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RESEARCH ARTICLE

A precision analysis and determination of the technical requirements of an atom interferometer for gravity measurement

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The influence of the wave-front curvature of Raman pulses on the measurement precision of gravitational acceleration in atom interferometry is analysed by the method of a transmission matrix. It is shown that the measurement precision of gravitational acceleration is largely dependent on the spot size of the Raman pulse, the temporal interval between Raman pulses and the optical path difference of the two counter-propagating Raman pulses. Moreover, the influence of Doppler frequency shift on the precision is discussed. In order to get a certain measurement precision, the requirement for the accuracy of frequency scanning of the Raman pulse to compensate for the Doppler frequency shift is obtained.

Keywords atom interferometry, measurement precision, gravitational acceleration, wave-front curvature, Doppler shift

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1 Introduction

The wave nature of cold atom cloud and Bose-Einstein condensate is a hot topic of research. For examples, the high-contrast interference fringes of Bose-Einstein condensate was observed [1], the quantum tunneling of Bose-Einstein condensed atoms in the optical lattices is investigated [2−4], the Josephson effect for photons in two weakly linked micro cavities containing ultracold atoms was studied [5], the quantized vortices of Bose-Einstein condensate are created [6−9], and so on. The wave nature of cold atoms provides some practical applications. For instance, atom interferometer manipulated by two-photon Raman pulses is an important method to measure some fundamental physical constants with high precision, such as the gravitational acceleration constant [10], the fine structure constant [11], and so on. In 1999, Chu's group obtained a precision of 3×10^{-9}g in absolute gravitational acceleration measurement by using an atom interferometer [12]. In 2002, Kasevich group used two atom interferometers to achieve a precision of $40E$ ($1E = 10^{-9}/\mathrm{s}^2$) in the measurement of the gravitational acceleration gradient [13]. In 2006, the Jet Propulsion Laboratory of NASA improved the precision to $30E$ [14].

The Raman pulses in the atom interferometer were regarded as plane waves in the previous researches, but as a matter of fact, they are closer to Gaussian beams. Furthermore, the atom cloud itself has finite size, and the phases of the optical field imprinted in each atom are different. In this paper, we will consider the effect of wave-front curvature of the Raman pulses on an atom interferometer. In addition, atoms will be accelerated due to gravity falling down in the process of interference; therefore, the Doppler frequency shifts will be different at different times. If this difference is not compensated for, the frequency difference between the Raman pulses will influence the measurement precision of the gravity. The influence of Doppler shift will also be discussed in this article.

2 Influences of wave-front curvature of the Raman pulses

The general process of the Raman pulse style atom interference could be described as follows: a three level atom, with state $|1\rangle$ and state $|3\rangle$ as two sub-states corresponding to the two hyperfine ground states of the atom, and state $|2\rangle$ corresponding to the excited state. Atom clouds initially in state $|1\rangle$ are divided into two equal parts in state $|1\rangle$ and state $|3\rangle$ after interacting with the first $\pi/2$ Raman pulse. Atoms transferred to state $|3\rangle$ acquire momentum $\hbar k_{\text{eff}}$ ($\boldsymbol{k}_{\text{eff}} = \boldsymbol{k}_1 - \boldsymbol{k}_2$, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are wave vectors of Raman pulses) and will be accelerated and separated from atoms remaining in state $|1\rangle$. After a time T of atoms in free propagation, the π Raman pulse exchanges the atom states between $|1\rangle$ and $|3\rangle$. Moreover, a $|1\rangle \to |3\rangle$ transition increases the atom momentum by $\hbar k_{\text{eff}}$, and a reverse transition reduces the same amount of momentum. After another T, the two parts of the divided atoms recombine. Simultaneously, the second $\pi/2$ Raman pulse makes the atoms split again to induce the interference.

In order to calculate influences of wave-front curvature of the Raman pulses, we built the model shown in Fig. 1. In the direction of z(the direction of the gravity field), a linearly polarized Raman beam coming from an optical fiber is collimated by a lens, then transformed into left circularly polarized light by a $\lambda/4$ wave plate. The beam passes through another $\lambda/4$ wave plate and is reflected by a mirror, becoming into right circularly polarized, which forms a pair of counter-propagation Raman beams. The $\pi/2$–π–$\pi/2$ Raman pulse sequence interacts with atoms at the positions z_1, z_2 and z_3 respectively. f is the focal length of the lens which determines the size of the Raman beam. L_1 is the distance between the position where the first pair of Raman beams interact with the atom group and the lens, while L_2 is the distance from the position of the first pair of Raman beams to the mirror.

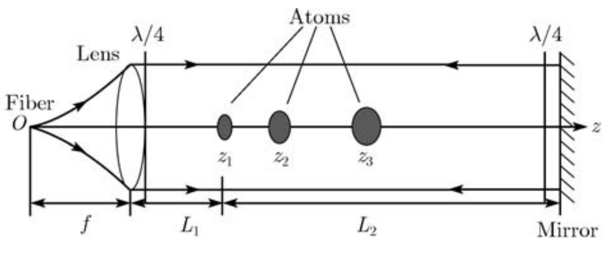

Fig. 1 The schematics of the propagation of the Raman beam.

We assume that the Raman beams could be described as Gaussian beams. At $z = 0$, the field distribution of a Gaussian beam could be given as follows:

$$E(x, y, 0) = E_0 \exp\left(-\frac{\mathrm{i}\pi r^2}{\lambda q_0}\right) \tag{1}$$

where $q_0 = \mathrm{i}\pi w_0^2/\lambda = iz_0$, while w_0 is the beam waist of the Gaussian beam and z_0 is the Rayleigh distance. The field at a given z could be expressed as [15]:

$$E(x, y, z) = E_0 \exp\left[-\frac{\mathrm{i}\pi r^2}{\lambda q(z)}\right] \tag{2}$$

where $q(z)$ is the radius of the complex curvature of the beam at the position z. According to the ABCD law for Gaussian beams, $q_1(z_1)$ and $q_1'(z_1)$ are given by:

$$\frac{1}{q_1} = \frac{C_1 q_0 + D_1}{A_1 q_0 + B_1}, \quad \frac{1}{q_1'} = \frac{C_1' q_0 + D_1'}{A_1' q_0 + B_1'} \tag{3}$$

where $\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}$, $\begin{pmatrix} A_1' & B_1' \\ C_1' & D_1' \end{pmatrix}$ are the ray transfer matrices of the forward and reverse propagating from $z = 0$ to $z = z_1$, respectively, given by

$$\begin{pmatrix} \mathrm{A}_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A_1' & B_1' \\ C_1' & D_1' \end{pmatrix} = \begin{pmatrix} 1 & L_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & L_1 + L_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \tag{4}$$

From Eq. (2), we can deduce the phase difference that the two counter-propagating Raman beams imprint on atoms at z_1, z_2 and z_3 as follows:

$$\phi_i = \alpha_i r_i^2, \ i = 1, \ 2, \ 3 \tag{5}$$

where

$$\begin{aligned} \alpha_i &= \frac{\pi}{\lambda}\left[\mathrm{Re}\left(\frac{1}{q_i'}\right) - \mathrm{Re}\left(\frac{1}{q_i}\right)\right] \\ &= \frac{\pi}{\lambda}\left(\frac{A_i' C_i' z_0^2 + B_i' D_i'}{A_i'^2 z_0^2 + B_i'^2} - \frac{A_i C_i z_0^2 + B_i D_i}{A_i^2 z_0^2 + B_i^2}\right) \\ & \qquad i = 1, \ 2, \ 3 \end{aligned} \tag{6}$$

In the process of atom interference, the size of the atom cloud is enlarged due to the thermal motion and atomic collisions. Let v_x, v_y denote the lateral velocity of the atoms. The atom cloud falls freely with zero initial longitudinal velocity, as shown in Fig. 2. σ_0 is the initial radius of the atom cloud, t is the time of free fall of the atom cloud before encountering the first $\pi/2$ Raman pulse, and T is the temporal interval between pulses. At the positions z_1, z_2 and z_3, the radius of the atom cloud are given by the following formulas:

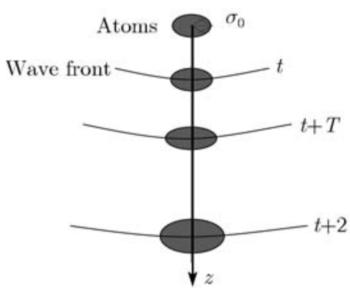

Fig. 2 The schematics for the positions, where the Raman beams interact with the atom cloud.

$$\begin{aligned} r_1^2 &= (v_x t + \sigma_0)^2 + (v_y t + \sigma_0)^2 \\ r_2^2 &= [v_x (t+T) + \sigma_0]^2 + [v_y (t+T) + \sigma_0]^2 \\ r_3^2 &= [v_x (t+2T) + \sigma_0]^2 + [v_y (t+2T) + \sigma_0]^2 \end{aligned} \tag{7}$$

From theoretical analysis of an atom interferometer, the relation between the probability of atoms on state $|1\rangle$ and the gravitational acceleration is $P = \left[1+\cos\left(k_{\text{eff}} g T^2 + \Delta\varphi\right)\right]/2$, where $\Delta\varphi$ is the phase difference due to all kinds of factors. The phase difference imprinted on the atoms due to the wave-front curvature of Raman pulses is given by:

$$\Delta\varphi = \phi_1 - 2\phi_2 + \phi_3 = \alpha_1 r_1^2 - 2\alpha_2 r_2^2 + \alpha_3 r_3^2 \tag{8}$$

The atoms have a Boltzmann distribution with respect to position and velocity as follows:

$$f(x, y, v_x, v_y) = A \exp\left(-\frac{x^2+y^2}{2\sigma_0}\right)\exp\left[-\frac{m\left(v_x^2+v_y^2\right)}{2k_{\text{B}}T_{\text{temp}}}\right] \tag{9}$$

So that the mean phase difference due to the wave-front curvature of Raman pulses over the whole atom cloud can be expressed as:

$$\begin{aligned} \langle\Delta\varphi\rangle &= \frac{\iiiint f(x,y,v_x,v_y)\cdot\Delta\varphi\cdot \mathrm{d}x\mathrm{d}y\mathrm{d}v_x\mathrm{d}v_y}{\iiiint f(x,y,v_x,v_y)\,\mathrm{d}x\mathrm{d}y\mathrm{d}v_x\mathrm{d}v_y} \\ &= \frac{2}{m}\{m(\alpha_1-2\alpha_2+\alpha_3)\sigma_0^2+[-2T^2(\alpha_2-2\alpha_3) \\ &\quad +t^2(\alpha_1-2\alpha_2+\alpha_3)+4tT(-\alpha_2+\alpha_3)]k_{\text{B}}T_{\text{temp}}\} \end{aligned} \tag{10}$$

Therefore, the relation between measurement precision of gravitational acceleration and the phase difference imprinted into atoms due to the wave-front curvature of the Raman pulse is given by

$$\Delta g = \frac{\langle\Delta\varphi\rangle}{k_{\text{eff}}T^2} \tag{11}$$

Based on Eq. (11), we can calculate the dependence of measurement precision on the focal length of lens f, the distance L_1, the distance L_2 and the temporal interval between pulses T. In our calculation, the initial radius and the temperature of the atom cloud is $\sigma_0 = 1.5$ mm and $T_{\text{temp}} = 5$ μk respectively, the initial beam waist of the Gaussian beam coming out from the optical fiber is $w_0 = 2.5$ μm, and the time before the free falling atom cloud encounters the first pair of Raman pulses is $t = 20$ ms.

Figure 3 shows the relation between the gravity measurement precision Δg and the focal length of lens f under the parameters $T = 50$ ms, $L_1 = 0.15$ m and $L_2 = 1.5$ m. From Fig. 3, we can see that as the focal length gets longer, the gravity measurement precision gets better. Since the size of the Raman beams is proportional to the focal length of the lens, that means that for a larger beam size, the gravity measurement precision becomes higher. If we intend to get a measurement precision better than 10^{-9} m/s^2, f should be larger than 0.17 m, with the corresponding size of the Raman beam being around $w = 1.6$ cm.

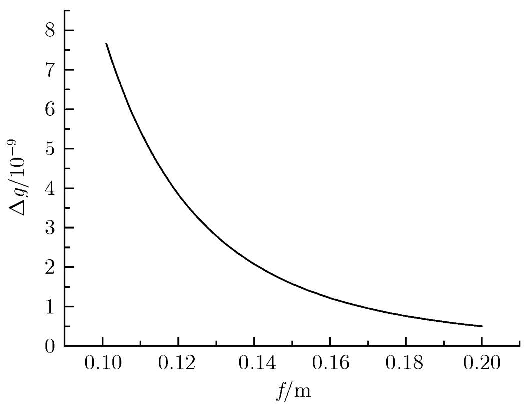

Fig. 3 The relation between the measurement precision (Δg) and the focal length of the lens (f).

Figure 4 shows the relation between the gravity measurement precision Δg and the position where the first pulses interact with the atoms when $T = 50$ ms, $w = 1.6$ cm and $L_2 = 1.5$ m. From Fig. 4, we can conclude that the position where the first pulses interact with the atoms has a small influence on the gravity measurement precision.

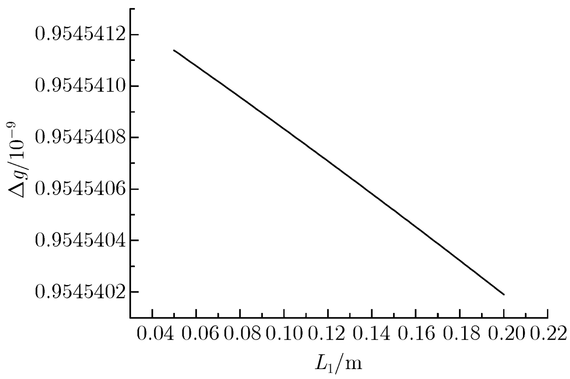

Fig. 4 The relation between the measurement precision (Δg) and the position where the first pulses interact with the atom cloud (L_1).

Figure 5 shows the relation between the gravity measurement precision and the position of the mirror L_2 which is used to reflect the Raman beams under the parameters $T = 50$ ms, $w = 1.6$ cm and $L_1 = 0.15$ m. The optical path difference between the two counter-propagating Raman pulses depends on the position of the mirror directly. From Fig. 5, we can see that the precision of gravity measurement is inversely proportional to the optical path difference. When the mirror is located at $L_2 = 1$ m, the gravity measurement precision equals

$0.6 \times 10^{-9}\text{m/s}^2$; for $L_2 = 2$ m, the gravity measurement precision equals $1.2 \times 10^{-9}\text{m/s}^2$.

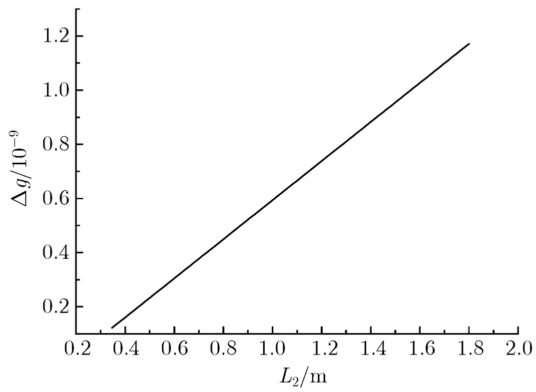

Fig. 5 The relation between the measurement precision (Δg) and the optical path difference of the two counter-propagating Raman pulses (L_2).

Figure 6 shows the relation between the gravity measurement precision and the temporal interval between Raman pulses T under parameters: $w = 1.6$ cm, $L_1 = 0.15$ m and $L_2 = 1.5$ m. From Fig. 6, we can find that when the interval time is longer, the free propagation of atoms covers a larger distance, thus leading to a higher measurement precision. The temporal interval also affects the size of the atoms' cloud. The longer time leads to a larger size of the atom cloud, thus the wave front of Raman beams imprinted on the atoms has a great influence on the precision of gravity measurement. We can see from Fig. 6 that the gravity measurement precision is not square inverse proportional to the interval time.

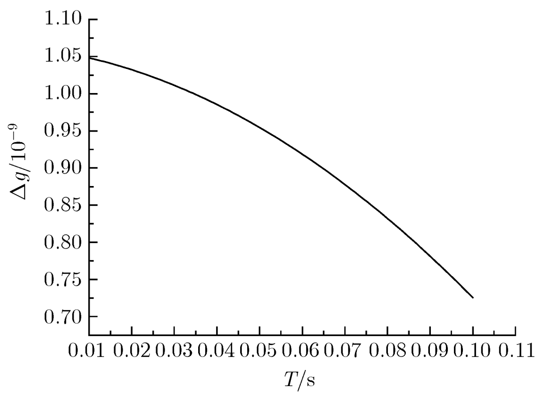

Fig. 6 The relation between the measurement precision (Δg) and the temporal interval between pulses (T).

3 Influences of Doppler shift

In the process of atom interference, the atom cloud is accelerated by gravity. Thus, the amount of Doppler shift of the atoms is different at different positions z_1, z_2 and z_3, where the Raman beams interact with atoms. If the difference between these frequency shifts is not well compensated, the frequency difference between the two Raman pulses will differ from that between the two ground states of the atoms. In the following, we will discuss the influence of the Doppler shift on the measurement precision of the gravitational acceleration.

According to the conservation of energy, we can obtain the relation between the detuning of Raman beams $\omega_1 - \omega_2$, the frequency difference between the two ground states of the atoms $\omega_e - \omega_f$, the Doppler shift and the photon recoil shift as follows [16, 17]:

$$\delta(t) = \omega_1 - \omega_2 - (\omega_e - \omega_f) = k_{\text{eff}} \cdot v_0 + k_{\text{eff}} g t + \frac{\hbar k_{\text{eff}}^2}{2m} \tag{12}$$

where $k_{\text{eff}} \cdot v_0 + k_{\text{eff}} g t$ is the Doppler frequency shift and the last term is the photon recoil shift. We can see from Eq. (12) that for a different time t, one Doppler frequency shift $k_{\text{eff}} g t$ is changed. In the experiment, in order to compensate for the frequency difference due to the varying Doppler shift, the frequency of Raman beams should be scanned in a certain range.

Suppose that the frequency scanning in the experiment is $f' = \alpha t + f_0$, where $\alpha$and f_0 are the slope coefficient and initial value of the frequency scanning respectively. The overall time of the π/2–π–π/2 pulse sequence interacting with the atoms is $t' = 2T + 4\tau$, where τ is the time during which the pulses interact with the atoms. So that the measurement precision of the gravitational acceleration corresponding to the Doppler shift is:

$$\begin{aligned}\Delta g &= \frac{\{\delta - 2\pi[\alpha(2T + 4\tau) + f_0]\}\tau}{k_{\text{eff}} T^2} \\ &= \frac{\tau}{k_{\text{eff}} T^2}\Bigg[(2T + 4\tau)(k_{\text{eff}} g - 2\pi\alpha) \\ &\quad + \left(\frac{\hbar k_{\text{eff}}^2}{2m} + k_{\text{eff}} \cdot v_0 - 2\pi f_0\right)\Bigg]\end{aligned} \tag{13}$$

Under typical experimental parameters: the interaction time being $\tau = 8$ μs, the temporal interval between pulses being $T = 50$ ms, and the velocity of atoms being $v_0 = 15.7$ cm/s, we find that if one intends to confine the value of Δg below $1 \times 10^{-9}\text{m/s}^2$, the value of α must be chosen between 25 004.227 kHz/s and 25 004.243 kHz/s. It requires that the precision of the frequency scanning be in the order of 10 Hz/s. At the same time, the value of f_0 has to be chosen between 414 621.46 Hz and 414 623.06 Hz, which requires that the precision of frequency scanning used to compensate for the Doppler shift is in the order of 1 Hz.

4 Summary

In this paper, we calculated the influence of wave-front curvature of the Raman beams on the measurement precision of gravitational acceleration, treating the Raman beam as a Gaussian beam. We found that the precision is proportional to the spot size of the Raman beams. When the optical path difference between the two counter-propagating Raman pulses is decreased, the measurement precision increases. We also found that a longer temporal interval between pulses would play a positive role on the measurement precision. Furthermore, we took into account the variation of the atom Doppler frequency shift at different times during free falling of the atom cloud, thus obtaining the relation between the measurement precision of gravitational acceleration and the accuracy of frequency scanning to compensate for the Doppler frequency shift. All these results are relevant in experiments on the atom interferometer.

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