Front. Phys. China, 2009, 4(2): 231–234
DOI 10.1007/s11467-009-0049-z
RESEARCH ARTICLE

Condensation and evolution of a space–time network

Qiao BI (毕桥)[1,3](✉), Li-li LIU (刘莉丽)[1,3], Jin-qing FANG (方锦清)[2,3]

[1] *Department of Physics, Science School, Wuhan University of Technology, Wuhan 430070, China*
[2] *China Institute of Atomic Energy, P.O. Box 275-27, Beijng 102413, China*
[3] *International Noble Academy, 1075 Ellesmere Road, Toronto, M1P 5C3 Canada*
E-mail: biqiao@gmail.com

Received October 28, 2008; accepted January 20, 2009

In this work, we try to propose in a novel way, using the Bose and Fermi quantum network approach, a framework studying condensation and evolution of a space–time network described by the Loop quantum gravity. Considering quantum network connectivity features in Loop quantum gravity, we introduce a link operator, and through extending the dynamical equation for the evolution of the quantum network posed by Ginestra Bianconi to an operator equation, we get the solution of the link operator. This solution is relevant to the Hamiltonian of the network, and then is related to the energy distribution of network nodes. Showing that tremendous energy distribution induces a huge curved space–time network may indicate space time condensation in high-energy nodes. For example, in the case of black holes, quantum energy distribution is related to the area, thus the eigenvalues of the link operator of the nodes can be related to the quantum number of the area, and the eigenvectors are just the spin network states. This reveals that the degree distribution of nodes for the space–time network is quantized, which can form space–time network condensation. The black hole is a sort of result of space–time network condensation, however there may be more extensive space–time network condensations, such as the universe singularity (big bang).

Keywords Loop quantum gravity, spin network, complex network, quantum network, black hole

PACS numbers 04.60.Pp, 04.70.Dy, 05.45.Tp, 89.75.2k

1 Introduction

The greatest challenging topic in contemporary fundamental physics is coordinating general relativity, quantum mechanics and quantum gravity research. This study will cause us to have profound changes in the concept of space, time and matter structure, and achieve a quantum revolution. At present, for the quite advanced theory of quantum gravity, there are two forms. First, quantization of general relativity, canonical quantum gravity belongs to this. Second, quantization of a classical theory which is deferent from general relativity, the general relativity is its low-energy limit. The super-string/M theory belongs to this. Loop quantum gravity [1] is the current representative form of canonical quantum gravity. Canonical quantum gravity is one only when there exist gravitational effects of quantum gravity. Comparing with the superstring/M theory, it does not include other different interactions. Its basic concept is to apply standard quantization procedures to general relativity, while general relativity is written in the canonical (Hamiltonian) form. According to historical development canonical quantum gravity can be divided into two stages, simple quantum gravity and loop quantum gravity. Roughly speaking, the former occurred before 1986, and the latter occurred after 1986. In simple quantum gravity, there is renormalization difficulty for UV divergence, and Loop quantum gravity has currently become the representative form.

According to the basic spirit of Loop quantum gravity, space–time is the gravitational field and space–time is quantized in the Plank scale; a space–time network is also formed, which uses the quantum of the volume as a node, connected through the quantum of the adjacent area. The basis of the space time network can be described by the spin network. Physical space time can be expanded by the spin networks as basis. The evolution of spin networks can be studied by calculating the spinfoam. Spinfoam is the main method for studying evolution of spin networks, based on the Feynman propagation in the Loop quantum gravity. The currently studied complex network theoretical method [2] is usually irrelevant to the study of the space–time network. However, recently, Bianconi and Barabási have related the Bose–Einstein condensation to the complex network research [3], and have conducted analogy research, inspiring us to think that the space–time network may be the product of mesh points self-organizing to form a complex network with the network topology and the evolution fully reflecting network's kinetic property.

In this work, we try to use the Bose and Fermi quantum network approach to present a framework for studying condensation and evolution of the space–time network [1]. Considering quantum network connectivity features, we introduce a link operator, and through extending the complex network equation for the evolution of th quantum network in the literature [3] to an operator equation, we get the solution of the link operator.

This solution is relevant to the Hamiltonian of the network, and then is related to the energy distribution of network nodes. Showing that tremendous energy distribution induces a huge curved space–time network may have space time condensation in high-energy nodes. For example, in the case of black holes, quantum energy distribution is related to the area, thus the eigenvalues of the link operator of the nodes can be related to the quantum number of the area, and the eigenvectors are just the spin network states. This reveals that the degree distribution of nodes for the space–time network is quantized, which can form the space–time network condensation. The black hole is a sort of space–time network condensation, however there may be a more extensive network of space–time setting, for example, the universe singularity (big bang).

2 Connectivity of the spin network

Let a graph Γ to be given be immersed in the manifold Σ, for which an ordering and an orientation have been chosen. Denote the nodes as the end points of the oriented curves in Γ, and joined by links l. Then j_l is an assignment of an irreducible representation to each link l and i_n is an assignment of an intertwiner to each node n. The intertwiner i_n associated with a node is between the representations associated with the links adjacent to the node. Γ becomes a collection of nodes n with links l. The triplet $S \equiv (\Gamma, i_n, j_n)$ is called a spin network embedded in Σ. The physical space–time is a combination of spin networks. More precisely, j_n represents unitary irreducible representations and i_n represents a basis in the space of the intertwiners between the representations adjacent to the nodes n. The area of a surface cutting n links of the spin network with $l = 1, \cdots, n$ quantized labels j_l is given by the eigenvalue of the area operator, $8\pi\gamma \frac{h}{2\pi} G \sum_n \sqrt{j_n (j_n + 1)}$, where γ is the Immirzi parameter which is a free dimensionless constant and G is the Newton constant. Considering the quantum property of the connectivity for nodes of the spin network, we introduce a link (degree) operator of node $\boldsymbol{K}$ which acts on the combination of the spin network states $|S\rangle = \sum_s \xi_s |s\rangle$ and gives the eigenvalue j_n by

$$\boldsymbol{K}|S\rangle = \tau^{f(j_n)}|S\rangle \tag{1}$$

where f is a function of j_n, ξ_s is the coefficient expanding $|s\rangle$ as $|S\rangle$ and $|s\rangle = (\Gamma, i_n, \alpha_n, \beta_n)$ is given by the spin network $s \equiv (\Gamma, i_n, j_n)$ in such way that $\langle U|\, j, \alpha, \beta\rangle = \left(R^j(U)\right)^\beta_\alpha$ forms an orthonormal basis in the Hilbert space $L_2[G, \mathrm{d}U]$), and introduces a spin network state $|s\rangle$ by the Peter–Weyl theorem [1].

Extending the rate equation of connectivity in Ref. [2] to the link operator $\boldsymbol{K}$, a dynamical equation for $\boldsymbol{K}$ can be constructed by

$$\frac{\partial \boldsymbol{K}}{\partial \tau} = m_+ \frac{\mathrm{e}^{\beta H}\boldsymbol{K}}{\mathrm{Tr}\,(\mathrm{e}^{\beta H}\boldsymbol{K})} - m_- \frac{\mathrm{e}^{-\beta H}\boldsymbol{K}}{\mathrm{Tr}\,(\mathrm{e}^{-\beta H}\boldsymbol{K})} \tag{2}$$

where τ is introduced as a proper time in this relativity system, the spin network state is associated with a general three-dimensional spacelike hypersurface, m_+ and m_- are proportional (suitable) coefficients (constant or function) for making Eq. (2) hlod, and H is a constrained Hamiltonian of the system given by

$$\begin{aligned} H &= \int N\mathrm{Tr}\,(F \wedge \{\boldsymbol{V}, A\}) \\ &= p_\tau + H_0 \end{aligned} \tag{3}$$

here, F (curvature) and A (complex field) are limits of the holonomy operators of small paths, { } is the Poisson bracket serving as quantum commutator, $\boldsymbol{V}$ is the quantum volume operator, and H_0 is a nonrelativistic Hamiltonian, and p_τ is the energy part of H [1, 5].

In the thermodynamical limit, $\mathrm{Tr}\big(\mathrm{e}^{\beta H}\boldsymbol{K}\big)$ and $\mathrm{Tr}(\mathrm{e}^{-\beta H}\boldsymbol{K})$ are supposed to tend to

$$\mathrm{Tr}\left(\mathrm{e}^{\beta H}\boldsymbol{K}\right) \to m_+ \tau \mathrm{e}^{\beta\mu_+} \tag{4}$$

$$\mathrm{Tr}\left(\mathrm{e}^{-\beta H}\boldsymbol{K}\right) \to m_- \tau \mathrm{e}^{-\beta\mu_-} \tag{5}$$

where μ_+, μ_- are suitable constants which are related to the asymptotic behavior of the network. This is so because $\mathrm{Tr}\big(\mathrm{e}^{\beta H}\boldsymbol{K}\big)$ and $\mathrm{Tr}\big(\mathrm{e}^{-\beta H}\mathbf{K}\big)$ can be assumed to be self-average and converge to their mean value and grow linearly in time under suitable conditions [4].

Thus, Eq. (2) becomes

$$\frac{\partial \boldsymbol{K}}{\partial \tau} = \left[\mathrm{e}^{\beta(H-\mu_+)} - \mathrm{e}^{-\beta(H-\mu_-)}\right] \frac{\boldsymbol{K}}{\tau} \tag{6}$$

and the solution of the equation therefore is given by

$$\boldsymbol{K} = \tau^{f(H)} \tag{7}$$

where $f(H)$ is defined by

$$f(H) = e^{\beta(H-\mu_+)} - e^{-\beta(H-\mu_-)} \tag{8}$$

This operator solution is correct and can be proved. In fact, by replacing $\boldsymbol{K} = \tau^{f(H)}$ into Eq. (6), we immediately have

$$\frac{\partial \tau^{f(H)}}{\partial \tau} = \left[e^{\beta(H-\mu_+)} - e^{-\beta(H-\mu_-)}\right] \frac{\tau^{f(H)}}{\tau} \tag{9}$$

which results in

$$f(H)\,\tau^{f(H)-1} = f(H)\,\frac{\tau^{f(H)}}{\tau} = f(H)\,\tau^{f(H)-1} \tag{10}$$

Therefore the solution allows one to obtain the mean value $\langle \boldsymbol{K} \rangle$ as:

$$\langle \boldsymbol{K} \rangle = (m_+ + m_-)\left(\frac{\tau}{\tau'}\right)^{f(E+M)} \tag{11}$$

with the initial condition

$$\langle \boldsymbol{K} \rangle_0 = m_+ + m_- \tag{12}$$

where $E+M$ represents the eigenvalue of $H = p_\tau + H_0$, E is related to the energy in the nodes which is the eigenvalue corresponding to p_τ, M is an eigenvalue corresponding to H_0, and relevant tracing calculation is performed with respect to the eigen-states of H; here, the eigen-state of H is assumed to be the combination of the spin network states $|S\rangle$ for consistence with the definition (1). This is reasonable because a certain combination of the spin network states can be constructed as an eigen-state of H although a single spin network state usually is not an eigen-state of H. When H acts on this eigen-state $|S\rangle$, $\langle \boldsymbol{K} \rangle$ can represent the general connectivity of the space–time network.

3 Condensation of the space–time network

In the Schwarzschild black hole environment, the energy E is related to the area A through [6]

$$E = \sqrt{\frac{A}{16\pi G^2}} \tag{13}$$

Thus, the mean value $\langle \boldsymbol{K} \rangle$, through Eq. (11), becomes

$$\langle s|\,\boldsymbol{K}\,|s\rangle = (m_+ + m_-)\left(\frac{\tau}{\tau'}\right)^{f\left(M+\sqrt{\frac{A}{16\pi G^2}}\right)}$$
$$= (m_+ + m_-)\left(\frac{\tau}{\tau'}\right)^{f\left(M+\sqrt{\frac{\frac{h}{2\pi}\gamma\sqrt{j_n(j_n+1)}}{2G}}\right)} \tag{14}$$

where the spin network state $|s\rangle$ is used for simplicity and is an eigen-state corresponding to E since E is related to A (the spin network state is the eigen-state of the area operator in Loop quantum gravity). In this case M is an action of H_0 to $|s\rangle$, which may be a function or number. However, $f\left(M + \sqrt{\frac{1}{2G}\frac{h}{2\pi}\gamma\sqrt{j_n(j_n+1)}}\right)$ is a function related to the quantum number $\sqrt{j_n(j_n+1)}$ or j_n. This shows that $\langle s|\,\boldsymbol{K}\,|s\rangle$ is quantized, with the quantum number of the link as $\sqrt{j_n(j_n+1)}$, where j_n is determined by eigenvalues of the area operator $\boldsymbol{A}$ with respect to spin network state s and represents the quantum number of the area, whether M is a function or a number. Hence, one of the properties of the quantized links of nodes is that there are possibly many links labelled by quantum numbers between two adjacent nodes with relevant quantum weights, while the classical link between the same two nodes is just one link with (or without) a classical weight.

Consequently, one can get

$$\frac{m}{m+m'} = \int \mathrm{d}(E+M)\,p(E+M) \cdot \frac{e^{\beta(E+M-\mu_+)}}{1-e^{\beta(E+M-\mu_+)}+e^{-\beta(E+M-\mu_-)}} \tag{15}$$

and

$$\frac{m'}{m+m'} = \int \mathrm{d}(E+M)\,p(E+M) \cdot \frac{e^{-\beta(E+M-\mu_+)}}{1-e^{\beta(E+M-\mu_+)}+e^{-\beta(E+M-\mu_+)}} \tag{16}$$

Here, the ratio $\frac{m}{m+m'}$ or $\frac{m'}{m+m'}$ represents a fraction of m or m' links attached or detached to nodes in the evolution of the network. The former represents the increase of the links on the node, and the second one represents the decrease of links on the node. When $\mu_+ = \mu_- = 0$ and $E \longrightarrow 0$, it is

$$\frac{m}{m+m'} = \frac{m'}{m+m'} = \int \mathrm{d}M p(M) \tag{17}$$

which shows that the quantized connectivity of the space–time network, in the very low energy condition, evolves to either an increase or decrease in time as a mixed process of attaching or detaching to nodes. While $\mu_+ = \mu_- = 0$ and $E \longrightarrow \infty$, it is not difficult to find $\frac{m'}{m+m'} = 0$, and one gets

$$\frac{m}{m+m'} \longrightarrow -\int \mathrm{d}E p(E) \tag{18}$$

This shows, in this situation that there is only a link increase process on the node while the detached process disappears! This reveals that a kind of condensation of the space–time network around the huge energy nodes may happen because there is only a monotonic attached link process in the evolution of the space–time network. For example, in the (Schwarzschild) black hole surroundings, there is a sort of space–time network condensation on the singularity with $E \longrightarrow \infty$. Furthermore, in the universal space–time scale, there may be a more extensive space–time network condensation, such as when the universe condenses itself as a single (many) singularity, a single (many) big bang.

4 Connectivity distribution

The connectivity distribution $P(\langle \boldsymbol{k} \rangle)$ is given by the sum of the probabilities of the nodes with energy E and connectivity $\langle \boldsymbol{k} \rangle$. Following the idea in Ref. [2], considering the degree of the nodes in our model where k changes to the mean value $\langle \boldsymbol{K} \rangle$ due to the quantum property of the link for the space–time network, we have to sum over all the modes with energy lower or higher than a threshold defined as $\epsilon_s = \frac{\mu_+ + \mu_-}{2}$, when $\langle \boldsymbol{k} \rangle < m + m'$ or $\langle \boldsymbol{k} \rangle > m + m'$, respectively,

$$\begin{aligned}
P(\langle \boldsymbol{k} \rangle) &= \theta(\langle \boldsymbol{k} \rangle - \langle \boldsymbol{k} \rangle_0) \int_{\epsilon_s > \frac{\mu_+ + \mu_-}{2}} \mathrm{d}(E+M)\, p(E+M) \frac{1}{\frac{\partial \langle \boldsymbol{k} \rangle}{\partial \tau}} \\
&\quad + \theta(\langle \boldsymbol{k} \rangle_0 - \langle \boldsymbol{k} \rangle) \int_{\epsilon_s < \frac{\mu_+ + \mu_-}{2}} \mathrm{d}(E+M)\, p(E+M) \frac{1}{\frac{\partial \langle \boldsymbol{k} \rangle}{\partial \tau}} \\
&= \theta(\langle \boldsymbol{k} \rangle - m - m') \frac{1}{m+m'} \int_{\epsilon_s > \frac{\mu_+ + \mu_-}{2}} \mathrm{d}(E+M)\, p(E+M) \frac{1}{f(E+M)} \left(\frac{\langle \boldsymbol{k} \rangle}{m+m'} \right)^{-1-\frac{1}{f(E+M)}} \\
&\quad + \theta(m+m' - \langle \boldsymbol{k} \rangle) \frac{1}{m+m'} \int_{\epsilon_s < \frac{\mu_+ + \mu_-}{2}} \mathrm{d}(E+M)\, p(E+M) \frac{1}{f(E+M)} \cdot \frac{1}{f(E+M)} \left(\frac{\langle \boldsymbol{k} \rangle}{m+m'} \right)^{-1-\frac{1}{f(E+M)}}
\end{aligned} \tag{19}$$

If $\mu_+ = \mu_- = 0$ and $E \to \infty$, it is possible that $\frac{1}{f(E+M)} = 0$, and $\int_{\epsilon_s > \frac{\mu_+ + \mu_-}{2}} \mathrm{d}(E+M)\, p(E+M) \frac{1}{f(E+M)}$ is finite (convergent and $\neq 0$), which results in

$$\begin{aligned}
P(\langle \boldsymbol{k} \rangle) &= \theta(\langle \boldsymbol{k} \rangle - m - m') \frac{1}{m+m'} \\
&\quad \cdot \int_{\epsilon_s > \frac{\mu_+ + \mu_-}{2}} \mathrm{d}(E+M)\, p(E+M) \\
&\quad \cdot \frac{1}{f(E+M)} \left(\frac{\langle \boldsymbol{k} \rangle}{m+m'} \right)^{-1} \sim B \frac{1}{\langle \boldsymbol{k} \rangle}
\end{aligned} \tag{20}$$

where the coefficient B is defined by

$$B = \int_{\epsilon_s > \frac{\mu_+ + \mu_-}{2}} \mathrm{d}(E+M)\, p(E+M) \frac{1}{f(E+M)} \tag{21}$$

and the integral variable is supposed to be irrelevant to $\langle \boldsymbol{k} \rangle$. This result means that the connectivity distribution $P(\langle \boldsymbol{k} \rangle)$ in the above approximations obeys the power law, and the space–time network is scale-free; the connectivity distribution $P(\langle \boldsymbol{k} \rangle)$ is inversely proportional to the increase of the connectivity $\langle \boldsymbol{k} \rangle$. This means the probability of $\langle \boldsymbol{k} \rangle$ tending to infinity is quite small if value of B is tiny. However, if B is big or is a function of $\langle \boldsymbol{k} \rangle$, then the situation of $P(\langle \boldsymbol{k} \rangle)$ is complicated, then, the relevant space–time network may not be scale-free.

5 Conclusions

The quantum connectivity of the space–time network can be calculated by introducing the link operator. The mean value of the link operator is a function of the eigenvalue for the energy part of the Hamiltonian operator of the space–time network. In the Schwarzschild black hole situation, the mean value of the link operator is a function of the quantum number of the area operator with respect to the spin network states, and the connectivity is quantized. Moreover, there is a sort of condensation of the space–time network around its extreme high energy nodes. In the condensation progression there exists only a monotonic attached link process in the evolution of the network, which may result in the singularity of the universe. The connectivity distribution can be represented by Eq. (20), which may or may not obey the power law.

Acknowledgements This work was supported by the grants from NSERC, CIPI, MMO, CITO, Wuhan University of Technology, and the National Natural Science Foundation of China (Grant No. 60874087).

References

1. C. Rovelli, Quantum Gravity, Cambridge: Cambridge University Press, 2004
2. R. Albert and A.-L. Barabasi, Rev. Mod. Phys., 2002, 74: 47
3. G. Bianconi, Phys. Rev. E, 2002, 66: 056123
4. G. Bianconi and A.-L. Barabasi, Phys. Rev. Lett., 2001, 86: 5632
5. T. Thiemann, Modern Canonical Quantum General Relativity, Cambridge: Cambridge University Press, 2007
6. C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation, San Francisco: W. H. Freeman and Company, 1973