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RESEARCH ARTICLE

Energy spectrum for a atrongly correlated network and local magnetism

Li-li LIU (刘莉丽)[1], **Qiao BI** (毕桥)[1,2] (✉)

[1] *Department of Physics, Science School, Wuhan University of Technology, Wuhan 430070, China*
[2] *Center for Advanced Nanotechnology, University of Toronto, Toronto, ON M5S 3E4, Canada*
E-mail: biqiao@gmail.com

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In this work, we consider a quantum strongly correlated network described by an Anderson s - d mixing model. By introducing the Green function on the projected formalism of the Schrieffer and Wolf transformation, the energy spectrum of the system can be obtained. Using this result we calculate the survivability distribution of the network and discuss the local magnetism in the network, which shows that the survivability is an important statistical characteristic quantity not just to reflect the network topological property but also dynamics.

Keywords strongly correlated system, quantum network, Green function

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1 Introduction

Nowadays, studying strongly correlated systems is an important direction that crosses many physical domains. The energy spectrum analysis for the strongly correlated system is crucial for studying the physical properties of these systems. However, the traditional perturbative method is not easy to use for getting the energy spectrum of these systems because these systems are strongly correlated. Using the retarded Green function approach, the spectrum analysis for the Anderson s - d mixed model has been achieved [1], but the energy spectrum for its Schrieffer and Wolf transformation is still unclear. In this work, we introduce a quantum strongly correlated network described by an Anderson s - d mixed model [2]. By using the Green function approach on the projected formalism of the Schrieffer and Wolf transformation, the energy spectrum of the network can be obtained based on the average field approach. This allows one to analyze some physical properties of the quantum network, such as average energy and the local magnetism in the nodes. As a demonstration, we calculate the connectivity distribution of the network based on the concept of the fermionic network [3], and discuss the local magnetism affected by the difference of the energy on the nodes.

2 Physical model

We consider a quantum network described by the Anderson s - d mixed model which mixes the impure atoms in a disordered matrix [2], i.e. the model is generally defined as a graph which is a collection of vertices or sites and unordered edges or bonds connecting certain distinct pairs of vertices [5]. Concretely speaking, in this network, a node of the network is composed of an electron confined in a quantum dot as an impurity. These nodes of the network are arranged as a topological disorder system without long distance order. For simplicity, this local electron is still called as d electron (as the original Anderson meaning). The free Hamiltonian of the nodes is described by $\sum_\sigma E_d n_{d\sigma}$, where $n_{d\sigma} = d_\sigma^+ d_\sigma$, d_σ^+ is a creation operator of the d electron with spin σ. The strong correlation between each node (quantum dot) is considered as the link of the network, and the corresponding Hamiltonian is supposed as $U n_{d\uparrow} n_{d\downarrow} = \dfrac{U}{2} \sum_\sigma n_{d\sigma} n_{d\bar{\sigma}}$, where $U \gg 1$ is defined as a correlated energy, and the spins are given by $\sigma = \uparrow, \downarrow$ and $\bar{\sigma} = -\sigma$. The environmental (or control) field of the network is composed of many conductive electrons between quantum dots. For simplicity, we call them as s electrons, and their free

Hamiltonian is given by $\sum_{k,\sigma} E_k n_{k\sigma}$, $n_{k\sigma} = C^+_{k\sigma} C_{k\sigma}$, where $C^+_{k\sigma}$ is a creation operator of the s electron with wave vector k and spin σ. The interaction taking place between d electron confined in the quantum dot and s electron moving outside the quantum dot is described by $\sum_{k,\sigma} V_{kd} \left(C^+_{k\sigma} d_\sigma + d^+_\sigma C_{k\sigma}\right)$. Considering there is no effect of external magnetic field and ignoring the s - d reciprocity, we define $V_{kd} = V$. Hence, the total Hamiltonian operator for the system can be expressed by

$$H = H_0 + H_1 \tag{1}$$

with

$$\begin{aligned} H_0 &= H_d + H_k + H_u \\ &= \sum_\sigma E_d d^+_\sigma d_\sigma + \sum_{k,\sigma} E_k C^+_{k\sigma} C_{k\sigma} + \frac{U}{2} \sum_\sigma n_{d\sigma} n_{d\bar{\sigma}} \end{aligned} \tag{2}$$

where

$$H_1 = \sum_{k,\sigma} V \left(C^+_k d_\sigma + d^+_\sigma C_k\right) \tag{3}$$

3 Projection transformation

In Anderson mixed model, we notice that as a result of d electron not coupling with the s electron, when $V = 0$, there are two energy levels of the impurity respectively, E_d and $E_d + U$, which correspond on three kinds of different electronic configurations in this network. This allows the nodes of the network to be possible in three combined status: $0(E_0 = 0)$, $1(\sigma = \uparrow$ or $\downarrow$, $E_{1\sigma} = E_d)$, and 2 ($\sigma = \uparrow$ and $\downarrow$, $2E_d + U$) occupations. Following the Schrieffer and Wolf transformation [6, 7], we put the Anderson model into a Hilbert space and introduce three projection operators to divide the total Hilbert space as three subspaces:

$$P_0 = (1 - n_{d\uparrow})(1 - n_{d\downarrow}) \tag{4}$$

$$P_1 = n_{d\uparrow}(1 - n_{d\downarrow}) + n_{d\downarrow}(1 - n_{d\uparrow}) \tag{5}$$

$$P_2 = n_{d\uparrow} n_{d\downarrow} \tag{6}$$

where P_n, $n = 0, 1, 2$ corresponds upon $0, 1, 2$ occupations, and satisfies the fundamental rule of the projection operator:

$$P_n^2 = P_n \tag{7}$$

$$P_n P_{n'} = 0 \quad (n \neq n') \tag{8}$$

Assume that the total wave function Ψ in the Hilbert space is composed by ψ_0, ψ_1, ψ_2 in these three subspaces, respectively, then the Schrödinger equation $H\Psi = E\Psi$ can be written as:

$$\begin{pmatrix} H_{00} & H_{01} & H_{02} \\ H_{10} & H_{11} & H_{12} \\ H_{20} & H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix} = E \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix} \tag{9}$$

where $H_{nn'} = P_n H P_{n'}$, in terms of the Anderson model, it is impossible to produce and eliminate two electronic interactions simultaneously, hence

$$H_{02} = H_{20} = 0 \tag{10}$$

Other off-diagonal matrix elements (such as H_{01} or H_{12}) can be obtained using the Pauli principle and the Fermionic operator equality $n^2_{d\sigma} = n_{d\sigma}$,

$$H_{01} = P_0 H P_1 = V \sum_{k,\sigma} C^+_{k\sigma} (1 - n_{d\sigma}) d_\sigma \tag{11}$$

$$H_{12} = P_1 H P_2 = V \sum_{k,\sigma} C^+_{k\sigma} d_\sigma n_{d\sigma} \tag{12}$$

At the same time the matrix is Hermitian, so

$$H_{10} = H^+_{01}, \quad H_{21} = H^+_{12} \tag{13}$$

When the nodes are in the 1 occupation, the local extra spin appear, which allow the network to show the type of local magnetic effect, as a self-organized system. Thus, by cancelling ψ_0 and ψ_2 from Eq. (9) and considering Eqs. (11) and (12), a kinetic equation is given by

$$\Theta \psi_1 = E_1 \psi_1 \tag{14}$$

where Θ is introduced by

$$\Theta = H_{11} + H_{12} (E - H_{22})^{-1} H_{21} + H_{10} (E - H_{00})^{-1} H_{01} \tag{15}$$

which represents the effective Hamiltonian of single occupation. So far, we have not made any approximation. When $V = 0$, we can replace $(E_{k'} - E_d - U)$ with $(E - H_{22})$, and $(E_d - E_k)$ with $(E - H_{00})$. In the second order of approximation with respect to V, we have

$$\begin{aligned} H_{12} (E - H_{22})^{-1} H_{21} = &\sum_{k,k',\sigma,\sigma'} \frac{V^2}{U + E_d - E_{d'}} \\ &\cdot (-1) C^+_k C_{k'} d_\sigma n_{d\sigma} d^+_{\sigma'} n_{d\sigma'} \end{aligned} \tag{16}$$

$$\begin{aligned} H_{10} (E - H_{00})^{-1} H_{01} = &\sum_{k,k',\sigma,\sigma'} \frac{V^2}{E_d - E_k} \\ &\cdot (-1) C^+_k C_{k'} d^+_{\sigma'} (1 - n_{d\sigma'}) d_\sigma (1 - n_{d\sigma'}) \end{aligned} \tag{17}$$

Considering $n_d = n_{d\sigma} + n_{d\bar{\sigma}} = 1$, and introducing notation for the spinning of a half:

$$\widehat{S}^z = \frac{1}{2} \left(d^+_\sigma d_\sigma - d^+_{\bar{\sigma}} d_{\bar{\sigma}}\right) \tag{18}$$

$$\widehat{S}^+ = d^+_\sigma d_{\bar{\sigma}} \tag{19}$$

$$\widehat{S}^- = d^+_{\bar{\sigma}} d_\sigma \tag{20}$$

Eqs. (16) and (17) are written as:

$$H_{12} (E - H_{22})^{-1} H_{21} = \sum_{k,k'} \frac{V^2}{U + E_d - E_{k'}}$$

$$\left[\left(\widehat{S}^z C_k^+ C_{k'} + \widehat{S}^- C_k^+ C_{k'}\right) - \frac{1}{2} C_k^+ C_{k'}\right] \tag{21}$$

and

$$H_{10}\left(E - H_{00}\right)^{-1} H_{01} = \sum_{k,k'} \frac{V^2}{E_k - E_d} \cdot \left[\left(\widehat{S}^z C_k^+ C_{k'} + \widehat{S}^- C_k^+ C_{k'}\right) + \frac{1}{2} C_k^+ C_{k'}\right] \tag{22}$$

By considering Eqs. (21), (22), and simplifying H_{11} as $H_{11} \approx H_0 = \sum_k E_k C_k^+ C_k$, Eq. (15) can be written as:

$$\Theta = H_0 + H_p + H_{sd} \tag{23}$$

where H_p is the scattering energy of the impurity,

$$H_p = \sum_{k,k'} J_{kk'} C_k^+ C_{k'} \overset{U\to\infty}{\longrightarrow} J \sum_{k,k'} C_k^+ C_{k'} \tag{24}$$

and H_{sd} represents the interaction with the impurity relating spinning degree-of-freedom,

$$\begin{aligned} H_{sd} =& -\sum_{k,k'} J_{kk'} \left(\widehat{S}^z C_k^+ C_{k'} + \widehat{S}^- C_k^+ C_{k'}\right) \overset{U\to\infty}{\longrightarrow} \\ & -J \sum_{k,k'} \left(\widehat{S}^z C_k^+ C_{k'} + \widehat{S}^- C_k^+ C_{k'}\right) \end{aligned} \tag{25}$$

This is so because the s electron locates around the Fermi surface, and can replace E_F with E_k and $E_{k'}$. Concerning $U + E_d > E_F > E_d$, one has $J_{kk'} < 0$, where in the strong correlation condition, $U \to \infty$, the interaction has no relation with k, k', which results in

$$\begin{aligned} J_{kk'} &= \frac{V^2}{2} \left\{\frac{1}{E_k - E_d} - \frac{1}{U + E_d - E_{k'}}\right\} \overset{U\to\infty}{\longrightarrow} J \\ &= -\frac{V^2}{|E_d - E_F|} < 0 \end{aligned} \tag{26}$$

Therefore, by introducing projected operator P and $Q = 1 - P$, using the above projected formalism (the Schrieffer and Wolf transformation) and neglecting the scattering energy H_p, an effective Hamiltonian (the intermediate [8, 9]) Θ can be described by

$$\begin{aligned} \Theta =& \sum_{k,\sigma} E_k C_{k\sigma}^+ C_{k\sigma} - \left\{J \sum_{k,k'} \widehat{S}^z \left(C_{k\sigma}^+ C_{k'\sigma} - C_{k\bar{\sigma}}^+ C_{k'\bar{\sigma}}\right) \right. \\ & \left. + J \sum_{k,k'} \widehat{S}^+ C_{k\bar{\sigma}}^+ C_{k'\sigma} + J \sum_{k,k'} \widehat{S}^- C_{k\sigma}^+ C_{k'\bar{\sigma}}\right\} \\ =& H_0 - (H_1 + H_2 + H_3) \end{aligned} \tag{27}$$

4 Green function formalism

Introducing an equation of motion for two-time retarded Green function, one gets

$$\langle\langle A(t); B(t')\rangle\rangle = -\mathrm{i}\theta(t - t') \langle [A(t), B(t')]\rangle \tag{28}$$

By differentiating the quotient of time, we get

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\langle\langle A(t); B(t')\rangle\rangle = \delta(t-t')\langle [A, B]\rangle + \langle\langle [A, H] \mid B(t')\rangle\rangle \tag{29}$$

Taking the Fourier transformation

$$\langle\langle A \mid B\rangle\rangle_\omega = \int \mathrm{d}t \mathrm{e}^{\mathrm{i}\omega(t-t')} \langle\langle A(t); B(t')\rangle\rangle \tag{30}$$

one gets a general formula of the Green function as:

$$\omega\langle\langle A \mid B\rangle\rangle_\omega = \langle [A, B]_+\rangle + \langle\langle [A, H] \mid B\rangle\rangle_\omega \tag{31}$$

This allows one to obtain an equation of motion for the Green function $\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle$ by

$$\omega\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega = \langle [d_\sigma, d_\sigma^+]_+\rangle + \langle\langle [d_\sigma, \Theta] \mid d_\sigma^+\rangle\rangle_\omega \tag{32}$$

where d electron is a fermion, which satisfies anticommutation relation with $[d_\sigma, d_\sigma^+]_+ = 1$. Now considering the second term in the Eq. (32), using the Bosons and the Fermions commutation relation in quantum mechanics, and some related operations, one gets

$$\begin{aligned} [d_\sigma, \Theta] &= [d_\sigma, H_0] - [d_\sigma, H_1] - [d_\sigma, H_2] - [d_\sigma, H_3] \\ &= -\frac{J}{2} \sum_{k,k'} (C_{k\sigma}^+ C_{k'\sigma} - C_{k\bar{\sigma}}^+ C_{k'\bar{\sigma}}) d_\sigma \\ &\quad - J \sum_{k,k'} C_{k\bar{\sigma}}^+ C_{k'\sigma} d_{\bar{\sigma}} \end{aligned} \tag{33}$$

This allows Eq. (32) to be written as:

$$\begin{aligned} &\omega\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega \\ &= 1 - \langle\langle \frac{J}{2} \sum_{k,k'} (C_{k\sigma}^+ C_{k'\sigma} - C_{k\bar{\sigma}}^+ C_{k'\bar{\sigma}}) d_\sigma \mid d_\sigma^+\rangle\rangle_\omega \\ &\quad - \langle\langle J \sum_{k,k'} C_{k\bar{\sigma}}^+ C_{k'\sigma} d_{\bar{\sigma}} \mid d_\sigma^+\rangle\rangle_\omega \end{aligned} \tag{34}$$

Introducing new operators:

$$\widehat{A}^z = \frac{1}{2} \sum_{k,k'} (C_{k\sigma}^+ C_{k'\sigma} - C_{k\bar{\sigma}}^+ C_{k'\bar{\sigma}}) \tag{35}$$

$$\widehat{A}^+ = \sum_{k,k'} C_{k\sigma}^+ C_{k'\bar{\sigma}}$$

$$\widehat{A}^- = \sum_{k,k'} C_{k\bar{\sigma}}^+ C_{k'\sigma}$$

Eq. (34) becomes

$$\omega\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega = 1 - J\langle\langle \widehat{A}^z d_\sigma \mid d_\sigma^+\rangle\rangle_\omega - J\langle\langle \widehat{A}^- d_{\bar{\sigma}} \mid d_\sigma^+\rangle\rangle_\omega \tag{36}$$

where $[\widehat{A}^+, \widehat{A}^-]$, $[\widehat{A}^+, \widehat{A}^z]$ and $[\widehat{A}^-, \widehat{A}^z]$ are calculated by commutation relation:

$$[\widehat{A}^+, \widehat{A}^-] = \sum_{k,k'} (C_{k\sigma}^+ C_{k'\sigma} - C_{k\bar{\sigma}}^+ C_{k'\bar{\sigma}}) \delta_{kk'} = 2\widehat{A}^z \delta_{kk'} \tag{37}$$

and

$$[\widehat{A}^+, \widehat{A}^z] = -\widehat{A}^+\delta_{kk'} \tag{38}$$

$$[\widehat{A}^-, \widehat{A}^z] = \widehat{A}^-\delta_{kk'} \tag{39}$$

Therefore, the above Eq. (36) can be solved by the following procedure:

(1) The second term in Eq. (36) can be expanded as:

$$\omega\langle\langle \widehat{A}^z d_\sigma \mid d_\sigma^+\rangle\rangle_\omega = \langle[\widehat{A}^z d_\sigma, d_\sigma^+]_+\rangle + \langle\langle[\widehat{A}^z d_\sigma, \Theta] \mid d_\sigma^+\rangle\rangle_\omega \tag{40}$$

Furthermore,

$$[\widehat{A}^z d_\sigma, \Theta] = \frac{1}{2}\sum_{\substack{k,k'\\k\neq k'}} E_k(C_{k\bar\sigma}^+ C_{k'\bar\sigma} - C_{k\sigma}^+ C_{k'\sigma})d_\sigma - 2J(\widehat{A}^z)^2 d_\sigma \tag{41}$$

Eq. (40) is written as:

$$\omega\langle\langle \widehat{A}^z d_\sigma \mid d_\sigma^+\rangle\rangle_\omega = \left\langle \widehat{A}^z \right\rangle + \frac{1}{2}\sum_{\substack{k,k'\\k\neq k'}} E_k\langle C_{k\bar\sigma}^+ C_{k'\bar\sigma} - C_{k\sigma}^+ C_{k'\sigma}\rangle\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega - 2J\left\langle \widehat{A}^z \right\rangle^2 \langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega \tag{42}$$

(2) The third term in Eq. (36) can be expanded as:

$$\omega\langle\langle \widehat{A}^- d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega = \langle[\widehat{A}^- d_{\bar\sigma}, d_\sigma^+]_+\rangle + \langle\langle[\widehat{A}^- d_{\bar\sigma}, \Theta] \mid d_\sigma^+\rangle\rangle_\omega \tag{43}$$

where denoting

$$[\widehat{A}^- d_{\bar\sigma}, \Theta] = -\sum_{k,k'(k\neq k')} E_k C_{k\bar\sigma}^+ C_{k'\sigma} \tag{44}$$

Using the same way, Eq. (43) can be written as:

$$\omega\langle\langle \widehat{A}^- d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega = -\sum_{k,k'(k\neq k')} E_k\langle C_{k\bar\sigma}^+ C_{k'\sigma}\rangle\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega \tag{45}$$

which is related to the Green function $\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle$ and enable us to establish the $\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle$ equation of motion.

(3) The Green function $\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega$can be calculated by

$$\omega\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega = \langle[d_{\bar\sigma}, d_\sigma^+]_+\rangle + \langle\langle[d_{\bar\sigma}, \Theta] \mid d_\sigma^+\rangle\rangle_\omega \tag{46}$$

where $[d_{\bar\sigma}, \Theta]$ can be calculated:

$$[d_{\bar\sigma}, \Theta] = \frac{J}{2}\sum_{k,k'}(C_{k\sigma}^+ C_{k'\sigma} - C_{k\bar\sigma}^+ C_{k'\bar\sigma})d_{\bar\sigma} - J\sum_{k,k'} C_{k\sigma}^+ C_{k'\bar\sigma} d_\sigma \tag{47}$$

which enables Eq. (46) to become

$$\omega\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega = J\langle\langle \widehat{A}^z d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega - J\langle\langle \widehat{A}^+ d_\sigma \mid d_\sigma^+\rangle\rangle_\omega \tag{48}$$

(4) Using the same way as Eq. (42), the third term $\langle\langle \widehat{A}^z d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega$ in Eq. (48) can be written as:

$$\omega\langle\langle \widehat{A}^z d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega = \langle[\widehat{A}^z d_{\bar\sigma}, d_\sigma^+]_+\rangle + \langle\langle[\widehat{A}^z d_{\bar\sigma}, \Theta] \mid d_\sigma^+\rangle\rangle_\omega \tag{49}$$

where

$$[\widehat{A}^z d_{\bar\sigma}, \Theta] = \frac{1}{2}\sum_{k,k',(k\neq k')} E_k(C_{k\bar\sigma}^+ C_{k'\bar\sigma} - C_{k\sigma}^+ C_{k'\sigma})d_{\bar\sigma} + 2J\left(\widehat{A}^z\right)^2 d_{\bar\sigma} \tag{50}$$

This allows Eq. (49) to become

$$\omega\langle\langle \widehat{A}^z d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega = \frac{1}{2}\sum_{k,k',(k\neq k')} E_k\langle C_{k\bar\sigma}^+ C_{k'\bar\sigma} - C_{k\sigma}^+ C_{k'\sigma}\rangle\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega + 2J\langle\widehat{A}^z\rangle^2\langle\langle d_{\bar\sigma} \mid d_\sigma^+\rangle\rangle_\omega \tag{51}$$

The second term in the Eq. (48) can also be expanded as:

$$\omega\langle\langle \widehat{A}^+ d_\sigma \mid d_\sigma^+\rangle\rangle_\omega = \langle[\widehat{A}^+ d_\sigma, d_\sigma^+]_+\rangle + \langle\langle[\widehat{A}^+ d_\sigma, \Theta] \mid d_\sigma^+\rangle\rangle_\omega \tag{52}$$

it is just

$$[\widehat{A}^+ d_\sigma, \Theta] = -\sum_{k,k',(k\neq k')} E_k C_{k\sigma}^+ C_{k'\bar\sigma} d_\sigma \tag{53}$$

Therefore Eq. (51) can be written as:

$$\begin{aligned}&\omega\langle\langle \widehat{A}^+ d_\sigma \mid d_\sigma^+\rangle\rangle_\omega\\ &= \langle\widehat{A}^+\rangle - \langle\langle\sum_k E_k C_{k\sigma}^+ C_{k'\bar\sigma} d_\sigma \mid d_\sigma^+\rangle\rangle_\omega\\ &= \langle\widehat{A}^+\rangle - \sum_{k,k'(k\neq k')} E_k\langle C_{k\sigma}^+ C_{k'\bar\sigma}\rangle\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega\end{aligned} \tag{54}$$

Now, combining Eqs. (36), (42), (45), (48), (51) and (54), one can obtain the Green function $\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega$:

$$\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega = \frac{(\omega - J\langle\widehat{A}^z\rangle)(\omega^2 - M) - N}{(\omega^2 - 2J^2\langle\widehat{A}^z\rangle^2)^2 - \frac{J^2}{4}\sum_{\substack{k,k',\sigma\\k\neq k'}} E_k^2\langle C_{k\sigma}^+ C_{k'\sigma}\rangle} \tag{55}$$

where we introduce symbols:

$$M = 2J^2\langle\widehat{A}^z\rangle + \frac{J}{2}\sum_{k,k',(k\neq k')} E_k\langle C_{k\bar\sigma}^+ C_{k'\bar\sigma} - C_{k\sigma}^+ C_{k'\sigma}\rangle \tag{56}$$

and

$$N = J^2\sum_{k,k',(k\neq k')} E_k^2\langle C_{k\bar\sigma}^+ C_{k'\sigma}\rangle\langle\widehat{A}^+\rangle \tag{57}$$

In order to obtain the function pole, the denominator of $\langle\langle d_\sigma \mid d_\sigma^+\rangle\rangle_\omega$ must be equal to zero, which allows one to have the function pole as:

$$\omega = \left| 2J^2\langle \widehat{A}^z\rangle^2 \pm \frac{J}{2}\sum_{\substack{k,k',\sigma\\ k\neq k'}} E_k\langle C^+_{k\sigma}C_{k'\sigma}\rangle \right|^{\frac{1}{2}} \tag{58}$$

This allows the energy of d electrons to be given by

$$E_{d\sigma} = \left| 2J^2\langle \frac{1}{2}\sum_{k,k'}(C^+_{k\sigma}C_{k'\sigma} - C^+_{k\bar{\sigma}}C_{k'\bar{\sigma}})\rangle^2 \pm \frac{J}{2}\sum_{\substack{k,k',\sigma\\ k\neq k'}} E_k\langle C^+_{k\sigma}C_{k'\sigma}\rangle \right|^{\frac{1}{2}} \tag{59}$$

The obtained energy formula shows that the energy spectrum $E_{d\sigma}$ is related to the strong correlation coupling J which is a kind of function related to E_d, and interaction between d electrons and s electrons. However, $E_{d\sigma}$ is independent on individual k, k', and only changes with σ since $\frac{1}{2}\sum_{k,k'}(C^+_{k\sigma}C_{k'\sigma} - C^+_{k\bar{\sigma}}C_{k'\bar{\sigma}})^2$ change's with σ.

5 Spectral function of Green function

We now turn into the expression $\langle\langle d_\sigma \mid d^+_\sigma\rangle\rangle_\omega$ following standard form, which gives

$$\langle\langle d_\sigma \mid d^+_\sigma\rangle\rangle_\omega = \frac{A_1}{\omega - E_0} + \frac{A_2}{\omega + E_0} + \frac{A_3\omega + A_4}{\omega^2 - E_1^2} \tag{60}$$

where $\omega = E_0$ or $\omega^2 = E_1^2$ is a pole of Eq. (55). According to the Green function's spectral theorem,

$$\langle AB\rangle = \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty}\frac{\langle\langle A \mid B\rangle\rangle_{\omega+\mathrm{i}\eta} - \langle\langle A \mid B\rangle\rangle_{\omega-\mathrm{i}\eta}}{\mathrm{e}^{\beta\omega}+1}\mathrm{d}\omega \tag{61}$$

the average value $\langle d_\sigma d^+_\sigma\rangle$ represents the number of s electrons with spin σ in the nodes and can be calculated by

$$\begin{aligned}
&\langle d_\sigma d^+_\sigma\rangle \\
&= \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty}\frac{\langle\langle d_\sigma \mid d^+_\sigma\rangle\rangle_{\omega+\mathrm{i}\eta} - \langle\langle d_\sigma \mid d^+_\sigma\rangle\rangle_{\omega-\mathrm{i}\eta}}{\mathrm{e}^{\beta\omega}+1}\mathrm{d}\omega \\
&= \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty}\left[\frac{-2\mathrm{i}\eta A_1}{(\omega-E_0)^2+\eta^2} + \frac{-2\mathrm{i}\eta A_2}{(\omega+E_0)^2+\eta^2}\right. \\
&\quad \left. + \frac{A_3(\omega+\mathrm{i}\eta)+A_4}{(\omega+\mathrm{i}\eta)^2-E_1^2} - \frac{A_3(\omega-\mathrm{i}\eta)+A_4}{(\omega-\mathrm{i}\eta)^2-E_1^2}\right]\frac{1}{\mathrm{e}^{\beta\omega}+1}\mathrm{d}\omega
\end{aligned} \tag{62}$$

where A_1, A_2, A_3, and A_4 are defined by

$$\begin{aligned}
A_1 &= \frac{(E_0^2 - M)\left(E_0 - J\langle\widehat{A}^z\rangle\right) - N}{2\left(E_0^2 - E_1^2\right)E_0} \\
A_2 &= \frac{(E_0^2 - M)\left(E_0 - J\langle\widehat{A}^z\rangle\right) - N}{2\left(E_0^2 - E_1^2\right)E_0} \\
A_3 &= \frac{M - E_1^2}{E_0^2 - E_1^2} \\
A_4 &= \frac{N - J\langle\widehat{A}^z\rangle\left(M + E_1^2\right)}{E_0^2 - E_1^2}
\end{aligned} \tag{63}$$

$$E_0 = \left| 2J^2\langle\widehat{A}^z\rangle^2 + \frac{J}{2}\sum_{\substack{k,k',\sigma\\ k\neq k'}} E_k\langle C^+_{k\sigma}C_{k'\sigma}\rangle \right|^{\frac{1}{2}}$$

$$E_1 = \left| 2J^2\langle\widehat{A}^z\rangle^2 - \frac{J}{2}\sum_{\substack{k,k',\sigma\\ k\neq k'}} E_k\langle C^+_{k\sigma}C_{k'\sigma}\rangle \right|^{\frac{1}{2}}$$

and

$$\widehat{A}^z = \frac{1}{2}\sum_{k,k'}(C^+_{k\sigma}C_{k'\sigma} - C^+_{k\bar{\sigma}}C_{k'\bar{\sigma}}) \tag{64}$$

Using a theorem of residues in the complex function $\int_{-\infty}^{\infty} f(\omega)\mathrm{d}\omega = 2\pi\mathrm{i}\sum_k \mathrm{Re}sf(\omega_k)$, through long term calculation, and taking the limit, finally we can obtain

$$\begin{aligned}
\langle d_\sigma d^+_\sigma\rangle &= \lim_{\eta\to 0} A_1[\mathrm{e}^{\beta(E_0+\mathrm{i}\eta)}+1]^{-1} \\
&\quad + \lim_{\eta\to 0} A_2[\mathrm{e}^{\beta(-E_0+\mathrm{i}\eta)}+1]^{-1} \\
&\quad + \lim_{\eta\to 0}(-A_3E_1 - A_4)\left[\frac{[\mathrm{e}^{\beta(E_1-\mathrm{i}\eta)}+1]^{-1}}{2E_1}\right. \\
&\quad \left. + \frac{[\mathrm{e}^{\beta(E_1+\mathrm{i}\eta)}+1]^{-1}}{2E_1}\right] \\
&\quad + \lim_{\eta\to 0}(-A_3E_1 + A_4)\left[\frac{\mathrm{e}^{\beta(E_1+\mathrm{i}\eta)}[\mathrm{e}^{\beta(E_1+\mathrm{i}\eta)}+1]^{-1}}{2E_1}\right. \\
&\quad \left. + \frac{\mathrm{e}^{\beta(E_1-\mathrm{i}\eta)}[\mathrm{e}^{\beta(E_1-\mathrm{i}\eta)}+1]^{-1}}{2E_1}\right] \\
&= (\mathrm{e}^{\beta E_0}+1)^{-1}(A_1 + A_2E_0) - A_3E_0E_1 \\
&\quad + \frac{(\mathrm{e}^{\beta E_1}+1)^{-1}}{E_0}A_4\left(\mathrm{e}^{\beta E_1}-1\right)
\end{aligned} \tag{65}$$

From this we can see that the mean number of up or down spin for d electrons is related with that of c electrons. When the spin is up ($\sigma = \uparrow$), the above formula can be calculated as:

$$\begin{aligned}
\langle d_\uparrow d^+_\uparrow\rangle &= (\mathrm{e}^{\beta E_0}+1)^{-1}(A_{1\uparrow} + A_{2\uparrow}E_0) - A_{3\uparrow}E_0E_1 \\
&\quad + \frac{(\mathrm{e}^{\beta E_1}+1)^{-1}}{E_0\uparrow}A_{4\uparrow}\left(\mathrm{e}^{\beta E_1}-1\right)
\end{aligned} \tag{66}$$

where the relevant parameter is given by

$$\begin{aligned}
A_{1\uparrow} &= \frac{(E_0^2 - M_\uparrow)\left(E_0 - J\langle\widehat{A}^z_{\uparrow\downarrow}\rangle\right) - N_\uparrow}{2\left(E_0^2 - E_1^2\right)E_0} \\
A_{2\uparrow} &= \frac{(E_0^2 - M_\uparrow)\left(E_0 - J\langle\widehat{A}^z_{\uparrow\downarrow}\rangle\right) - N_\uparrow}{2\left(E_0^2 - E_1^2\right)E_0}
\end{aligned} \tag{67}$$

$$A_{3\uparrow}=\frac{M_\uparrow-E_1^2}{E_0^2-E_1^2}$$

$$A_{4\uparrow}=\frac{N_\uparrow-J\langle\widehat{A}^z_{\uparrow\downarrow}\rangle\left(M_\uparrow+E_1^2\right)}{E_0^2-E_1^2}$$

$$\begin{aligned}M_\uparrow=&2J^2\langle\widehat{A}^z_{\uparrow\downarrow}\rangle+\frac{J}{2}\sum_{k,k'(k\neq k')}E_k(\langle C^+_{k\downarrow}C_{k'\downarrow}\rangle\\&-\langle C^+_{k\uparrow}C_{k'\uparrow}\rangle)\end{aligned}\tag{68}$$

$$N_\uparrow=J^2\sum_{k,k'(k\neq k')}E_k^2\langle C^+_{k\downarrow}C_{k'\downarrow}\rangle\langle\widehat{A}^+_{\uparrow\downarrow}\rangle$$

and

$$\langle\widehat{A}^z_{\uparrow\downarrow}\rangle=\frac{1}{2}\sum_{k,k'}\left(\langle C^+_{k\uparrow}C_{k'\uparrow}\rangle-\langle C^+_{k\downarrow}C_{k'\downarrow}\rangle\right)$$

$$\langle\widehat{A}^+_{\uparrow\downarrow}\rangle=\sum_{k,k'}\langle C^+_{k\uparrow}C_{k'\downarrow}\rangle\tag{69}$$

In the same way, while the spin is down ($\sigma=\downarrow$), $\left\langle d_\downarrow d^+_\downarrow\right\rangle$ is calculated as

$$\begin{aligned}\langle d_\downarrow d^+_\downarrow\rangle=&\left(\mathrm{e}^{\beta E_0}+1\right)^{-1}\left(A_{1\downarrow}+A_{2\downarrow}E_0\right)-A_{3\downarrow}E_0E_1\\&+\frac{\left(\mathrm{e}^{\beta E_1}+1\right)^{-1}}{E_0}A_{4\downarrow}\left(\mathrm{e}^{\beta E_1}-1\right)\end{aligned}\tag{70}$$

Both cases show that the electronic number of up spin is not equal to that of down spin, $\langle d_\uparrow d^+_\uparrow\rangle\neq\langle d_\downarrow d^+_\downarrow\rangle$, hence, the local magnetism can appear under this condition.

6 Application in network

The above model constructed iteratively by the creation and destruction of the d electrons (nodes) with interaction of the environmental s electrons can be specialized as a kind of fermionic network. Generally speaking, in this model, at each time step, a new node (d electron confined in an impurity) with m (or m') links (strongly correlated to m confined d electrons) are added (or cancelled) to the network. In terms of the idea of fermionic network in the Refs. [3, 4], the new node has an energy and a new link is attached, with probability

$$P_j\propto\mathrm{e}^{\beta E_j}k\left(t|\,E_j,t_j\right)$$

to node j arriving at the interface network at time t_j, with energy E_j and survivability $k\left(t|\,E_j,t_j\right)$ at time t. Therefore, the continuous equation describing the time evolution of the survivability $k\left(t|\,E_j,t_j\right)$ of the nodes is expressed by

$$\frac{\partial k\left(t|\,E_j,t_j\right)}{\partial t}=m\frac{\mathrm{e}^{\beta E_j}k\left(t|\,E_j,t_j\right)}{\sum_l\mathrm{e}^{\beta E_j}k\left(t|E_l,t_l\right)}\tag{71}$$

with the initial condition

$$k_0=k\left(t|\,E_j,t\right)\tag{72}$$

By using the asymptotic behavior in the thermodynamic limit

$$\sum_k\mathrm{e}^{\beta E_j}k\left(t|\,E_k,t_k\right)=Z\longrightarrow\langle Z\rangle\longrightarrow mt\mathrm{e}^{\beta\mu}\tag{73}$$

Eq. (71) can be reduced as

$$\frac{\partial k\left(t|\,E_j,t_j\right)}{\partial t}=m\mathrm{e}^{\beta(E_j-\mu)}t_j\frac{k(t|E_j,t_j)}{t}\tag{74}$$

whose solution is found as

$$k\left(t|\,E_j,t_j\right)=k_0m\left(\frac{t}{t_j}\right)^{f(E_j)}\tag{75}$$

with

$$f\left(E_j\right)=\mathrm{e}^{\beta(E_j-\mu_i)}\tag{76}$$

where $f\left(E_i\right)$ is the dynamic exponent which depends on the energy E_i, μ_i is a chemical potential and β is the inverse temperature β. Therefore, the degree distribution is given by

$$P\left(k\right)=\int\mathrm{d}Ep\left(E\right)\frac{1}{\dfrac{\partial k}{\partial t}}\sim Bk^{-1}\tag{77}$$

where

$$B=\int\mathrm{d}Ep\left(E\right)\tag{78}$$

which demonstrates that the fermionic network system is scale-free if B is irrelevant to k or quite small. This allows one to find a novel way to study the local magnetism existence for the network. In fact, using the obtained survivability $k\left(t|\,E_{d\uparrow},t_{d\uparrow}\right)$ for up spin and $k\left(t|\,E_{d'\downarrow},t_{d'\downarrow}\right)$ for down spin, one can find that the local magnetism existence can be judged by $k\left(t|\,E_{d\uparrow},t_{d\uparrow}\right)\stackrel{?}{=}k\left(t|\,E_{d'\downarrow},t_{d'\downarrow}\right)$, since the survivability reflects the number of the up spins or down spins in the interface of the network. This interface evolves with time to construct the part of the network, hence the number of the up spins or down spins in the interface can reflect the total number of the up spins or down spins as the nodes in the network. That means if $k\left(t|\,E_{d\uparrow},t_{d\uparrow}\right)=k\left(t|\,E_{d'\downarrow},t_{d'\downarrow}\right)$, there is no local magnetism, otherwise, $k\left(t|\,E_{d\uparrow},t_{d\uparrow}\right)\neq k\left(t|\,E_{d'\downarrow},t_{d'\downarrow}\right)$, there exist the local magnetism. The local magnetism exists if the number of the up spins and down spins are not equal in the interface of the network [4],

$$N_\uparrow=\sum_{t_{d\uparrow}}k\left(t|\,E_{d\uparrow},t_{d\uparrow}\right)\neq N_\downarrow=\sum_{t_{d'\downarrow}}k\left(t|\,E_{d'\downarrow},t_{d'\downarrow}\right)\tag{79}$$

Both cases can be expressed by

$$k\left(t|\,E_{d\uparrow},t_{d\uparrow}\right)=k_0m\left(\frac{t}{t_{d\uparrow}}\right)^{f(E_{d\uparrow})}$$

$$\begin{cases} \neq k\,(t|E_{d'\downarrow}, t_{d'\downarrow}) = k_0 m \left(\dfrac{t}{t_{d'\downarrow}}\right)^{f(E_{d'\downarrow})}, \\ \quad \text{for } E_{d\uparrow} \neq E_{d'\downarrow}, t_{d\uparrow} = t_{d'\downarrow} \\ = k\,(t|E_{d'\downarrow}, t_{d'\downarrow}) = k_0 m \left(\dfrac{t}{t_{d'\downarrow}}\right)^{f(E_{d\downarrow})}, \\ \quad \text{for } E_{d\uparrow} = E_{d'\downarrow}, t_{d\uparrow} = t_{d'\downarrow} \end{cases} \tag{80}$$

The above result shows that the origin of the local magnetism existence actually depends on $k\,(t|\,E_{d\uparrow}, t_{d\uparrow}) \neq k\,(t|\,E_{d'\downarrow}, t_{d'\downarrow})$ which is related to the energy of the local nodes of the network. It is the difference of d electron energy for up spin and down spin, $E_{d\uparrow} \neq E_{d'\downarrow}$, distributed in the local region, to result in the difference of $k\,(t|\,E_{d\uparrow}, t_{d\uparrow})$ and $k\,(t|\,E_{d'\downarrow}, t_{d'\downarrow})$, that introduces the local magnetism for the system.

7 Conclusion

The energy spectrum for the d electrons in the Schrieffer and Wolf transformation for the quantum network described by the Anderson d - s model is calculated using the Green function approach. The obtained energy formula shows that it is related to the strongly coupling number J and the interaction between d and s electrons through spins. The local magnetism of the model can be studied by using the spectral function of Green function, which shows how the energy of the system influence the number of up or down spin. Moreover, the given survivability of the fermionic network model can be used to study the local magnetism. The local magnetism existence actually depends on the difference of d electron energy for up spin and down spin. $E_{d\uparrow} \neq E_{d'\downarrow}$ distributed in the local nodes of the network can introduce the difference of the survivability for the up or down spins. This results in local magnetism in the network, which shows that survivability is an important statistical characteristic quantity not only to reflect the fermionic network topological property but also dynamics.

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References

1. P. W. Anderson, Phys. Rev., 1961, 124: 41
2. J. R. Schrieffer and P. A. Wolff, Phys. Rev., 1966, 149: 491
3. G. Bianconi, arXiv: cond-mat/0204506v2
4. G. Bianconi, Phys. Rev. E, 2002, 66: 056123
5. D. Baeriswyl, D. K. Campbell, J. M. P. Carmelo, F. Guinea, and E. Louis, The Hubbard Model, New York: Pienum Press, 1995
6. J. R. Schrieffer and P. A. Wolff, Phys. Rev., 1966, 149: 491
7. L. Z. Zheng, Solid State Theory, Beijing: Higher Education Press, 2002 (in Chinese)
8. I. Antoniou, Y. Melnikov, and Q. Bi, Physica A, 1997, 246: 97
9. Q. Bi, H. E. Ruda, M. S. Zhang, and X. H. Zeng, Physica A, 2003, 322: 345