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REVIEW ARTICLE

A quantitative assessment of stochastic electrodynamics with spin (SEDS): Physical principles and novel applications

Giancarlo CAVALLERI[1], Francesco BARBERO[1], Gianfranco BERTAZZI[1], Eros CESARONI[1], Ernesto TONNI[1], Leonardo BOSI[2], Gianfranco SPAVIERI[3](✉), George T. GILLIES[4]

[1] *Dipartimento di Matematica e Fisica, Università Cattolica del Sacro Cuore, via Musei 41, 25121 Brescia, Italy*
[2] *Politecnico di Milano (Polo Regionale di Lecco), Dipartimento di Fisica, piazza L. da Vinci 32, 20133 Milano, Italy*
[3] *Centro de Física Fundamental, Facultad de Ciencias, Universidad de Los Andes, Mérida, 5101 Venezuela*
[4] *School of Engineering and Applied Science, University of Virginia, Charlottesville, VA 22904-4746, USA*
E-mail: g.cavalleri@dmf.unicatt.it, leonardo.bosi@polimi.it, spavieri@ula.ve, gtg@cms.mail.virginia.edu

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Stochastic electrodynamics (SED) without spin, denoted as pure SED, has been discussed and seriously considered in the literature for several decades because it accounts for important aspects of quantum mechanics (QM). SED is based on the introduction of the nonrenormalized, electromagnetic stochastic zero-point field (ZPF), but neglects the Lorentz force due to the radiation random magnetic field $\boldsymbol{B}_r$. In addition to that rather basic limitation, other drawbacks remain, as well: i) SED fails when there are nonlinear forces; ii) it is not possible to derive the Schrödinger equation in general; iii) it predicts broad spectra for rarefied gases instead of the observed narrow spectral lines; iv) it does not explain double-slit electron diffraction patterns. We show in this short review that all of those drawbacks, and mainly the first most basic one, can be overcome in principle by introducing spin into stochastic electrodynamics (SEDS). Moreover, this modification of the theory also explains four observed effects that are otherwise so far unexplainable by QED, i.e., 1) the physical origin of the ZPF, and its natural upper cutoff; 2) an anomaly in experimental studies of the neutrino rest mass; 3) the origin and quantitative treatment of $1/f$ noise; and 4) the high-energy tail ($\sim 10^{21}$ eV) of cosmic rays. We review the theoretical and experimental situation regarding these things and go on to propose a double-slit electron diffraction experiment that is aimed at discriminating between QM and SEDS. We show that, in the context of this experiment, for the case of an electron beam focused on just one of the slits, no interference pattern due to the other slit is predicted by QM, while this is not the case for SEDS. A second experiment that could discriminate between QED and SEDS regards a transversely large electron beam including both slits obtained in an insulating wall, where the ZPF is reduced but not vanished. The interference pattern according to SEDS should be somewhat modified with respect to QED's.

Keywords quantum mechanics, Aharonov–Bohm effect, spin, electrodynamics

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Contents

1 Introduction

Any *bona fide* attempt to derive quantum mechanics (QM) from classical physics is of interest because QM is based on the postulated Schrödinger equation without any intuitive approach and with a coexistent breaking of continuity with classical physics. That is why QM, and in particular quantum electrodynamics (QED), as famously stated by Feynman [1], “cannot be understood.” (His exact quotation was, “It is my task to convince you not to turn away because you do not understand it. You see, my physics students do not understand it either. That is because I do not understand it. Nobody does.”)

Many researchers, unsatisfied with this state of affairs, have tried to derive some of the principles of QM through classical stochastic processes. Considering that one of the tasks of a scientific contribution is to stimulate thinking and further research by widening the scenario of alternative physical interpretations of natural phenomena, we propose the present review, where we report the relevant best attempts. It is not a question of favoring the classical versus the quantum interpretation or vice versa. It is a question of allowing a deeper understanding by explaining and scrutinizing the phenomenon from a wider point of view and context. We believe that the present review, by reanalyzing important issues connected to the classical and quantum scenarios such as the stability of atoms, two-slit particle diffraction, the origin of the high-energy tail of cosmic radiation, etc., complies with the mentioned task.

From a general perspective, there have been four classical approaches to QM, namely: i) stochastic mechanics, shortly revised in Section 2; ii) classical statistical mechanics (Section 3); iii) stochastic electrodynamics (SED) (Section 4); and iv) SED with spin (SEDS) (Section 5). We discuss each of them in what follows, showing that the first two approaches have only a historical interest but are nevertheless presented here for completeness.

The third approach, i.e., pure SED, developed during the past three decades, has been used to develop nonstandard alternative descriptions of quantum phenomena, as seen from a classical point of view and as the result of the behavior of particles undergoing stochastic microscopic processes under the influence of the so-called zero-point field (ZPF). However, as pointed out by some scientific journals, pure SED has a number of well-known drawbacks that make it untenable. Furthermore, if the radiation pressure of the ZPF is taken into account, it appears that all results derived from SED can be called into question.

The fourth approach under consideration is SED with spin (SEDS). It eliminates the drawbacks of pure SED. The key of its success is a radical change of the equation of motion for the electrically charged particles constituting matter. In fact, the spin motion, meant as a circular motion at the speed of light because of self-reaction, leads to a highly nonisotropic inertial mass, that is infinite for a force lying on the plane of the spin orbit, and minimum for a force parallel to the spin axis $\hat{\boldsymbol{n}}$. Special relativity (SR), which is not present at the real particle level, is reaffirmed and arises because one usually refers to the ideal center around which the particle revolves, and not to the spin microscopic motion. The main consequence of the new equation of motion is that the radiation pressure vanishes if $\hat{\boldsymbol{n}}$ remains fixed. Under that consequence of SEDS, the classical balance between radiated and absorbed power, typical of pure SED, leads to the Bohr condition for the ground level of a hydrogen atom. In Section 4.2, we recall other achievements of pure SED, all obtained by neglecting the Lorentz force due to the random e.m. field of the ZPF. That neglecting is not accounted for by pure SED, which possesses also the other four drawbacks mentioned in the Abstract. On the contrary, all the drawbacks of pure SED are overcome by SEDS.

As a novelty we add here the derivation of Bohr's complete condition, including the excited states. The predictions of SEDS are the same with QM, and even with QED when the ZPF is not modified (at frequencies below plasma's) by boundary conditions. Moreover, SEDS can also explain four phenomena presently unexplainable by QED, namely: 1) the physical origin of the ZPF, and its natural upper cutoff; 2) an anomaly in experimental studies of neutrino rest mass [2]; 3) the origin and quantitative studies of $1/f$ noise [3, 4]; and 4) the origin of the high-energy tail of cosmic radiation. The above phenomena are due to the presence of a real ZPF, whose real existence is proved by the observability of the relevant experiments. On the contrary, the ZPF is renormalized in QED, and that is why QED may not, at least so far, explain the above four phenomena.

When some phenomenon depends on a ZPF locally modified by boundary conditions, SEDS and QED predict different results, as in the case of diffraction of single electrons passing only through one of two slits. In Section 5 two relevant experiments are proposed, which could discriminate between the two theories. Those experiments are novel in design, and could provide auspicious tests of theory if carried out.

We close the review with a concise statement of the conclusions (Section 6) reached in the five previous sections.

2 Stochastic mechanics

Nelson [5] derived the Schrödinger equation starting from a Brownian motion due to hypothetical "zerons". In order to eliminate the inevitable friction present in any Brownian motion (which prevents a motion by inertia), he introduced " forward and backward" derivatives and two different local velocities for the local probability density. By contrast, in any macroscopic stochastic process one has a single *local* velocity and this occurs even for QM. Moreover, Gilson [6] has shown that the diffusion coefficient D introduced by Nelson [5] cannot vanish for $\Delta t \to 0$ or, equivalently, the velocity appearing in the continuity equation cannot be equal to the velocity appearing in the propagator. Kracklauer [7] has emphasized that the Brownian motion used by Nelson implies a mean square spreading, after a time Δt from the initial condition when the spreading is zero, proportional to Δt, whereas the final result of Nelson, i.e., the Schrödinger equation, implies a spreading proportional to Δt^2 (called ballistic spreading, or without friction, whereas a Brownian motion is dominated by friction). For other technical criticisms to Nelson's paper [5], see Appendix B of Ref. [8]. De la Peña-Auerbach and Cetto [9] have shown that Nelson's results can be saved only if not intended as based on classical physics, contrary to what was asserted by Nelson himself in his Introduction. In this interpretation, Nelson's findings are simply a new recipe for first quantization and are unrelated to classical physics, as emphasized by his definition of acceleration, based on forward and backward derivatives. Moreover, photons are extraneous since a Brownian motion cannot be advocated for them. As a consequence, all QED is extraneous to Nelson's approach. Other papers in the so-called stochastic mechanics followed, but the above-mentioned drawbacks remained. The most prominent adherants of Nelson's approach, e.g., De la Peña-Auerbach and Cetto, switched to stochastic electrodynamics (SED) which, as shown in Section 4, has physical bases, contrary to stochastic mechanics.

3 Classical statistical mechanics

In the preceding section we assessed only the pioneering work of stochastic mechanics, since all the subsequent work on it only added some applications without remedying the lack of physical bases and the other above-mentioned drawbacks. In what now follows, we examine the last, and the best, of the papers trying to make some contact with QM starting from classical statistical mechanics. The considered paper is the one of Carati and Galgani (CG) [10] who have studied energy exchange in collisions between diatomic molecules. CG have proved that Newton's law leads, for the vibrational energy, to a functional relation between mean value and variance having an analytic form equal to the one found by Einstein in connection with Planck's law. The authors pointed out that such a fact might be of interest for an understanding of the relations between classical and quantum mechanics, but limited themselves to a few marginal remarks. The main one is that CG [10] have found a quasi-thermodynamic formula for the mean energy U of a system of a large number N of oscillators of the same frequency ω very far from equilibrium that is similar to the Planck distribution. But the final equilibrium between the wall of the cavity and the e.m. modes inside the cavity should be governed by Rayleigh–Jeans (RJ) law. However, equilibrium is reached rapidly for "small" ω values (i.e., in usual terms, for $\hbar\omega \ll kT$) at which the Planck distribution is practically equal to the RJ distribution. Vice versa, for $\hbar\omega \gg kT$ the equilibrium towards the RJ distribution would require huge times [11] much larger than the age of the universe, because the cosmic background radiation (CBR) has an excellent Planck distribution. Obviously, if only those time constants were active, the distribution obtainable by CG [10] would remain negligible for $\hbar\omega \gg kT$. The point is that there also are much shorter time scales τ, sharply separated by those above considered, as emphasized by CG in Refs. [12, 13]. It is because of those τ that even the high frequencies can be excited and the power spectral density acquires a finite value even for $\hbar\omega \gg kT$, so as to become of a Planck-like type. That kind of meta-equilibrium implies a variation from the initial state at $T \approx 0$, towards the power spectral density of true equilibrium, i.e., towards the RJ law. Suppose that T be further increased and that CG's main remark remains valid even if the initial energies of the oscillators (the e.m. modes in the cavity) for $\hbar\omega \gg kT$ are no longer negligible. The short time constants τ can produce a new rapid variation towards the RJ law (which is the equilibrium distribution) so that a new Planck-like law is obtained, corresponding to a larger temperature. Suppose now that the temperature is decreased. The rapid variation due to the short time constant τ cannot go against the evolution towards a larger probability since, for $\hbar\omega \gg kT$, the RJ distribution at the smaller T_2 is still larger than the high ω tail of the Planck distribution at a larger T_1. Notice that the evolution towards *smaller* probability should be steady-state in order to reproduce a new Planck-like distribution. It is not a question of some single *fluctuations*, but it would be a constant trend against the entropy increase, i.e., against the second principle of thermodynamics. Let us exemplify this concept in Fig. 1, where we take $T_2 = T_1/2$. The RJ distribution at T_2 is still much higher than the Planck distribution for $\hbar\omega \gg kT$. Consequently, an evolution towards maximum probability would displace $\rho_P(\omega \gg kT/\hbar, T_1)$ towards $\rho_{RJ}(\omega \gg kT/\hbar, T_2)$. Any displacement would be impossible if only the very long time scale is taken into account, but it is possible by means of the much shorter time scale τ. Unfortunately, the displacement due to the short time scale is in the wrong direction, unless one goes against the evolution towards maximum probability, i.e., against the second law of thermodynamics.

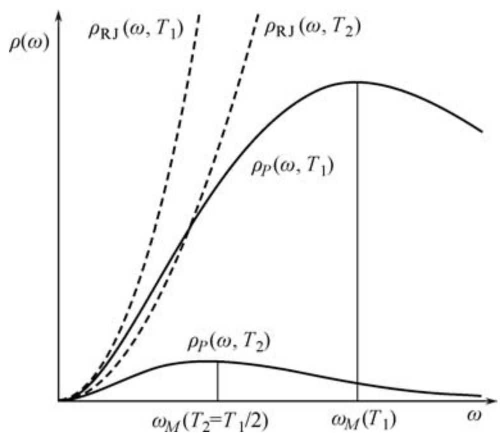

Fig. 1 A thermal transition from an initial temperature T_1 to a $T_2 < T_1$ (in the figure we have taken $T_2 = T_1/2$) should imply, according to the rapid variations pointed out by Carati and Galgani, a variation towards the maximum probability represented by the Rayleigh–Jeans (RJ) distribution at T_2. For $\hbar\omega \gg kT$ the RJ distribution at $T_2 = T_1/2$ is still higher than the Planck distribution at T_1. Consequently, the rapid variations leading to a metaequilibrium, would displace the high frequency tail upwards (i.e., towards the RJ distribution at T_2 that is higher than the Planck distribution at both T_1 and T_2), contrary to what occurs in reality.

In Conclusion, the Carati–Galgani theory [10] keeps its validity as far as its main result is concerned, i.e., the functional relationship between mean and variance having an analogical form equal to the one found by Einstein in connection with Planck's law. However, CG's main remark regarding the derivation of the Planck spectrum is valid only if starting from an initial temperature $T = 0$. It could even explain the transients if the temperature is increased, but it fails completely if T decreases.

4 Pure stochastic electrodynamics (SED)

Pure SED arose to answer the crucial question raised in the second half of the 19th century, i.e., the stability of the atoms. Classically, an electron must radiate in its orbital motion because it undergoes an acceleration. QM does not answer that question in a qualitative way. It simply solves the Schrödinger equation. In pure SED the radiated power has to be balanced by the power absorbed from the ubiquitous, stochastic e.m. field due to the radiation of all the particles of the universe. The balance is reported in the next subsection, premising some elementary concepts regarding the power spectral density, more simply called "spectrum".

4.1 Spectrum of the stochastic, ubiquitous, e.m. field denoted zero-point field (ZPF)

The power spectral density, or more simply, the spectrum, of e.m. radiation can be represented as:

$$\rho(\omega) = \frac{\mathrm{d}^4 K}{\mathrm{d}V \mathrm{d}\omega} = \frac{\mathrm{d}U}{\mathrm{d}\omega} \tag{1}$$

with K, V, ω, and $U = \mathrm{d}^3K/\mathrm{d}V$ denoting energy, volume, angular frequency, and energy density, respectively. The average force $\langle \boldsymbol{F} \rangle$ on a charged harmonic oscillator of mass m, electric charge e, having proper angular frequency ω_0, translating with average velocity $\langle \boldsymbol{v} \rangle$, and subject to the e.m. power spectral density $\rho(\omega)$, is given by the Einstein–Hopf formula [14]:

$$\langle \boldsymbol{F} \rangle = -\frac{4}{5}\pi^2 \frac{e^2}{mc^2} \langle \boldsymbol{v} \rangle \left[\rho(\omega_0) - \frac{\omega_0}{3} \left| \frac{\mathrm{d}\rho(\omega)}{\mathrm{d}\omega} \right|_{\omega=\omega_0} \right] \tag{2}$$

We notice that only for $\rho(\omega) = A\omega^3$ the force is null for every value of ω_0, hence allowing a "motion by inertia", at least for a harmonic oscillator. In particular, as shown by Boyer [15], a spectrum $\rho(\omega) = A\omega^3$ is also the only one to be relativistically invariant for any ω value.

Assuming a proportionality constant A that is given by

$$A = \frac{\hbar}{2\pi^2 c^3} \tag{3}$$

the power spectral density turns out to be

$$\rho(\omega) = \frac{\hbar\omega^3}{2\pi^2 c^3} \tag{4}$$

coinciding with the ZPF of QED. However, $\rho(\omega)$ is strongly divergent for $\omega \to \infty$, so that the ZPF is renormalized in QED. Yet, in the presence of a gravitational field, i.e., in a Riemannian space, $\rho(\omega)$ cannot be renormalized, and this implies difficulties for general relativity (GR). In fact, even truncating the ZPF at the minimum possible value, the mass energy density in the universe would be 10^{120} times that observed, a grave conflict indeed. In fact, GR predicts that the universe underwent an expansion (after the big bang) up to a maximum radius less than an atomic radius. After the short expansion (exhausted in one nanosecond), the universe would collapse into a black hole!

Filament theory (FT) [16, 17] which is a developmental effort that is presently in progress, leads to a gravitational theory different from general relativity. FT gives the same results of GR up to and including those of second order, which are the only ones that can be detected at present. But it is radically different, because the ZPF of FT has no effect on gravitation. Consequently, in FT (and in the consequent SEDS, that will be studied in Section 5) the ZPF is taken as real, i.e., as nonrenormalized. We will also see that it has a natural reduction at very high frequencies.

4.2 The bases of SED

The basis of pure SED is just the assumption of the spectrum given by Eq. (4) as a boundary condition. For all the rest, SED coincides with classical electrodynamics with the addition of the stochastic, or random, electric field $\boldsymbol{E}_r$ and magnetic field $\boldsymbol{B}_r$. Since we know the spectrum (4) of the energy density of $\boldsymbol{E}_r$ and $\boldsymbol{B}_r$, it is convenient to express them as a Fourier superposition of plane waves with random phases $\theta_s(\boldsymbol{k})$

$$\boldsymbol{E}_r(\boldsymbol{r},t)=\lim_{L\to\infty}\sum_{s=1}^{2}\int\tilde{\boldsymbol{E}}_s(\boldsymbol{k},t)\,\mathrm{e}^{\mathrm{i}[\omega_k t-\boldsymbol{k}\cdot\boldsymbol{r}+\theta_s(\boldsymbol{k})]}\mathrm{d}^3\boldsymbol{k} \quad (5)$$

where the summation is over the two polarizations implied for e.m. transverse waves, $\boldsymbol{k}=\hat{\boldsymbol{k}}\omega/c$ is the wave vector (or propagator because it is along the propagation direction), and "Re" means that we have to take the real part. A similar expression holds for $\boldsymbol{B}_r$. The energy density (per unit volume) of the random fields can be expressed, with the use of Eq. (4), as:

$$U_r=\frac{E_r^2+B_r^2}{8\pi}=\frac{E_r^2}{4\pi}=\int_0^{\infty}\mathrm{d}\omega\rho(\omega)=\int_0^{\infty}\mathrm{d}\omega\frac{\hbar\omega^3}{2\pi^2c^3} \quad (6)$$

and using their Fourier antitransforms

$$\begin{aligned}\frac{\langle E_r^2\rangle}{4\pi}&=\lim_{L\to\infty}\frac{1}{8L^3}\int_{-L}^{L}\frac{E_r^2}{4\pi}\\&=\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty}\mathrm{d}^3\boldsymbol{r}\sum_{s=1}^{2}\sum_{s'=1}^{2}\int\mathrm{d}^3\boldsymbol{k}\\&\quad\cdot\int\mathrm{d}^3\boldsymbol{k}'\frac{\tilde{\boldsymbol{E}}_s(\boldsymbol{k},t)\cdot\tilde{\boldsymbol{E}}^*_{s'}(\boldsymbol{k}',t)}{4\pi}\exp\big\{\mathrm{i}\big[(\omega_k-\omega_{k'})t\\&\quad+(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{r}+\theta_s(\boldsymbol{k})-\theta_{s'}(\boldsymbol{k}')\big]\big\}\\&=\sum_{s=1}^{2}\sum_{s'=1}^{2}\int\mathrm{d}^3\boldsymbol{k}\int\mathrm{d}^3\boldsymbol{k}'\frac{\tilde{\boldsymbol{E}}_s(\boldsymbol{k},t)\cdot\tilde{\boldsymbol{E}}^*_{s'}(\boldsymbol{k}',t)}{4\pi}\\&\quad\cdot\delta_{ss'}\delta^3(\boldsymbol{k}'-\boldsymbol{k})\\&=\sum_{s=1}^{2}\int_{-\infty}^{+\infty}\frac{\tilde{E}_s^2(\boldsymbol{k},t)\mathrm{d}^3\boldsymbol{k}}{4\pi}\\&=\int_0^{\infty}\tilde{E}^2(\boldsymbol{k},t)k^2\mathrm{d}k\\&=\int_0^{\infty}\tilde{E}_\omega^2(\omega,t)\left(\frac{\omega}{c}\right)^2\frac{\mathrm{d}\omega}{c}\end{aligned} \quad (7)$$

the last but one side being obtained because $\tilde{\boldsymbol{E}}_1\cdot\tilde{\boldsymbol{E}}_2=0$, so that $\tilde{E}_1^2+\tilde{E}_2^2=\tilde{E}^2$.

Identifying the integrands of Eqs. (6) and (7), we obtain

$$E_\omega^2=\frac{\hbar\omega}{2\pi^2}\ \text{ or }\ E_\omega=\frac{1}{\pi}\sqrt{\frac{\hbar\omega}{2}} \quad (8)$$

The same value holds for B_ω.

Using the orthogonal unit vectors $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{k}}$ which lie in the directions of the electric field and wave propagation vectors, respectively, we can write

$$\boldsymbol{E}_s(\boldsymbol{k},t)=\hat{\boldsymbol{\varepsilon}}_s\sqrt{\frac{\hbar\omega}{2\pi^2}} \quad (9)$$

and

$$\begin{aligned}\boldsymbol{B}_r(\boldsymbol{r},t)=&\lim_{L\to\infty}\sum_{s=1}^{2}\int\hat{\boldsymbol{k}}\times\hat{\boldsymbol{\varepsilon}}_s\sqrt{\frac{\hbar\omega}{2\pi^2}}\\&\cdot\exp\{\mathrm{i}[\omega_k t-\boldsymbol{k}\cdot\boldsymbol{r}+\theta_s(\boldsymbol{k})]\}\,\mathrm{d}^3\boldsymbol{k}\end{aligned} \quad (10)$$

The equation of motion used by SED is that of Abraham–Lorentz

$$m\ddot{\boldsymbol{r}}-\frac{2e^2}{3c^3}\dddot{\boldsymbol{r}}=e\left[\boldsymbol{E}+\boldsymbol{E}_r+\frac{\boldsymbol{v}}{c}\times(\boldsymbol{B}+\boldsymbol{B}_r)\right] \quad (11)$$

where the second term at the l.h.s. represents the radiation damping. In honour of the authors whose efforts are most typically identified with this part of the work, the latter expression is called the Brafford–Marshall equation.

The action on the charge e due to $\boldsymbol{E}_r$ is effective only on a charged harmonic oscillator, and the Fourier component $E_k(\omega_0)$ (where ω_0 is the proper angular frequency of the harmonic oscillator) contributes a lot to the power communicated to it. On the contrary, the effect of $\boldsymbol{E}_r$ on a free particle is only a tiny diffusion that increases logarithmically with the time t [8]. In a cathodic television set, that diffusion is of the order of an atomic size [8], and that is why it is unobservable on the TV screen. In fact, the velocity $\boldsymbol{v}_r$ acquired by an electron in a half oscillation of a train of sinusoidal oscillations of $\boldsymbol{E}_r$ is, on an average, reduced to zero at the end of the subsequent half oscillation. By contrast, the effect of $c^{-1}e\boldsymbol{v}\times\boldsymbol{B}_r$ on a free particle is huge. It is of the kind already found by Einstein for any stochastic process, i.e., the random impulses due to whole trains of $\boldsymbol{B}_r$ oscillations tend to increase the particle kinetic energy proportionally to the time t. If the e.m. spectrum is that of a cavity, and the ZPF is eliminated (Einstein did not know the ZPF), as soon as a charged particle acquires a given velocity with respect to the conducting walls of the cavity, the particle undergoes a friction force given by Eq. (2). Finding the average dynamic equilibrium, Einstein reattained Planck's black body spectrum. But if the e.m. spectrum contains the ZPF expressed by Eq. (4), the kinetic energy increase is so rapid that an atom would become ionized in a nanosecond [18], and a free charged particle in vacuum would become a highly energetic cosmic ray ($\sim 10^{20}$ eV) in ten centimeters [19]. That catastrophe implied that good results cannot be easily obtained by SED, and many skilful authors and researchers abandoned it. On the other side, the extremely great effect of the Lorentz force with $\boldsymbol{B}_r$ appeared to be a good reason to renormalize the ZPF, as is done in QED.

The curious fact is that, by disregarding the effect of $\boldsymbol{B}_r$, some results in agreement with QM have been obtained. By that limitation, those results are

- The stability of the atoms [20];
- The black body spectrum [14, 21]. With the same treatment of Rayleigh–Jeans, but with the inclusion of the ZPF (see Fig. 2), the Planck spectrum is found superimposed to the ZPF, i.e.,

$$\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3}\left[\frac{1}{2} + \frac{1}{\exp(-\hbar\omega/kT)}\right]$$

- The intuitive explanation of the Casimir effect [15], i.e., the attraction of two conducting plates (with no electric charge), due to the e.m. pressure of the ZPF, that is larger outside the plates;
- The Van der Waals forces between macroscopic objects, and between polarizable particles [15];
- The specific heats of oscillators and rotators [15];
- The fluctuations in thermal radiation [15];
- The third law of thermodynamics [15];
- The harmonic oscillator with radiative corrections [22, 23];
- The diamagnetic susceptibilities [24];
- The thermal effects of acceleration [25] (the Unruh–Davis effect).

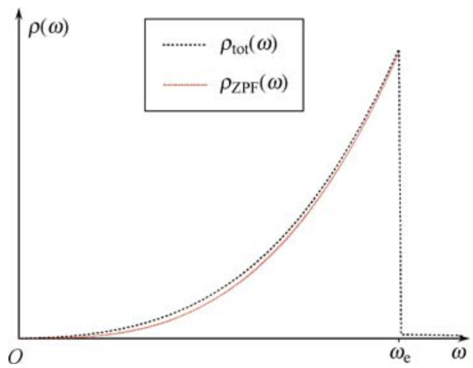

Fig. 2 In SED the Planck spectrum is superimposed to the zero point field (ZPF), represented as $\rho_{\mathrm{ZPF}}(\omega)$, giving the total spectrum $\rho_{\mathrm{tot}}(\omega)$.

Notice that the last result 10 has been published after the discoveries of the self-ionization of atoms [18], and of the huge diffusion–acceleration of a free particle that, in vacuum, should acquire an energy $\sim 10^{20}$ eV in a few nanoseconds [19]. Actually, some researchers went on with pure SED because it leads to some new qualitative results, as the origin of the extremely high energy tail of cosmic radiation unexplainable by usual QED [26, 27]. Indeed, Einstein had already shown that the kinetic energy of a particle subject to random impulses increases linearly with time unless a frictional force arises due to the stochastic process itself. However, if the stochastic process is that of the ZPF, the frictional force vanishes and the kinetic energy of a charged particle steadily increases until the particle undergoes a collision, which is very rare in intergalactic space. This serves to explain the huge observed energies of cosmic rays, which can be up to 10^{21}eV. In this case there is no modification of the ZPF because of the boundary conditions, and that is why QED in the usual time-asymmetric formulation (in which the unmodified zero point field is subtracted) does not predict this effect. By SED, Rueda [26, 27] predicted not only the existence of this new effect, but also the correct slope of the very high energy tail of the cosmic ray distribution function versus energy. Unfortunately, the intensity of the acceleration mechanism turned out to be too intense, so that an electron would become a cosmic ray in an oscilloscope tube [19]!

The above mechanism of acceleration, discovered by Einstein, further clarified by Boyer, and applied to the cosmic radiation by Rueda [26, 27], is universal, i.e., valid for any stochastic process. It is therefore untenable to attempt to eliminate it, although that possibility has been examined by De la Peña-Auerbach in the Cali Conference on SED [28]. Unfortunately, some workers (e.g., Rueda) accepted that line of thought, and discontinued work on the origin of the high-energy tail of the cosmic radiation. At the same time, he, together with Puthoff, Cole, and Haisch [29–32], applied pure SED to find the inertia of an e.m. energy contained in an accelerated box. Their calculations are mainly based on the $\boldsymbol{B}_r$ Lorentz force, just what Rueda discarded following De La Peña. However, as repeatedly noted, the $\boldsymbol{B}_r$ Lorentz force makes pure SED untenable. Moreover, in a recent paper [33] Levin has shown that all the results cited in Refs. [29–32] are linked to incorrect physical and mathematical assumptions associated with taking a nonrelativistic approach.

4.3 Balance between the radiated power, and that absorbed from the random electric field $\boldsymbol{E}_r$

A harmonic oscillator, having mass m and proper angular frequency ω_0, absorbs a power from a stochastic field $\boldsymbol{E}_r$, exerting on it a random force $e\boldsymbol{F}_r$ expressed by

$$P_{\mathrm{abs}} = \frac{2}{3}\frac{\pi^2 e^2}{m}\rho(\omega_0) \tag{12}$$

Let us consider a hydrogen atom according to the Bohr model. In the presence of the real, ubiquitous ZPF, the electron absorbs a power from the random $\boldsymbol{E}_r$ whose spectrum is that given by Eq. (4). A circular motion is equivalent to two harmonic oscillators. We derive from Eqs. (4) and (12)

$$P_{\rm abs}|_{\rm circ} = 2\frac{2}{3}\frac{\pi^2}{m}e^2\frac{\hbar\omega_0^3}{2\pi^2c^3} \tag{13}$$

The power radiated by the electron is given by the Larmor formula

$$P_{\rm rad}|_{\rm circ} = \frac{2}{3}\frac{e^2}{c^3}a^2 = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{v^2}{R}\right)^2 \tag{14}$$

with $a = v^2/R$ as the centripetal acceleration.

Equating the radiated and absorbed powers, i.e., equating Eq. (13) to Eq. (14), and using $\omega = v/R$, we obtain

$$mvR = \hbar \tag{15}$$

which is the Bohr condition, allowing the calculation of the most probable atomic radius. Eq. (15) also leads to the uncertainty principle.

It is remarkable that the simple balance between radiated and absorbed power answers an old question relevant to the stability of the atom. QM does not answer it, being limited instead to mathematical solutions of the Schrödinger equation. Notice that the absorbed power considered here only contains that associated with the random electric field $\boldsymbol{E}_r$, and disregards the effects of radiation pressure, due to the combined effect of $\boldsymbol{E}_r$ and $\boldsymbol{B}_r$. This is a strong limitation of pure SED. The reason why the random impulses due to the radiation pressure are strongly reduced comes from SEDS examined in Section 5, and in particular from Eqs. (37) and (38).

4.4 Excited states

Excited states have never been obtained in SED, and we present them as a new development in this field. Actually, even this is an achievement of SEDS, which explains why the Lorentz force due to $\boldsymbol{B}_r$ causes a small acceleration on a spinning particle, so that, as far as the simple balance between absorbed and radiated power is concerned, we can neglect $\boldsymbol{B}_r$.

In the preceding, for the dynamic balance $P_{\rm abs} = P_{\rm rad}$ that led to Eq. (15), we considered pure circular motion, which cannot exist because of the random action of the ZPF. Although the ZPF, after disregarding its radiation pressure, only weakly modifies an orbit during a single revolution, in the long run the orbit becomes elliptical with slow variations of eccentricity, major axis, and even of the orbit plane. The Bohr radius R_1 is only the most probable value to find the electron in a thin spherical shell around R_1.

Let us now consider the case that a ZPF fluctuation has produced a small variation of an initially circular orbit, transforming it into an elliptical orbit, represented in polar coordinates r (distance) and θ (angle) by

$$r = \frac{R}{1-\epsilon\cos\theta} \tag{16}$$

If the eccentricity ϵ is much less than 1, Eq. (16) is equivalent, to within second order terms in ϵ, to

$$\begin{aligned} x &= R\cos\theta + \epsilon R\cos 2\theta \\ y &= R\sin\theta + \epsilon R\sin 2\theta \end{aligned} \tag{17}$$

In fact, it is

$$\begin{aligned} r &= \sqrt{x^2+y^2} = R\sqrt{1+\epsilon^2+2\epsilon\cos\theta} \\ &\simeq R + \epsilon R\cos\theta \simeq \frac{R}{1-\epsilon\cos\theta} \end{aligned} \tag{18}$$

In a Keplerian motion there is conservation of angular momentum $\mathit{\Gamma}$, so that ω depends only on the distance r, with

$$\omega(r) = \frac{\mathit{\Gamma}}{mr^2} \tag{19}$$

Putting Eqs. (18) into (19) and defining $\omega_0 = \mathit{\Gamma} m^{-1}R^{-2}$ as the angular frequency associated with radius R, we have to first order in ϵ

$$\omega = \omega_0 - 2\epsilon\omega_0\cos\theta \tag{20}$$

Solving Eq. (20) iteratively in θ, we derive to first order

$$\theta = \int \omega \mathrm{d}t \simeq \omega_0 t - 2\epsilon\sin(\omega_0 t) \tag{21}$$

Substituting this equation into the polar equation for the ellipse, given by Eq. (16), we obtain the trajectory as a function of time t. Since in Eq. (16) the term $\cos\theta$ is multiplied by ϵ and we are limiting our calculation to first order in $\epsilon \ll 1$, we can neglect the second term.

With $\theta = \omega_0 t$, the first term on the r.h.s. of Eq. (21) represents the main circular motion. By analogy to planetary motions, we term it the deferent. On it, there is a second circular motion (ϵ times the first one) considered as an epicycle. Since we take $\epsilon \ll 1$, the motion is practically circular, so that the radiated power remains unaltered. What drastically changes is the absorbed power since now there are four harmonic oscillators. The epicycle rotates with angular velocity 2ω with respect to the laboratory [see Eq. (17)]. However, since the epicycle rotates around a point that, in turn, rotates with ω, what is effective for the absorbed power is the relative frequency $2\omega - \omega = \omega$, i.e., the same frequency as the one of the deferent. Consequently, the absorbed power for the first excited state can be written as:

$$P_{\rm abs}^{\rm excited} = n\, P_{\rm abs}|_{\rm circ} \tag{22}$$

with $P_{\rm abs}|_{\rm circ}$ given by Eq. (13) and $n = 2$, corresponding to 2 planar motions (hence 4 harmonic oscillators with the same angular frequency ω_0).

The Bohr orbit corresponds to $n = 1$, i.e., to one planar motion (hence two harmonic oscillators). More generally, a periodical elliptical motion can be expanded in a Fourier series:

$$x = R\cos\theta + R\sum_{n=2}^{+\infty}\epsilon_n\cos(n\theta+\varphi_n)$$

$$y = R\sin\theta + R\sum_{n=2}^{+\infty}\epsilon_n\sin(n\theta+\varphi_n) \quad (23)$$

where φ_n are constant phases. Each additional term corresponds to a circular motion which, being relevant to the same electron, is epicycloidal. If we limit the model to $n = 3$, we have an epicycle rotating with angular velocity 3ω (in the approximation $\theta = \omega t$) on another epicycle rotating with angular velocity 2ω, in turn rotating on the deferent with angular velocity ω. The relative, effective frequencies for absorption from the ZPF are $3\omega - 2\omega = 2\omega - \omega = \omega$, i.e., the same of case $n = 2$. Because $\epsilon_i \ll 1$ in Eq. (23), the radiated power remains the same as for a circular orbit, i.e., it is still given by Eq. (14), whence

$$P_{\rm rad}^{\rm excited} \simeq P_{\rm rad}|_{\rm circ} \quad (24)$$

Equating the absorbed and radiated powers, from Eqs. (13), (14), (22), and (24) we obtain

$$mv_nR_n = n\hbar \quad (25)$$

i.e., Bohr's condition for quantization.

Let us examine the transition between two states. If the radius R of the orbit changes very slowly, we may consider it to be in quasi-equilibrium, so that $e^2/R^2 = mv^2/R$, i.e.,

$$v = \frac{e}{\sqrt{mR}} \quad (26)$$

Putting Eq. (26) into Eqs. (22) and (24), and using Eqs. (13) and (14), the radiated and absorbed powers are given by

$$P_{\rm abs}^{\rm excited} = \frac{2ne^5\hbar}{3c^3m^{\frac{5}{2}}R^{\frac{9}{2}}} \quad (27)$$

$$P_{\rm rad}^{\rm excited} = \frac{2e^6}{3c^3m^2R^4} \quad (28)$$

With a given value of n and for $P_{\rm abs}^{\rm excited} \simeq P_{\rm rad}^{\rm excited}$, as an average effect we have stable equilibrium for a given radius R_n. In fact, if $R = R_n + \delta R$ (with $\delta R \ll R_n$), we have $P_{\rm rad}^{\rm excited} > P_{\rm abs}^{\rm excited}$ and the radius R decreases. *Vice versa*, if we have $R = R_n - \delta R$, then $P_{\rm rad}^{\rm excited} < P_{\rm abs}^{\rm excited}$ and the radius R increases, as shown in Fig. 3.

There are, however, the fluctuations of the ZPF (besides its average effect), which can easily destroy the small amplitude ($\epsilon_n R$) of one of the epicyclic motions. In this case, $P_{\rm abs}^{\rm excited}$ loses two harmonic oscillators (passing from n to $n-1$) and we have $P_{\rm rad}^{\rm excited}$ sensibly larger than $P_{\rm abs}^{\rm excited}$. As a consequence, the electron motion becomes, on average, a spiral motion towards the lower most probable orbit $n-1$.

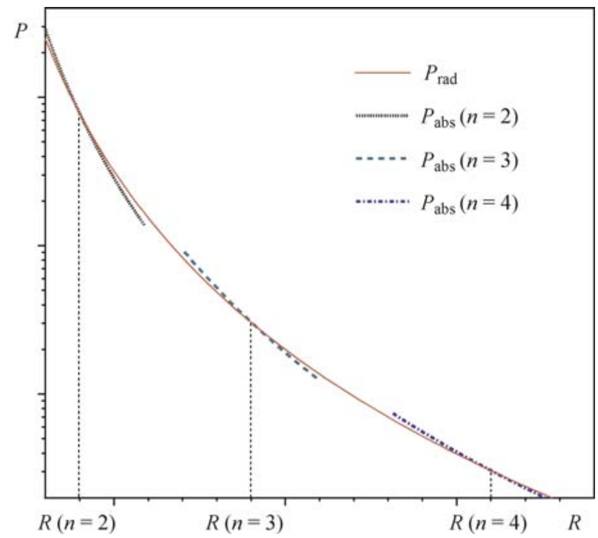

Fig. 3 Radiated power $P_{\rm rad}$ [10^{-7}J · s^{-1}] and absorbed power $P_{\rm abs}$ [10^{-7}J · s^{-1}] vs radius R [cm] of the electron circular orbit. For any number n of plane orbits (in QM terms, n is the principal quantum number) there is a stable point of intersection $P_{\rm abs}(n, R)\, P_{\rm rad}(n)$ if the average effect of the zero-point field (ZPF) is considered. The ZPF fluctuations concordant with the radiation damping cause the transition from n to $n-1$.

The net radiated energy is twice the one of the ZPF corresponding to the net observable weighted average frequency $\langle\omega\rangle$.

4.5 Other four drawbacks of pure SED

Even neglecting the Lorentz force due to the random fields, pure SED has four other additional drawbacks. The first is that SED works well only when there are linear forces. In the case of nonlinear forces, as in the quartic anharmonic oscillator, the results of pure SED are in disagreement with those of QM [34]. A second additional shortcoming which always troubled researchers is that SED implies broad spectra for radiation and absorption of rarefied gases, instead of the sharp observed lines! Indeed, according to SED, the quasi-elliptical orbit of an electron around a nucleus undergoes a maximum relative change of 10^{-5} during a revolution, if compared with the corresponding Keplerian orbit. Then, after 10^6 revolutions, the energy and, particularly, the revolution frequency can change radically. Although there is average equilibrium between the radiated and absorbed power for orbits not very different from Bohr's orbits, there is a net observable radiation for all the intermediate orbits with their large spread in frequency of revolution. For instance, in the passage from the second to the first orbit of Bohr, there is a classical spread in frequency of a factor of 8.

A third additional drawback of SED was the impossibility of explaining the diffraction of electrons, and a fourth one was that the Schrödinger equation has been derived in particular cases only. The impossibility of deriving the Schrödinger equation by pure SED when nonlinear forces are present (as in the case of atoms, where the Coulomb force is highly nonlinear) is related to the first additional drawback discussed above.

We will see in the next section that the introduction of spin, i.e., SEDS, eliminates all the drawbacks present in pure SED and, moreover, explains four effects so far unexplained by QED.

5 SED with spin (SEDS)

Spin was originally introduced by Pauli, who called it a "nonclassically explainable two valuedness".

When Goudsmit provided an intuitive model for it, it was as a rotation of an elementary particle around its own symmetry axis. However, that turned out to be an incorrect picture of the physical phenomenon, because an electron (or a quark), on the basis of scattering experiments at LEP, has a maximum size less than 10^{-19}m, and with that radius and a peripheral speed equal to that of light, the angular momentum would be less than $10^{-6}\hbar$.

5.1 A new conception of the spin motion

Schrödinger, solving the Dirac equation for an isolated electron, found a motion at the speed of light along a circular trajectory, having the Compton radius

$$R_e = 3.86 \times 10^{-13}\text{m} \tag{29}$$

Barut and Zanghi [35] showed that circular motion was the best interpretation for spin, but it seemed a contradiction having a particle moving at the speed of light without having infinite mass, and infinite e.m. radiation.

The complete solution comes from the filament theory [16, 17], where special relativity (SR) is not present at the particle level, but only with respect to the ideal point O around which the particle performs its "spin gyration".

SR is a consequence of that "gyration" [36]. What is usually considered as the electron velocity $\boldsymbol{v}$ is not the real velocity of the gyrating particle, but the one of the center O around which the electron revolves. The real motion of the particle is a helix, as shown in Fig. 4. The pitch of the helix depends on $v < c$. Only for a pitch tending to infinity will the speed of O (usually considered as the electron speed) tend toward the speed of light c.

Let us consider a reference frame F' at rest with O. For F' the gyration period is

$$T' = 2\pi R_e / c \tag{30}$$

For the frame F, at rest with the laboratory, O moves with a velocity $\boldsymbol{v}$ perpendicular to the plane of gyration

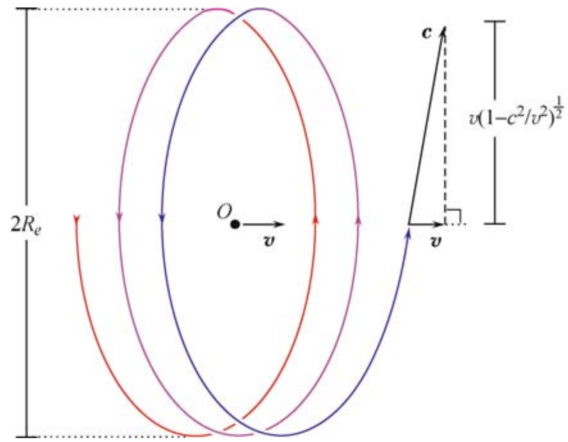

Fig. 4 If the center O of the gyration has velocity $\boldsymbol{v}$, the electron moves along a helix. With c being the local velocity of the electron, by the Pythagorean theorem we obtain Eq. (32), i.e., the time transformation of special relativity.

α. Since for F the electron moves with speed c along a helix, the period T to traverse a pitch is longer. Precisely, the component $c_\perp$ on α of its velocity is

$$c_\perp = \sqrt{c^2 - v^2} = c\sqrt{1 - v^2/c^2} \tag{31}$$

so that the period is

$$T = \frac{2\pi R_e}{c_\perp} = \frac{2\pi R_e}{c\sqrt{1 - v^2/c^2}} = \gamma T' \tag{32}$$

just as given by SR, but also as derived here from Galilean kinematics by the Pythagorean theorem.

With no SR at the particle level, the radiated power due to spin gyration can be derived from Eq. (14) where we put $v = c$

$$P^{\text{spin}}_{\text{rad}} = \frac{2}{3}\frac{e^2 c}{R_e^2} \tag{33}$$

R_e denoting the radius of the electron spin gyration, numerically expressed by Eq. (29). That power is $(c/v)^2 > 10^{12}$ times the power radiated by an electron in its motion of revolution around a proton in a hydrogen atom, and expressed by Eq. (14). Consequently, the ZPF is practically due to the spin gyration [36].

The power absorbed by the spin gyration from the ZPF can be derived from Eq. (13), where we put $\omega_0 = c/R_e$. It is

$$P^{\text{spin}}_{\text{abs}} = \frac{2}{3}\frac{e^2\hbar}{mR_e^3} \tag{34}$$

Equating Eq. (33) to Eq. (34) yields

$$mcR_e = \hbar \tag{35}$$

The spin axis $\hat{\boldsymbol{n}}$ can assume any direction, as shown in Fig. 5. In the presence of a magnetic field $\boldsymbol{B}$, the spin axis precesses around $\boldsymbol{B}$. What is called "spin up" is an $\hat{\boldsymbol{n}}$ distribution in a half sphere having $\boldsymbol{B}$ as a symmetry axis. For "spin down", the symmetry of the half-sphere is antiparallel to $\boldsymbol{B}$. The average value, the only one measurable, is

$$\Gamma_B = \hbar \int_0^{\pi/2} \mathrm{d}\theta \sin\theta \cos\theta = \frac{\hbar}{2} \tag{36}$$

which is the standard value. With the above distribution of $\hat{\boldsymbol{n}}$, it has been proved by Pitowsky [37] that the procedure used by John Bell to derive the famous inequalities leads to results in agreement with QM, thus eliminating the speculations regarding "superluminar speeds", and consequent nonlocality. There is no longer any seeming, or presumed, contrast [38] between special relativity and atomic physics (as interpreted by SEDS). Consequently, one can argue that SEDS is in some sense more robust than QM, because it predicts the same results as QM regarding the Aspect experiments [39], but avoids both the EPR paradoxes, and any necessity to have superluminal speeds.

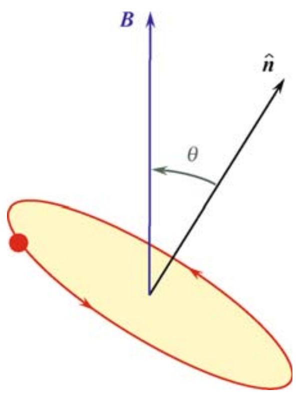

Fig. 5 What is called a "spin up" means a distribution of the spin axis $\hat{\boldsymbol{n}}$ in a half sphere having $\boldsymbol{B}$ as a symmetry axis.

5.2 The ZPF is caused by the spin radiation of all the particles of our expanding universe

As shown in Fig. 6, if all the centers of the electron spin gyrations were at rest with respect to the laboratory, the power spectral density would be a narrow spread around an almost Dirac delta function centered at $\omega_e = c/R_e$. However, if we consider spherical shells concentric with the observer in our expanding universe, the contribution of the shells decreases as their radii increase, because of the Döppler–Fizeau effect. The result is $\rho(\omega) \propto \omega^3$ for $\omega < \omega_e/10$ as shown in Ref. [40].

At the beginning of the universe the particles could not be in a steady-state condition because $\rho(\omega)$ did not grow *versus* ω as ω^3 for $\omega_e/10 < \infty$. Consequently, the spin radius decreased spiralling up to $\sim 10^{-16}$ of the present value of R_e, and $\rho(\omega) \propto \omega^3$ up to $\simeq 10^{15}\omega_e$. For $\omega > 10^{15}\omega_e$ the ZPF spectrum began to increase more mildly and, after a maximum value, decreases to zero at $\omega \simeq 10^{16}\omega_e$. At present, the departing from ω^3 dependence begins at lower ω values because of Döppler–Fizeau reduction of the upper cutoff, upper maximum, and of all parts of the spectrum appreciably different from $\propto \omega^3$.

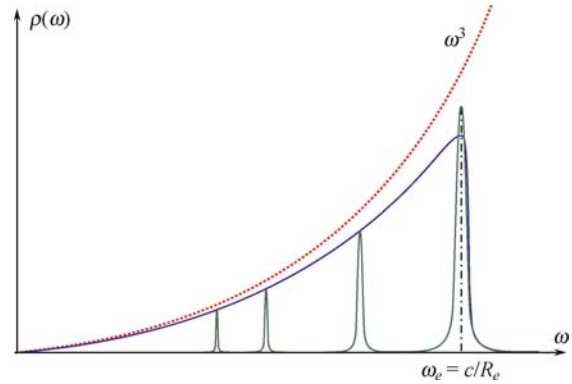

Fig. 6 If all the centers of the electron spin gyrations were at rest with respect to the laboratory, the power spectral density $\rho(\omega)$ would be a narrow spread centered at $\omega_e = c/R_e$. However, ours is an expanding universe, whence, at least for $\omega < 0.1\omega_e$, it is $\rho(\omega) \propto \omega^3$ due to the Döppler–Fizeau effect.

The ZPF with $\rho(\omega) \propto \omega^3$ in the observable region, the natural upper cutoff due to the maximum spin frequency before primordial recombination of particles with antiparticles, and the proportionality constant in terms of cosmological quantities [36], constitute the first fundamental result beyond the possibilities of QED.

The second result beyond QED is the explanation of an anomaly in an experiment regarding the neutrino rest mass [2].

The third result beyond QED regards the origin of electrical noise having power spectral density proportional to $1/f$ (when $f = \omega/2\pi$ denotes frequency). That long standing problem was studied successfully via the introduction of the ZPF. Not only did this lead to one explanation of the origin of $1/f$ noise, but also a dependence on the electron number density was found, which resulted in excellent agreement with the experimental findings for the case of ultrapure semiconductors [3, 4].

The fourth result unexplainable by QED, and mentioned in the Abstract, is shown in the next subsection.

5.3 A radically different equation of motion for a spinning particle

A spinning particle in a constant field $\boldsymbol{E}$ lying in the plane of spin gyration increases the spin radius in one half of the trajectory and decreases in the other half. The result is a null acceleration for the centre O of particle circular trajectory. Therefore, the particle can accelerate along its spin axis $\hat{\boldsymbol{n}}$, and the equation of motion, neglecting the self-reaction, is [2, 36]

$$m_* \boldsymbol{a} = \boldsymbol{F} \cdot \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \tag{37}$$

where m_* is the inertial mass when $\hat{\boldsymbol{F}} = \hat{\boldsymbol{n}}$ (indeed, in this case, we have $m_* \boldsymbol{a} = \boldsymbol{F}$). When an e.m. wave impinges on an electron, the electric field produces a velocity variation $\delta \boldsymbol{v} \propto \hat{\boldsymbol{n}}$, so that the additional acceleration due to the Lorentz force $\delta \boldsymbol{F}_L$ vanishes, since

$$\delta \boldsymbol{F}_L \cdot \hat{\boldsymbol{n}} = e\delta \boldsymbol{v} \times \boldsymbol{B} \cdot \hat{\boldsymbol{n}} \propto e\hat{n} \times \boldsymbol{B} \cdot \hat{\boldsymbol{n}} = 0 \tag{38}$$

Only when $\hat{\boldsymbol{n}}$ precesses, $\langle \delta \boldsymbol{F}_L \cdot \hat{\boldsymbol{n}} \rangle$ is no longer zero. The Einstein–Boyer–Rueda mechanism for the acceleration of an electron in the ZPF is strongly reduced, since an electron with spin is only sensitive to a ZPF frequency that is roughly equal to its precession frequency. This mechanism can still justify the existence of the most energetic cosmic rays, but the acceleration requires some thousands or millions of light years in the intergalactic space. This is the fourth result of SEDS beyond QED.

The quenching of the mechanism of acceleration due to the component $\boldsymbol{E}_r + (\boldsymbol{v}/c) \times \boldsymbol{B}_r$ in Eq. (11) also explains why good results for the atoms are obtained even when considering only the effect of $\boldsymbol{E}_r$. In this scenario, we no longer have self-ionization of atoms.

5.4 Narrow spectral lines in a stochastic process

We also overcome the impossibility of pure SED to explain the narrow spectral lines that are either emitted or absorbed by gases. The new equation of motion strongly reduces the random impulses due to radiation pressure of the ZPF, which would vanish in the absence of $\hat{\boldsymbol{n}}$ precession. However, the torques due to the extended spin orbit and due to the atomic nucleus produce a precession of $\hat{\boldsymbol{n}}$. As a consequence, the ZPF exerts random impulses on the precessing spinning (or, better, gyrating) electron, which strongly perturbs the regular spiraling motion that there would be if only classical radiation damping were present. The actual motion is similar to a spiral-like trajectory with a superimposed rapid diffusion.

In other words, while the classical spiraling motion requires $\sim 10^6$ revolutions to pass from an excited state (for instance the 2P state of an H atom) to the ground state (the 1S state for H), the rapid diffusion is such that the orbit major axis changes from 2P's to 1S's in only $\sim 10^2$ revolutions. Consequently, on the average, the complete transition takes $n \sim 10^6/10^2 = 10^4$ passages from one state to the other. The Fourier transform of the net radiated electric field is

$$\begin{aligned}\tilde{E}(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}t E(t) \exp(-\mathrm{i}\omega t) \\ &= \frac{1}{2\pi}\sum_{s=1}^{n-1}\int_{t_s}^{t_{s+1}} \mathrm{d}t E(t) \exp(-\mathrm{i}\omega t)\end{aligned} \tag{39}$$

where the time interval $t_2 - t_1$ corresponds to the first passage, and $t_n - t_{n-1}$ to the last passage. If we consider the Fourier transform corresponding to $t_2 - t_1$ only, we have a very broad spectrum, mainly contained between $\omega(2\mathrm{P})$ and $\omega(1\mathrm{S}) = 8\omega(2\mathrm{P})$. This would be the spectrum according to pure SED. If we now include the other passages, between the two states, the Fourier transform at a given intermediate ω increases by a factor $\sqrt{n}$, because the different waves have differently distributed ω values and random phases.

On the other hand, the radiated field $\tilde{E}(\langle\omega\rangle)$, calculated in correspondence with the average value $\langle\omega\rangle$ of each passage, increases as n, because $\langle\omega\rangle$ changes very little between one passage and another. The ratio $\tilde{E}(\langle\omega\rangle)/\tilde{E}(\omega)$ is roughly $\sqrt{10^4} = 10^2$ in the example considered here. In practice, $\tilde{E}(\langle\omega\rangle)$ is the only one observed, the other values of $\tilde{E}(\omega)$ are included in the background noise. Obviously, there is a Gaussian spread about $\langle\omega\rangle$, since the average value of each passage is slightly different from the others. However, if we consider $\mathcal{N}$ atoms that radiate, the Fourier transform $\tilde{E}_{\mathrm{tot}}(\omega)$ of all the radiated fields has a still sharper line around

$$\langle\omega\rangle_{\mathrm{tot}} = \frac{1}{\mathcal{N}n}\sum_{s=1}^{\mathcal{N}}\sum_{i=1}^{n}\omega_{is} \tag{40}$$

because it corresponds to $\mathcal{N}n$ passages.

5.5 Derivation of the Schrödinger equation, then correct solutions even when there are nonlinear forces

Through SEDS, which is SED with spin, it becomes possible to derive the Schrödinger equation for a single particle [41] and for many distinguishable particles [42], and these results have been discussed in a positive light in the "*News and Views*" section of *Nature* [43]. A more elaborate derivation of the Schrödinger equation was then produced [44], where additional nonlinear terms were added. The effect of the additional terms was shown to be a correction of $\sim 1\%$ to the Lamb shift [45].

In all cases, the Schrödinger equation can be derived when the ZPF is not modified by boundary conditions. In fact, for its derivation there must be no dissipation, i.e., no friction. According to the Einstein–Hopf Eq. (2), no friction is possible only if $\rho \propto \omega^3$ exactly. If at low frequencies (i.e., below the plasma frequency of the boundary material) the ZPF spectrum ρ is modified, a tiny friction in vacuum (at present undetected) should be present. That implies a modification of the Schrödinger equation if applied to spinning electrons having precession frequencies inside the modified ω interval. The effect on those electrons can also be huge if their precession frequencies are in resonance with some frequencies (of the spectrum) left among others completely quenched, as in the case examined in Section 6, where QM and SEDS lead to different predictions.

5.6 Diffraction of "photons" through two slits (Young experiment)

In the first quantization, i.e., by the Schrödinger equation, the e.m. field is not quantized, and it remains the classical one. That is just what is obtained by SEDS, where, as already pointed out for SED at the beginning of Section 4.2, the classical e.m. field is maintained, with the addition of classical, stochastic, ubiquitous ZPF brought about by the spin radiation (see Section 5.2). The interference term in the two slit diffraction (Young experiment) is immediately explained by the superposition of the fields radiated by the electrons lying on the edges of the two slits, and acted upon by a transversely large (so as to include the two slits), impinging e.m. wave.

Troubles afflict the interpretation of QED, where point-like photons appear as a consequence of second quantization. In fact, how can a point-like photon, passing through a slit, feel the presence of the other slit? There is only Feynman's inexplicability (see the Introduction), or Wheeler's smoking dragon just beyond the wall where the slits are obtained. Nevertheless, QED makes recourse to the complementarity principle and conceives the associated wave function, that has to be transversely large so as to include the two slits. However, if we have a transversely narrow beam passing through only one of the two slits, classical physics predicts no interference term, because there is no longer any e.m. field radiated by the electrons lying on the other slit. That prediction is confirmed by experiments, so that, in QED, the associated wave has to be intelligent, narrowing itself transversely in order to be limited to a single slit. Put in a humoristic way, a single photon passing through one of the slits should know whether other photons belonging to the same beam can, or not, pass even through the other slit, in order to transmit past and future information to its guiding, associate wave.

5.7 Diffraction of electrons through two slits in a conductive wall

Classically, electrons are considered as point-like. For point-like particles, not only in QED, but also in QM there is no qualitative explanation for the interference due to the other slit. They are left with Wheeler's smoking dragon. On the contrary, in SEDS there are the standing waves of the ZPF between the edges of the two slits. Obviously, their frequencies cannot exceed the plasma frequency ω_p, beyond which it is as if the conducting wall with the two slits was nonexistent, because it is completely transparent for $\omega > \omega_p$. For metals, ω_p is much higher than that producing the extreme observable peaks, so that the above limitation cannot be detected. Well, when an electron is close to one slit, it feels those standing waves of ZPF, and undergoes transverse impulses that bring about the interference aspect of the diffraction pattern.

Thus limited, the qualitative explanation seems to be achievable by pure SED. The point is that the conductive wall decreases the ZPF for all $\omega < \omega_p$, so that far from the wall the ZPF is stronger. Why are electrons insensitive to the more intense ZPF far from the wall? By pure SED the impulses due to the Lorentz force of the random field $\boldsymbol{B}_r$ should accelerate electrons up to $> 10^{20}$eV. The answer for SEDS has already been given in Section 5.3, showing that the Lorentz force due to $\boldsymbol{B}_r$ has no effect if the spin axis $\hat{\boldsymbol{n}}$ does not precede, as mathematically expressed by Eq. (38). Now the $\hat{\boldsymbol{n}}$ precession far from the walls is only due to $\boldsymbol{E}_r$, and it is very small [8], i.e., it occurs at low frequencies thus producing very small transversal impulses $\hbar\omega_e/2c$. It is only when an electron approaches one slit, that the induced charge on the wall, and mainly on one edge of the slit, exerts a torque on the extended spin orbit, that undergoes an effective precession. The precession frequency ω_n depends on the distance between the center of the spin orbit, and one edge of the slit through which the electron is passing by. When ω_n is close to the angular frequency of a ZPF standing wave, the electron undergoes a transverse impulse.

Let us now pass to the quantitative explanation. The ZPF is modified by the conducting wall up to the plasma frequency ω_p of the metal (see Fig. 7). The *spatial* Fourier transform of the ZPF *amplitude* with $\boldsymbol{E} = \boldsymbol{0}$ on the walls and $\boldsymbol{E} \neq \boldsymbol{0}$ within the slits ($2b$ apart from each other along the y axis) is (see Refs. [36, 46, 47])

$$\rho\,(k_y) = \frac{1}{2b}\int_{-b}^{+b} \mathrm{d}y\,(k_y b)\exp\,(\mathrm{i}k_y y) = \frac{\sin\,(k_y b)}{k_y b} \quad (41)$$

The corresponding spatial distribution of the energy modes allowed by the slits is proportional to $(k_y b)^{-2}\sin^2(k_y b)$, with intensity maxima for

$$k_y = 0 \text{ and } k_y b = \pi\,(n + 1/2)\ ,\text{ with } n = 0, 1, 2, 3, \ldots \quad (42)$$

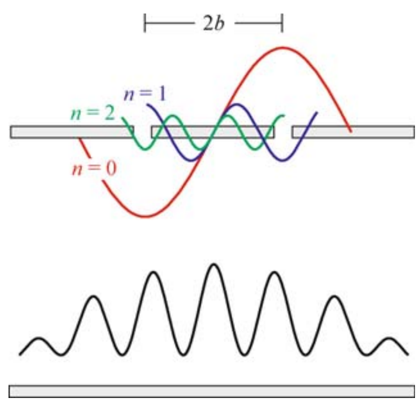

Fig. 7 The diffraction of electrons passing two slits is due to the ZPF modified by the conducting wall up to the plasma frequency of the metal. The Fourier transformed waves have been drawn for $n = 0, 1, 2$. According to Eq. (42), their amplitudes in correspondence of the right slit is $(-1)^n\,\pi^{-1}\,(n + 0.5)^{-1}$.

These are just the intensity maxima for a plane wave of either e.m. radiation or of a transversely large beam of electrons, according to QM. We have now to show that electrons modify their trajectory because of the ZPF action, and that the probability maxima of their arrivals on the screen correspond to those of the ZPF given by Eq. (42). In fact, when an electron approaches the wall with the slits, it induces an opposite charge on it. As said above, the induced charge exerts a force and a torque on the extended spin orbit, whose axis $\hat{\boldsymbol{n}}$ assumes a rather rapid precession [36]. As a result, the small range of frequency of the ZPF around the precession frequency ω_n of the electron spin gives a transverse impulse to the electron, in the amount of Ref. [47]

$$m\left\langle v_{\perp}^{2}\right\rangle^{1/2}=\frac{\hbar\omega_n}{2c} \tag{43}$$

If v is the speed of the electron, the consequent deviation is

$$\sin\theta=\pm\frac{\left\langle v_{\perp}^{2}\right\rangle^{1/2}}{v}=\pm\frac{\hbar\omega_n}{2mvc} \tag{44}$$

Now, ω_n depends on the distance r from the nearest edge and is therefore distributed from zero (for an electron passing through the middle of the slit) to a large maximum value when r is an atomic distance. Consequently, the intensity maxima are practically those of the spatial, squared amplitudes of the ZPF, given by Eq. (41). In other words, the transverse deviations of electrons passing a slit depend on the energy (hence momentum) per normal mode of the ZPF. The intensity of the deviated beams (i.e., of their crowding on a far screen) depends on the spatial density of modes allowed by the slits and given by Eqs. (41) and (42). With the use of Eqs. (42) and (44), the intensity maxima correspond to those of [36, 46]

$$\sin\theta_M=0 \text{ and } \sin\theta_M=\pm\frac{\hbar\pi\,(n+1/2)}{2bmv} \tag{45}$$

with $n=0,1,2,3,\ldots$, in agreement with experiments, and with QM when the electron beam is transversely large so as to include both slits.

5.8 On the interpretation of the Aharonov–Bohm (AB) effect

Let us insert a confined magnetic field $\boldsymbol{B}_0$ between the two slits. Theoretically, the confined $\boldsymbol{B}_0$ can be obtained by an indefinite solenoid. Practically, it is obtained by a toroidal magnet, and better so if it is a superconductor [48–50]. In both cases, the magnet can be placed either just before, or just after, the wall with the slits. When $\boldsymbol{B}_0$ is switched on, the pattern (on the screen) of the interference between the two slits undergoes a displacement, as if the electrons traversed $-\boldsymbol{B}_0$. This effect, foreseen by Aharonov and Bohm (AB) [51], has always been considered as nonlocal, because $\boldsymbol{B}_0=0$ outside the solenoid along the classical path of the electrons. According to Aharonov and Bohm it is the vector potential $\boldsymbol{A}$, and not $\boldsymbol{B}_0$ that is responsible for the phase change of the particle wave function and leads to the observable phase shift or interference pattern displacement.

There are other effects of the AB type discussed in the literatures [52–59] that, besides electrons, involve other particles that possess a magnetic dipole and/or an electric dipole. As shown by one of us (Refs. [60–62]) all these effects can be described in a unitary way and, as is the case for the AB effect, can be described by means of the Schrödinger equation. In all the effects of the AB type, the particles travel in a field- or force-free region of space [60–62]. Since there are no forces or fields acting locally on the particles, the effects of the AB type are considered to be nonlocal effects.

Nevertheless, several authors have tried to explain the observable phase shift arising in these effects as due to the action of local forces acting on the particles. We mention here two of such attempts, one due to Boyer [63, 64], and the other to two of us [65], where the variable field due to the moving electron is considered to interact with the magnet currents. The latter radiates and only a very small fraction of the radiated field acts again on the moving electrons. That tentative explanation was in the right direction but gave a negligible contribution.

Even accepting that the effects of the AB type are force- or field-free effects, there is no inconsistency with the SEDS formulation or point of view. In fact, as mentioned above, the Schrödinger equation has been derived from SEDS [41, 42] and, since this equation explains all the effects of the AB type [60–62], we conclude that they are compatible with the SEDS assumptions. It is worth recalling that in these effects the particles feel no forces or fields on the *average*. However, this does not exclude that, at the microscopic level, there are local interactions or forces that are being averaged to zero at the normal macroscopic level. The description of what goes on at the microscopic level is left for a future research paper.

Finally, since this is a review article, it is worth recalling a point that has not been emphasized in the literature and that recently has been considered by Spavieri [66]. This author shows that in order to observe these effects, one needs to compare the obtained interference pattern with a reference pattern. For example, in the AB effect, the pattern corresponding to the value $\boldsymbol{A}$ of the vector potential is formed on a screen where it is visible and then compared with the pattern corresponding to $\boldsymbol{A}_0$. The relative displacement of the two patterns, i.e., what is actually being measured in these effects, can thus be determined and is related to the AB phase shift $\Delta\phi$. It should be emphasized that the pattern corresponding to $\boldsymbol{A}$ is observed while the current in the solenoid is kept constant. The magnetic field outside the solenoid is zero and thus, the electrons travel in a field-free region of space. Similar conditions occur for the pattern corresponding to $\boldsymbol{A}_0$, which is being generally taken as $\boldsymbol{A}_0 = 0$.

However, it is not possible to determine the AB phase shift $\Delta\phi$ by observing solely the pattern corresponding to $\boldsymbol{A}_0$ or the pattern corresponding to $\boldsymbol{A}$. In order to observe the AB phase shift, both patterns must be created and compared during the experiment. This is equivalent to having the current of the solenoid changed and having the vector potential go from the value $\boldsymbol{A}_0$ to the value $\boldsymbol{A}$, or vice versa. Obviously, if $\boldsymbol{A}$ were to change with time, the electric field $\boldsymbol{E} = -\boldsymbol{\partial}_t\boldsymbol{A}/c$ would be created and the corresponding force $\boldsymbol{f} = e\boldsymbol{E}$ would act on the electron beam producing the phase variation. This fact has led to a better understanding of the properties of gauge invariance in the effects of the AB type and also to the discovery of a new effect of the AB type [66].

It should be clear now in what sense the AB effect is a field-free (or nonlocal) effect: in the sense that each interference pattern is formed and made visible on the screen separately, where it is then observed while keeping the solenoid current constant, so that the outside magnetic field is zero and no force acts on the classical path of the electrons.

At this point one could ask:

In a classical explanation of the AB effect, what is the force acting on the electrons that would be able to produce the observed AB phase shift?

The answer is given in Ref. [66] and consists in the following. In order to produce the relative displacement of the interference patterns related to the AB phase shift, an ideal local action on the electrons takes place when the solenoid current is changed and the vector potential goes from $\boldsymbol{A}_0$ to $\boldsymbol{A}$, or vice versa. Since $\Delta\phi$ is linked to the variation $\boldsymbol{A}_0 - \boldsymbol{A}$ it is possible to express $\Delta\phi$ in terms of the time integral of $\boldsymbol{f} = e\boldsymbol{E} = -\boldsymbol{\partial}_t\boldsymbol{A}/c$, the force that would act on the beam of charged particles if the intensity of the current in the solenoid were made to vary during the time interval t from the value $\boldsymbol{A}_0(t = 0)$ to the final value $\boldsymbol{A}(t)$. Of course, the time integral of this force corresponds precisely to the time integral of the local force acting on the electron beam, as assumed in the cited attempts [63, 64] and [65] to derive the AB effect classically.

5.9 Proposal of two experiments to discriminate between QM and SEDS

In Section 5.6, we have seen that the diffraction of e.m. waves through two slits (Young experiment) is correctly predicted in both QM and SEDS. The interference term (of the two slits) is present if the e.m. beam is transversely large, so as to include both slits. On the contrary, if the e.m. beam is transversely smaller than the mutual distance of the two slits, the interference term disappears (as also confirmed experimentally). With point-like "photons", the qualitative explanation in QED presents difficulties because there is no effective physical connection. Independently of that qualitative difficulty (only for QED), both SEDS and QED correctly predict the same results, as shown in Section 5.7. However, it would be wrong to think that the two theories are equivalent simply because in this case they predict the same right result. Actually, only SEDS gives an intuitive, clear, local explanation according to common sense.

Things are different if we consider electrons. In QED, photons and electrons should share the same behavior, in particular there would be no interference term if the electrons (as point-like particle) can pass only through one of the two slits. On the contrary, in SEDS the interference term should be always present, because any electron, always passing through one of the two slits, feels the ZPF waves due to the boundary conditions, i.e., the Fourier transformed waves of the ZPF being present only between the two slits (and not on the conducting wall). All the details are examined in Refs. [36, 46, 47].

A relevant experiment could therefore discriminate between QM and SEDS. The experiment can be done by a transmission electron microscope, where in the diffraction zone the electron beam is already narrow. The wall with the two slits, placed in the diffraction zone, has the possibility to be displaced by a micrometric skew. Initially, the beam impinges only on the wall, and nothing appears on the screen. When something appears, it is because part of the electron beam begins to pass through a single slit. At this point, if no interference term is present, QM is right, and SEDS is wrong. Vice versa, if it is immediately present. Displacing further the wall, the electron beam includes both slits. According to QM, the interference pattern should appear at this later stage, while for SEDS nothing new occurs.

The above first experiment has been discussed in detail in Ref. [47], where a second experiment of discrimination is proposed. It consists of performing the Young experiment with an insulating wall that has two slits in it. In that case, even with an electron beam that is transversely large enough to include the two slits, there should be, according to SEDS, some modifications of the intensities of the peaks other than the central one. In fact, the frequency-dependent component of the ZPF is not completely cancelled inside the wall. However, QM, which does not rely on the ZPF, predicts no difference for either a conducting or an insulating wall.

6 Conclusions

This paper is mainly aimed at giving the present state-of-the-art of the classical approaches to QM, or even to QED. There have been four reasonable approaches to it.

The first two, examined in Sections 2 and 3, have only a historical interest, and are discussed thoroughly in those sections.

The third approach, which was the focus of Section 4, is pure SED, which has been developed over 30 to 40, years. It is based on the assumption that there is an ubiquitous, stochastic e.m. field throughout space, coinciding with the zero-point field (ZPF) of QED. However, while the ZPF is considered as real, i.e., nonrenormalized in SED, it is renormalized in QED. The explanation of the stability of the atoms, as done in Section 4.3, is a proof of the presence of a real ZPF. Other proofs are the explanation of the anomaly in the measurements of the neutrino rest mass [2], and, more recently, of the origin of one kind of electrical noise observed in semiconductors and elsewhere, having power spectral density inversely proportional to the frequency [3, 4], the so called $1/f$ noise.

The above results, though, are obtained by SEDS treated in Section 5. The main cause of SEDS correctness is the new equation of motion (37) that, in the absence of spin axis precession, completely quenches the Lorentz force due to the random magnetic field $\boldsymbol{B}_r$. That result cannot be obtained by pure SED, and has to be postulated in order to obtain some good predictions, that, however, were already obtained in QM. On the contrary, new results are obtained by SEDS. First of all, as shown in Section 5.2 and in Fig. 6, the origin and limitation (i.e., the natural upper cutoff) of the ZPF, brought about by the spin radiation of all the particles in our expanding universe [40]. Spin is also the cause of SR as hinted in Section 5.1, and fully derived in Refs. [36, 40]. Spin also brings about QM, since, by it, it is possible to derive the Schrödinger equation (see Section 5.4).

The quenching of the Lorentz force due to $\boldsymbol{B}_r$ is not complete because the spin axis $\hat{\boldsymbol{n}}$ undergoes small random changes because of the ZPF. Consequently, the acceleration mechanism studied by Rueda [26, 27] is saved, although strongly reduced. A free particle in an intergalactic space can take millions of years to reach $\sim 10^{21}$ eV, while according to the original SED calculations of Rueda it could take only a few nanoseconds. Consequently, a free electron would acquire $\sim 10^{21}$ eV in cathodic TV set! The explanation of the high energy tail of cosmic rays (without leading to paradoxical behavior in a cathodic TV set) is therefore a new result of SEDS, unpredictable by QED, where the ZPF is renormalized.

In an atom, the action of the nucleus on the extended spin orbit causes precessions, so that $\boldsymbol{B}_r$ produces a diffusion that makes the average frequency of the revolving electron to be the only one observable, thus explaining the narrow spectral lines (Section 5.4).

A noticeable $\hat{\boldsymbol{n}}$ precession occurs also when an electron approaches one edge of a slit (see Section 5.7). That is why the transverse impulses of the ZPF are effective only in correspondence of the slits, and not far from them where the ZPF is more intense at the low frequencies. In fact, the boundary conditions on the wall with two slits eliminate the majority of the low frequencies of the ZPF. That fundamental fact cannot be accounted for by pure SED. As an electron approaches one slit, its precession frequency increases, and when it is equal to one of the standing e.m. ZPF waves between the two slits, it undergoes a transverse impulse. Notice that the transverse deviations should occur not only when the electron has equal probability to pass through either one or the other of the two slits, but also when it can pass through only one, known slit. That prediction is at variance with QM, so that the following experiment could discriminate between QM and SEDS. In a transmission electron microscope, we can focus the beam through only one slit. In that case, if no interference pattern due to the other slit appears, QM is right, and SEDS has to be rejected. Vice versa, if the interference pattern still occurs there is no agreement with QM.

Note that there is no contradiction between the above different prediction of SEDS with respect to QM, and the fact that the Schrödinger equation has been derived from SEDS. In fact, as emphasized in Section 5.7, the Schrödinger equation has been derived when the ZPF is not modified at frequencies smaller than plasma physics, because of boundary conditions. In the case of such a modification, the Schrödinger equation remains valid for frequencies larger than that of plasma, but no longer for smaller frequencies.

In Section 5.8 we discuss the Aharonov–Bohm effect, obviously in agreement with the prediction of the Schrödinger equation, derived from SEDS.

In the last subsection (Section 5.9), two experiments are proposed that could discriminate between QM and SEDS. The first considers electron diffraction with a beam passing through only one of the two slits. The second experiment has been proposed in Ref. [47]. It regards the diffraction of electrons in a transversely large beam including both slits, but obtained in an insulating wall. SEDS predicts some modifications of the intensities of the peaks other than the central one. Because of the fundamental nature of the possible outcomes, it would be very interesting if appropriate experimental technology, such as that developed by Tonomura *et al.* [48], could be brought to bear on at least one of such important experiments.

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