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RESEARCH ARTICLE

# Enhanced vacuum Rabi splitting and double dark states in a composite atom-cavity system

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The transmission spectrum of four-level atoms in a cavity is calculated. It is shown that the four separate peaks associated with normal mode splitting and intra-cavity double dark states can be observed simultaneously. The position and intensity of the four peaks can be controlled by the intensity of the third interacting light. Therefore, the enhancement of normal mode splitting by a third coupling light of the intra-cavity four-level atoms is developed.

**Keywords** vacuum Rabi splitting, transmission spectrum, electromagnetically induced transparency, four-level system, double dark states

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Normal mode splitting (also called vacuum Rabi splitting at the low-photo limit in an atom-cavity system) has been considered as an important effect of the quantum nature in cavity quantum electrodynamics (CQED). Eberly first predicted that the double-peak radiation spectrum of single two-level atom appeared in the presence of a cavity field [1]. Later, it is proved that this effect also remained for a collection of $N$ two-level intracavity atoms [2]. At the same time, the experimental studies of the vacuum Rabi frequency were carried out in the strong coupling regime of atom and cavity [3–7]. Up to now, the vacuum Rabi sidebands have been developed in many regimes, such as semiconductor quantum micro-cavity with quantum wells [8], photonic crystals [9], micro-disk micro-cavity [10], and hot-atoms ensemble [11]. Recently, the three-peak radiation spectrum with two vacuum Rabi sidebands and a central peak was observed in a system consisting of an optical cavity and three-level atoms [12, 13]. The observation of two distinct Rabi sidebands or "cavity polaritons" could be taken as the characteristic of the multi-atom enhanced coupling of atom-cavity, while the central peak represented the dark-state polariton as the combination of cavity and the phenomenon of electromagnetically induced transparency (EIT). Since the central peak has a smaller line-width than the natural line-width and the cavity line-width, it may be used for frequency stabilization, high-resolution spectroscopic measurements or other interesting applications such as control of optical multi-stability [14, 15].

In this paper, we present an analysis of the transmission spectrum of a cavity with four-level atoms; four transmission peaks with two Rabi sidebands and double central peaks are obtained. Different from previous theories and experiments, the third control laser is applied in addition to coupling and probe laser, which results in the enhanced Rabi splitting and two intra-cavity dark states. This paper is structured as follows. First, we calculate the nonlinear susceptibility of a homogenously-broadened medium of four-level atoms. Second, we qualitatively discuss the properties of Rabi sidebands and double intra-cavity dark states and make a comparison when the control laser is on or off.

Considering a homogeneously broadened medium inside an optical cavity, the four-level (upper levels $|a\rangle$ and $|d\rangle$, lower levels $|b\rangle$ and $|c\rangle$) N-type atomic system is shown in Fig. 1. The level $|a\rangle$ and $|b\rangle$ are coupled by a probe field of frequency $\nu_p$ with Rabi frequency $\Omega_p$ and detuning $\Delta_p = \omega_{ab} - \nu_p$, the level $|a\rangle$ is coupled to the level $|c\rangle$ by a strong coupling field with Rabi frequency $\Omega_c$. Here, we introduce the third control laser coupling to the $|d\rangle$ and $|c\rangle$ transition with Rabi frequency $\Omega_s$. The

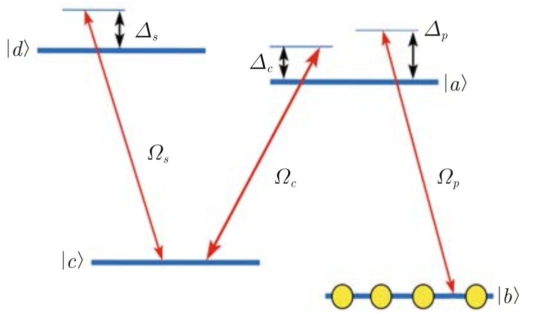


**Fig. 1** The diagram of four-level atomic system interacting with a coupling field, a control field and a probe field.

density-matrix equation of motion is

$$\begin{aligned}
\dot{\rho}_{ab} &= -(\mathrm{i}\Delta_p + \gamma_a)\rho_{ab} + \frac{\mathrm{i}}{2}\Omega_c\rho_{cb} + \frac{\mathrm{i}}{2}\Omega_p(\rho_{bb} - \rho_{aa}) \\
\dot{\rho}_{cb} &= -(\mathrm{i}\Delta_p + \gamma_{bc})\rho_{cb} + \frac{\mathrm{i}}{2}\Omega_c\rho_{ab} + \frac{\mathrm{i}}{2}\Omega_s\rho_{db} - \frac{\mathrm{i}}{2}\Omega_p\rho_{ca} \\
\dot{\rho}_{db} &= -(\mathrm{i}\Delta_p + \gamma_d)\rho_{db} + \frac{\mathrm{i}}{2}\Omega_s\rho_{cb} - \frac{\mathrm{i}}{2}\Omega_p\rho_{da} \\
\dot{\rho}_{ca} &= -\gamma_a\rho_{ca} + \frac{\mathrm{i}}{2}\Omega_c(\rho_{aa} - \rho_{cc}) + \frac{\mathrm{i}}{2}\Omega_s\rho_{da} - \frac{\mathrm{i}}{2}\Omega_p\rho_{cb} \\
\dot{\rho}_{da} &= -(\gamma_a + \gamma_d)\rho_{da} + \frac{\mathrm{i}}{2}\Omega_s\rho_{ca} - \frac{\mathrm{i}}{2}\Omega_p\rho_{db} - \frac{\mathrm{i}}{2}\Omega_c\rho_{dc} \\
\dot{\rho}_{dc} &= -\gamma_a\rho_{dc} + \frac{\mathrm{i}}{2}\Omega_s(\rho_{cc} - \rho_{dd}) - \frac{\mathrm{i}}{2}\Omega_c\rho_{da}
\end{aligned} \tag{1}$$

where $\gamma_a$ and $\gamma_d$ represent the decay rate of the upper levels $|a\rangle$ and $|d\rangle$ . Assuming that the atoms are initially in the ground level $|b\rangle$, and the probe light is much weaker than the coupling and control lights, we acquire the approximation: $\rho_{bb}^{(0)} = 1, \rho_{aa}^{(0)} = \rho_{cc}^{(0)} = \rho_{ca}^{(0)} = \rho_{da}^{(0)} = 0$. In the following calculation, we ignore the weak incoherent decay rate of the ground states $\gamma_{bc} = 0$. Therefore, the simplified density-matrix equation of motion is

$$\begin{aligned}
\dot{\rho}_{ab} &= -(\mathrm{i}\Delta_p + \gamma_a)\rho_{ab} + \frac{\mathrm{i}}{2}\Omega_c\rho_{cb} + \frac{\mathrm{i}}{2}\Omega_p \\
\dot{\rho}_{cb} &= \frac{\mathrm{i}}{2}\Omega_c\rho_{ab} - \mathrm{i}\Delta_p\rho_{cb} + \frac{\mathrm{i}}{2}\Omega_s\rho_{db} \\
\dot{\rho}_{db} &= \frac{\mathrm{i}}{2}\Omega_s\rho_{cb} - (\mathrm{i}\Delta_p + \gamma_d)\rho_{db}
\end{aligned} \tag{2}$$

The steady-state solution to Eq. (2) can be written as:

$$\rho_{ab} = \frac{\Omega_p(4(\gamma_d + \mathrm{i}\Delta_p)\Delta_p - \mathrm{i}\Omega_s^2)}{2(\gamma_d + \mathrm{i}\Delta_p)(-4\mathrm{i}\gamma_a\Delta_p + 4\Delta_p^2 - \Omega_c^2) - 2(\gamma_a + \mathrm{i}\Delta_p)\Omega_s^2} \tag{3}$$

The complex susceptibility $\chi$ is then obtained from the polarization [16]:

$$P = \varepsilon_0 E_p \chi \mathrm{e}^{-\mathrm{i}\nu_p t} + c.c. = 2N_a\wp_{ab}\rho_{ab}(t) + c.c. \tag{4}$$

where $N_a$ is the atom number density, and $\wp_{ab}$ is the dipole moment matrix element for the transition between $|a\rangle$ and $|b\rangle$.

Because $\chi = \chi' + \mathrm{i}\chi''$, where the real part $\chi'$ is responsible for the medium dispersion and the imaginary part $\chi''$ for the medium absorption, respectively, Eq. (3) could show us the function of the real and imaginary parts of the complex susceptibility, as shown in Fig. 2.

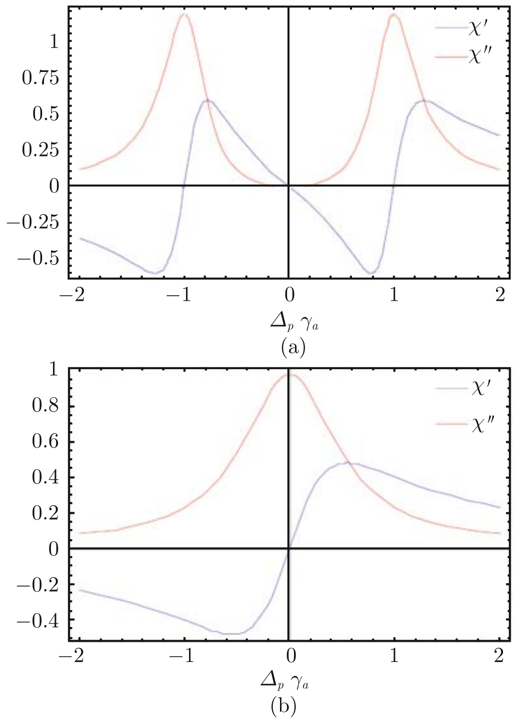


**Fig. 2** Dispersion and absorption properties vs. the detuning of the probe laser when the control laser on **(a)** or off **(b)**. *Blue lines*: dispersion; *Red lines*: absorption.

We define an average decay rate $\gamma_a$ as a unit for the following calculating. When the control laser is turned off, we have a transparency and large dispersion medium under the action of the strong coupling field at a zero probe detuning [Fig. 2(a)]: this is a normal situation of three-level EIT system. When the control laser is turned on, we find the probe spectrum changes from EIT to enhanced absorption at the zero probes detuning, meanwhile the dispersion curve changes its sign in Fig. 2(b). As a result, with the third control light turned on, the quantum coherence is switched from electromagnetically induced transparency to electromagnetically induced amplification.

Now we consider that a four-level atomic medium with complex susceptibility $\chi$ is placed inside a ring cavity with three mirrors, one of which is the input mirror with transmissivity $T_1 = 3\%$, and the transmissivity of the output mirror is $T_2 = 1.4\%$. Assuming that the atomic medium is included into a cell with a length $l = 5$ cm, and the cavity has a length $L = 40$ cm. The cavity is resonate with the probe light, and the transmission spectrum of the probe light is thus detected. In this case the probe field with amplitude $E_p$ and frequency $\nu_p$, propagating through the atomic cell, is modulated by a factor exp $(-\mathrm{i}k\chi l/2)$, in which $k = \nu/c$ is the wave number of the cavity field. In the following discussion, we make a small-gain approximation that the exponential may be expanded to first order, and the rate of the change of the cavity field due to the medium is approximated by $\delta E_p c/L$, here $L/c$ is the round-trip time. On the other hand, assuming that the cavity resonates with the atomic transition frequency $\omega_{ab}$, and the detuning of the probe field will be $\Delta_p = \omega_{ab} - \nu p$, so that an extra phase $\Delta_p L/c$ is accrued. We get the equation of the rate of the change of intra-probe field amplitude:

$$\dot{E} = \left(-\alpha + \chi''\frac{\nu_p l}{2L} + \mathrm{i}\Delta_p - \mathrm{i}\chi'\frac{\nu_p l}{2L}\right)E + \frac{c\sqrt{T_1}}{L}E_0 \quad (5)$$

Eq. (5) can be solved for the steady-state intra-cavity amplitude $E$, and then we obtain the total intensity transmission of the system:

$$T = \frac{T_1 T_2 c^2/L^2}{\left(\alpha - \chi''\dfrac{\nu_p l}{2L}\right)^2 + \left(\Delta_p - \chi'\dfrac{\nu_p l}{2L}\right)^2} \quad (6)$$

where $\alpha$ is the overall intra-cavity loss rate, considering the nonlinear attenuation of the ring cavity for different modes.

For reference, we introduce the vacuum Rabi frequency $g_p\sqrt{N_p} = \wp_{ab}[N_p\nu_p/(2\varepsilon_0\hbar V)]^{1/2}$, with $\wp_{ab}$ represents the atomic dipole moment, $\nu_p$ the frequency of probe laser, and $V, N_p$ the effective cavity mode volume and effective atomic number density, respectively, indicating the interaction strength between laser and atoms [17].

Substituting $g_p\sqrt{N_p}$ into Eq. (6) and ignoring the effect of decay rate $\gamma_a, \gamma_d$ and the overall intra-cavity loss $\alpha$, just to find the trend of the rule for the transmission spectrum, we get

$$T \sim \frac{\Delta_p^2(-4\Delta_p^2 + \varOmega_c^2 + \varOmega_s^2)^2}{[g_p^2 N_p(4\Delta_p^2 - \varOmega_s^2) + \Delta_p^2(-4\Delta_p^2 + \varOmega_c^2 + \varOmega_s^2)]^2} \quad (7)$$

From Eq. (7), we can easily find that there are four peaks in the transmission spectrum: if $\Delta_p$ is large, the position of two Rabi sidebands in this limit is given by

$$\Delta_p \sim \pm\sqrt{g_p^2 N_p + \varOmega_c^2/4 + \varOmega_s^2/4} \quad (8)$$

If $\Delta_p$ is small, namely, the probe field almost resonants with the atomic transition $|a\rangle \leftrightarrow |b\rangle$, the position of double dark states in this limit is given by

$$\Delta_p \sim \pm\varOmega_s/2 \quad (9)$$

Taking Eqs. (8) and (9) into Eq. (6), we get the transmission of two Rabi sidebands and double dark states respectively:

$$T_{\mathrm{side}} \sim \frac{4(4g_p^2 N_p + \varOmega_c^2 + \varOmega_s^2)}{16\alpha^2 g_p^2 N_p + 4\alpha^2\varOmega_c^2 + 4\alpha^2\varOmega_s^2 + \varOmega_s^4}$$

$$T_{\mathrm{dark}} \sim \frac{4}{4\alpha^2 + \varOmega_s^2} \quad (10)$$

If we consider a condition that the vacuum Rabi frequency $g_p\sqrt{N_p}$ is larger than the other two Rabi frequencies, as shown in Fig. 3, we take $\varOmega_s = 10$ for instance. As the vacuum Rabi frequency increases, the splitting of the two Rabi sidebands becomes large: it means that atoms exchange energy with cavity through photons more frequently. This effect is consistent with the result for the vacuum Rabi splitting in the normal two-level system. In Fig. 3, the other parameters are taken as: $\gamma_d = 0.8$, $\alpha = 0.5$, $\varOmega_p = 2$, $\varOmega_c = 6$.

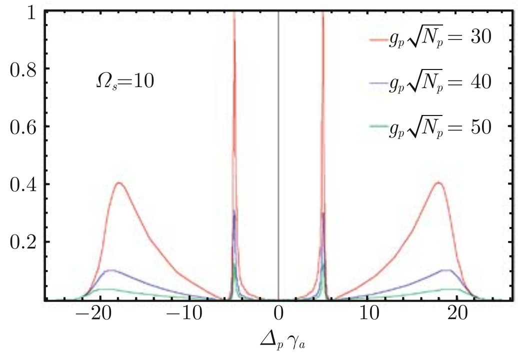


**Fig. 3** Transmission spectrum of the Vacuum Rabi splitting and double dark states vs. the detuning of the probe laser for different $g_p\sqrt{N_p}$.

We notice that if we apply $\varOmega_s = 0$, all the results mentioned above return to previous theories or experimental results of three-level system, therefore, we could easily obtain the corresponding three transmission peaks of three-level system.

It is clear that when the control laser intensity increases, the peaks of the two Rabi sidebands attenuate, and the width of the splitting is enhanced. As illustrated in Fig. 4, the Rabi sidebands attenuate and outspread slightly with the increase of the control laser, the space between these two sidebands becomes wide from $2\sqrt{g_p^2 N_p + \varOmega_c^2/4}$ to $2\sqrt{g_p^2 N_p + \varOmega_c^2/4 + \varOmega_s^2/4}$. Deferent from the Rabi sidebands, the double dark states exert a more simple relationship, determined barely by the Rabi frequency $\varOmega_s$. From Eqs. (9) and (10), we can easily hold the position and transmission of the double dark states. According to the Fig. 3, we note that although the Rabi sidebands change a lot, the double dark states stand still at $\Delta_p \sim \pm\varOmega_s/2$; however, if we fix the vacuum Rabi frequency and change the value of $\varOmega_s$, as shown in Fig. 4, the double dark states expand and attenuate apparently, which manifest that the variable $\varOmega_s$ can completely

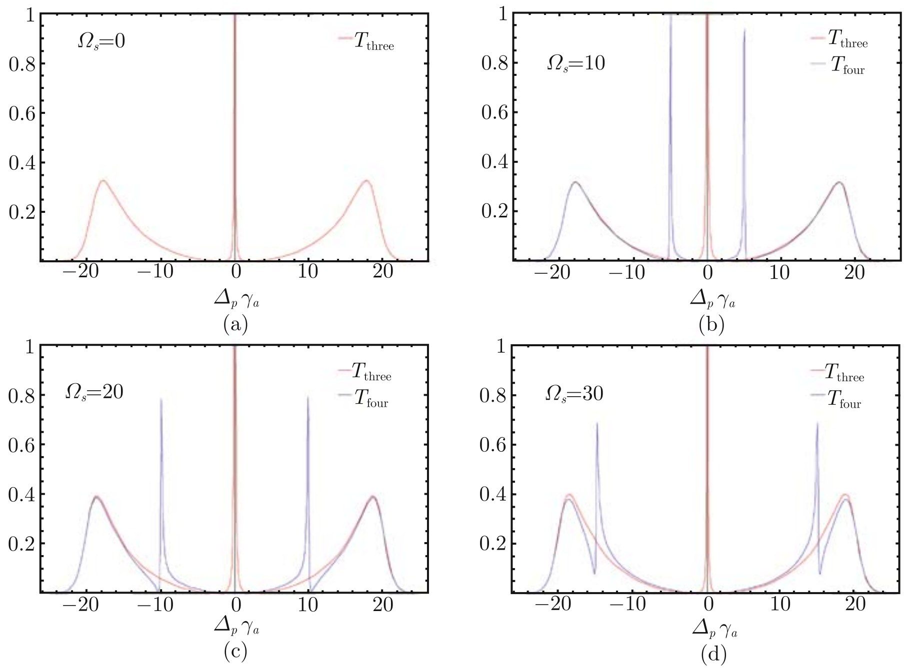


**Fig. 4** Transmission spectrum vs. the detuning of the probe field for different power of the control field and fixed $g_p\sqrt{N_p} = 30$. *Red lines*: three-level atom; *Blue lines*: four-level atom.

confirm the double dark states, position or intensity (for the loss $\alpha$ is determinate once we place the cavity well). It is possible for us to measure and manipulate these two quantum states through only one parameter. This tunable spectrum may be used to stabilize the cavity at a suitable position of detuning.

In view of the complexity of the CQED system, the system discussed in this paper may provide an applicable hot-atom system to observe the Rabi sidebands easily. Different from the three-level system, the four-level system can further enhance the Rabi sidebands splitting and the dark states. As shown in Fig. 5, when the Rabi frequency of control laser is large enough, the dark states disappear and the Rabi sidebands enhance. It is possible for us to observe the Rabi sidebands more clearly in the four-level system than in the three-level system.

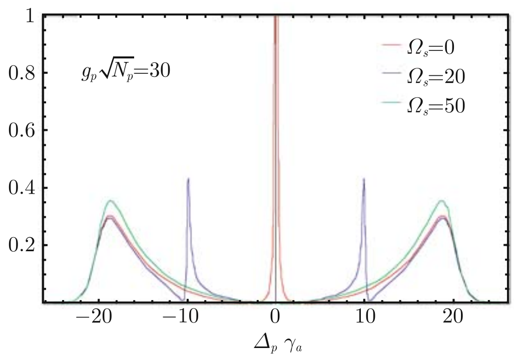


**Fig. 5** Transmission spectrum vs. the detuning of the probe field for different power of the control field and fixed $g_p\sqrt{N_p} = 30$.

In conclusion, we present a theory of cavity transmission spectrum of four-level N-type atoms. By controlling vacuum Rabi frequency $g\sqrt{N}$ and Rabi frequency $\Omega_s$, we could achieve the manipulation of the Rabi sidebands and the double intra-cavity dark states and observe the enhanced Rabi sidebands by sacrificing the power of the double dark states. Further theory calculation and experimental results will be presented hereafter.

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