \nabla(-k\nabla T) = Q$, where ρ is mass density, C is capacity, T is temperature, κ is thermal conductivity, Q is quantity of heat. In steady state, the thermal conductive equation can be simplified as: $\nabla(-k\nabla T) = Q$. It is clear that the steady state thermal conductive equation has invariance symmetry in other systems [8]. Hence, the invariance of the thermal conductive equation is a corollary of the invariance of Maxwell's equations, and we can realize a heat flux cloak and heat flux concentrator using the method of the coordinate transformation. The special structure may have actual applications, such as, the heat flux cloak can keep instrument and equipment from supercooling or overheating environment, and the heat flux concentrator can be adhered to solar cell plate to improve the efficiency of solar energy utilization.

2 Transformation formula

According to transformation media approach [1, 2], we suppose that the original space is a vacuum with thermal conductivity κ_0, and thermal conductivity κ of transformation media can be written as: $\kappa^{i'j'} = |\det(g^{i'j'})|^{-1/2} g^{i'j'} \kappa_0$, where the metric is given by $g^{i'j'} = \Lambda_k^{i'} \Lambda_k^{j'} \delta^{kl}$, and $\Lambda_k^{i'} = \frac{\partial x^{\alpha'}}{\partial x^{\alpha}}$ is the Jacobian transformation matrix, which is just the derivative of the transformed coordinates (x') with respect to the original coordinates (x). $|\det(g^{i'j'})|$ is the determinant of the matrix of $g^{i'j'}$.

2.1 Cloak

We schematically illustrate the geometries in Fig. 1(a). The transformation compresses the space in a square into another square clocking shell, where the exterior of the clocking shell is a side length $2a$, and the interior of the clocking shell is a side length $2b$. According to symmetry of geometries structure, we divided the transformation space into four area [Fig. 1(1,2,3,4)]. Then, we apply the method prescribed by Pendry *et al.* [1] to define the following mapping in the region 1 (shadowed area):

$$r' = \left(k_1 + \frac{b}{x}\right) r, \quad \theta' = \theta, \quad z' = z \tag{1}$$

where $k_1 = \frac{a-b}{a}$ represents the ratio of space compression. The thermal conductivity $\bar{\bar{\kappa}}$ can be expressed in a Cartesian coordinate system,

$$\bar{\bar{\kappa}} = \begin{pmatrix} \kappa_{xx} & \kappa_{xy} & 0 \\ \kappa_{yx} & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix} \kappa_0 \tag{2}$$

where $\kappa_{xx} = \frac{k_1}{k_2}$, $\kappa_{xy} = 0$, $\kappa_{yx} = \frac{by}{x^2 k_1}$, $\kappa_{yy} = \frac{(by/x)^2 + k_2^2}{k_1 k_2}$, $\kappa_{zz} = \frac{1}{k_1 k_2}$, and $k_2 = k_1 + b/x$.

We note that the thermal conductivity inside the transformation region are inhomogeneous and anisotropic. Because of the symmetrical property of the transformation shell, the thermal conductivity of the other three regions can be easily obtained in a similar way.

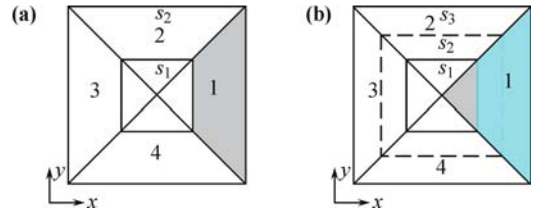

Fig. 1 Schematic diagram of the space transformation for the Square-shaped heat flux cloak and concentrator in the Cartesian coordinate system, where the structure is divided into four regions according to the symmetry. **(a)** Cloak model, **(b)** Concentrator model.

2.2 Concentrator

Similar to the heat flux cloak, the heat flux concentrator is illustrated in Fig. 2(b), which is composed of two-layered transformation media. In the outer layer, the exterior boundary is a square with side length $2a$, and the inner boundary is a square with side length $2b$. The inner layer is the square area with the side length $2b$. According to the symmetry, the structure is also divided into four regions.

We first consider the shadowed area (gray region) in Fig. 2(b), which corresponds to the inner layer. The corresponding mapping is

$$r' = k_1 r, \quad \theta' = \theta, \quad z' = z \tag{3}$$

where $k_1 = \frac{b}{c}$, which represents the ratio of space compression. The mapping means the regions s_1 and s_2 are compressed into the region s_1. The corresponding thermal conductivity $\bar{\bar{\kappa}}$ is then

$$\bar{\bar{\kappa}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/k_1^2 \end{pmatrix} \kappa_0 \tag{4}$$

Again, in the outer layer, we define the following mapping:

$$r' = \left(k_1 + \frac{k_2}{x}\right) r, \quad \theta' = \theta, \quad z' = z \tag{5}$$

where $k_1 = \frac{a-b}{a-c}$ and $k_2 = \frac{a(b-c)}{a-c}$. The mapping means the region s_3 is stretched into the regions s_2 and s_3. The corresponding thermal conductivity $\bar{\bar{\kappa}}$ is then

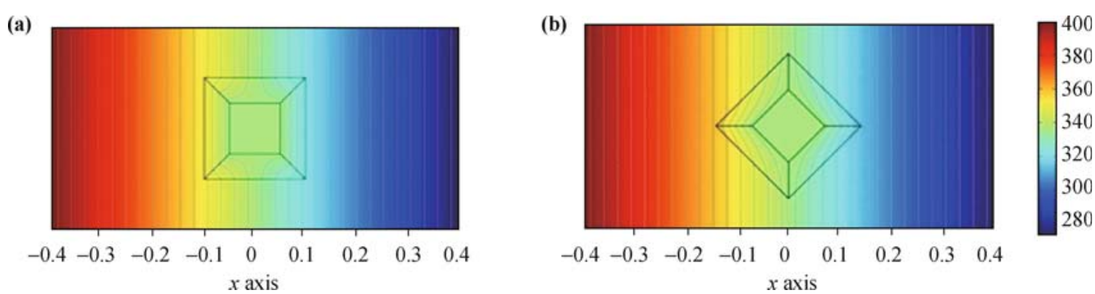

Fig. 2 The spatial distribution of the temperature and the isothermal lines (white lines) for heat flux cloaking. **(a)** The isothermal lines are parallel aligned to one side of the cloak, **(b)** Cloak rotated by $\pi/4$ with respect to the left boundary.

$$\bar{\bar{\kappa}} = \begin{pmatrix} \kappa_{xx} & \kappa_{xy} & 0 \\ \kappa_{yx} & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix} \kappa_0 \quad (6)$$

where $\kappa_{xx} = \frac{k_1}{k_2}$, $\kappa_{xy} = 0$, $\kappa_{yx} = \frac{k_2 y}{x^2 k_1}$, $\kappa_{yy} = \frac{(k_2 y/x)^2 + k_2^2}{k_1 k_2}$, $\kappa_{zz} = \frac{1}{k_1 k_2}$, and $k_3 = k_1 + k_2/x$.

The thermal conductivity inside the concentrator is also inhomogeneous and anisotropic. Because of the symmetrical property of the transparent shell, the thermal conductivity of the other three regions can be easily obtained in a similar way.

3 Simulation

To confirm the validity of the above concept, we have performed full wave simulations using the finite element method. In all numerical calculations, we choose a rectangular box, and set the right and left boundaries temperature at 273 K and 400 K, top and bottom boundaries are heat convection condition, respectively.

Figure 2 shows the results of the two-dimensional full-wave simulations of a square-shaped heat flux cloak, where we choose $a = 0.1$ m, $b = 0.075$ m and $c = 0.075$ m. The color map depicts the spatial distribution of the temperature along the x-direction. In addition, the isothermal lines are indicated by the white lines. In Fig. 2(a), the isothermal lines are parallel aligned to one of the sides of the cloak. As can be seen, although the square cloak possesses sharp corners, the isothermal lines are smoothly bent around the cloaked area, then completely restored outside the cloak shell. In Fig. 2(b), the cloak structure is rotated by an angle of 45° with respect to the left boundary. In this configuration, the isothermal lines are no longer parallel to any side of the square cloak. As before, the isothermal lines are completely restored after propagation through the cloak material, and the inner square is not sensed by heat flux. In order to illustrate the process of heat conduction in detail, Fig. 3(a) shows the conductive heat flux along the x-direction with $y = 0$. Because idiosyncratic effective conductivity is evident with the decrease of conductive heat flux in the inner region of the cloak, we can see that the values of conductive heat flux gradually reduce along the x-direction near the transformation region, and reduce to zero at the inner boundary. We also note that heat flux cloaks of arbitrary shape can also be designed by the same method in principle [12, 13]. This special heat cloak may have actual applications, such as, keeping instruments and equipment from a supercooling or overheating environment.

According to Eqs. (4) and (6), two-dimensional full-wave simulations of a square-shaped heat flux concentrator is carried out in Fig. 4, where we choose $a = 0.1$ m, $b = 0.05$ m and $c = 0.075$ m. The color map depicts the spatial distribution of the temperature along the z-direction. In addition, the isothermal lines are indicated by the white lines, and direction of heat flux is indicated by a black arrow. In Fig. 4(a), the isothermal lines are parallel aligned to one of the sides of the concentrator. It is obvious, that the isothermal lines are concentrated in the inner transformation region, and heat flux in the outer transformation region flows into the inner region for $x \leqslant 0$ and outflow inner region for $x > 0$. The intensity enhancement factor of the isothermal lines for the chosen structure depends upon the ratio k_1 of the Eq. (3), which is determined by square side-length b and c in the Fig. 1(b). Significantly stronger enhancements can be achieved by increasing the ratio k_1. In Fig. 4(b), the concentrator structure is rotated by an angle of 45° with respect to the isothermal lines. As can be seen, the isothermal lines are also strengthened in the inner transformation region, and heat flux in the outer transformation region flows into the inner region for $x \leqslant 0$ and outflow inner region for $x > 0$. Because of the rotational symmetry around the axis perpendicular to the x–y plane, the concentrator can focus heat fluxing from arbitrary directions. Figure 3(b) shows the distribution of the conductive heat flux along the x-direction with $y = 0$ in detail. We can see that the value of

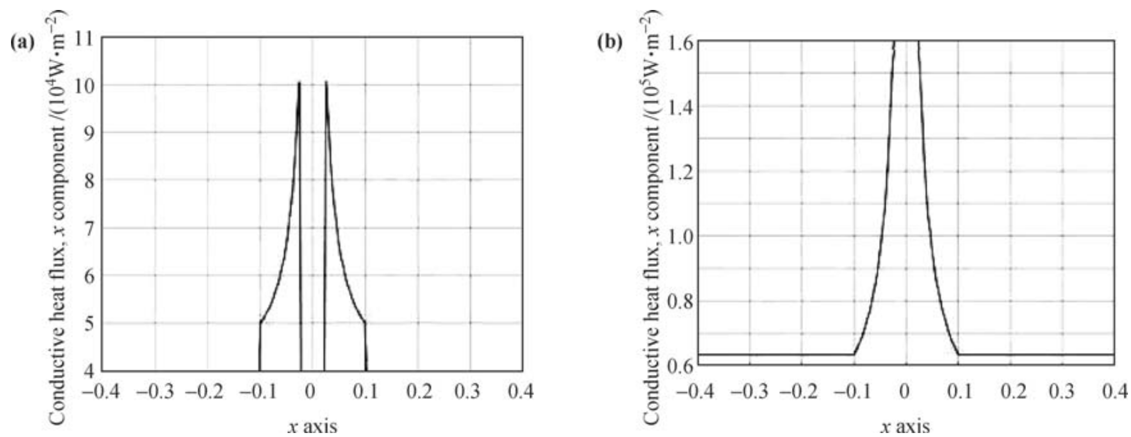

Fig. 3 The distribution of the conductive heat flux along x-direction with $y = 0$. **(a)** Heat flux cloak, **(b)** Heat flux concentrator.

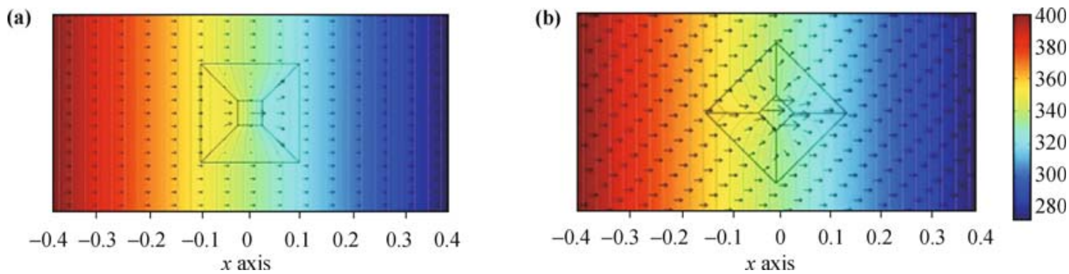

Fig. 4 The spatial distribution of the temperature and the isothermal lines (white lines) for heat flux concentrating. **(a)** The isothermal lines are parallel aligned to one side of the cloak, **(b)** Cloak rotated by $\pi/4$ with respect to the left boundary.

conductive heat flux increases in the transformation region and reaches a maximum value in the inner region. Although the thermal conductivity inside the concentrator is inhomogeneous and anisotropic, we think the heat flux concentrator can be adhered to solar cell plates to improve the efficiency of solar energy utilization.

4 Conclusion

According to the invariance symmetry in different coordinate systems of steady state thermal conductive equation, we have designed a square-shaped heat flux cloak and square-shaped heat flux concentrator. Using the method of coordinate transformation, heat flux cloaking and concentrating can be realized by the inhomogeneous and anisotropic thermal conductivity in the transformation region. Two-dimensional finite element simulations were performed to confirm the theoretical results. The special structure may have actual applications, such as, keeping instruments and equipment from a supercooling or overheating environment, and the heat flux concentrator can be adhered to solar cell plates to improve the efficiency of solar energy utilization.

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