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RESEARCH ARTICLE

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Quantum information density and network

Abstract We present a quantum information network in which quantum information density is used for performing quantum computing or teleportation. The photons are entangled in quantum channels and play a role of flying ebit to transmit interaction among the nodes. A particular quantum Gaussian channel is constructed; it permits photon-encoded information to transmit quantum signals with certain quantum parallelism. The corresponding quantum dynamical mutual information is discussed, and the controlling nodes connectivity by driving the network is studied. With regard to different driving functions, the connectivity distribution of the network is complicated. They obey positive or negative power law, and also influence the assortativity coefficient or the dynamical property of the network.

Keywords quantum information, Gaussian channel, complex network

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1 Introduction

The quantum information network is considered as having spatially separated nodes connected by quantum communication channels; the quantum communication and operation among different nodes allow the network to perform various quantum computations or communications nonlocally [1−4].

To physically implement this network, the network nodes can be thought of as clusters of trapped atoms, ions, photons or quantum dots [5]. Interactions between two different network nodes depend on the transmission of one or more photons through a single quantum channel. These channels constitute the links among the nodes and play an important role in the network. The competition for links is a common feature of the quantum network. Important progress has been shown recently in the Bose-Einstein condensation of the complex network [6]. The universal feature of these systems is that the nodes are self-organized into a complex network whose topology and evolution closely reflect the dynamical characteristics of the complex network [7−13].

What is interesting to consider here is that the quantum information density (QID) property and transmission capacity for the quantum network is related to the links of the network nodes. Furthermore, since environmental noise is unavoidably introduced into the network, an external driving function is necessary to propel the network towards the expected direction. How does this external driving function control the pattern of connections between the network nodes? All of these considerations will enable the examined quantum network to display certain complex behaviors. The paper is arranged as follows: in Section 2 and 3, we introduce quantum computing and quantum communication in the network using the QID approach directly. In Section 4, we construct a quantum Gaussian channel. In Section 5, we study properties of connections among network nodes by external driving functions induced by HUHPM. In Section 6, we discuss the connectivity distribution and its influence in the dynamical property of the network. Fi-

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nally, in Section 7, we give the summary and conclusion. Several detailed calculations are shown in the appendix.

2 QID computation in network

Suppose a quantum network node consists of a qubit, which is, for example, a trapped atom inside a high-Q cavity. The atom has standard "Λ" type internal state, such as alkali atom with $|0\rangle$ and $|1\rangle$ denoting hyperfine manifolds of the ground state. The qubits are formed by QID of the atoms and QID of the polarization-entangled photons. Assuming that the mixed state in the system has the spectral decomposition $\rho = \sum_j p_j\rho_j$, where ρ_j is the jth eigen-projector of the Hamilton for the test system. The spectral decomposition for QID, $\rho\ln\rho$, is:

$$I = \rho\ln\rho = \sum_j p_j \ln(p_j\rho_j) \equiv \sum_j I_j \tag{1}$$

Then, the starting qubits for quantum computing can use the mixture states in the network given by

$$I_A \otimes I_{A_1B_1} \otimes I_B = \alpha I_A^0 \otimes I_{A_1B_1} \otimes I_B + \beta I_A^1 \otimes I_{A_1B_1} \otimes I_B$$

where I_A and I_B express the QID in the nodes A and B, respectively,

$$I_A = \rho_A \ln\rho_A = p_A^0 \ln(p_A^0\rho_A^0) + p_A^1 \ln(p_A^1\rho_A^1) = \alpha I_A^0 + \beta I_A^1 \tag{2}$$

$$I_B = \rho_B \ln\rho_B = p_B^0 \ln(p_B^0\rho_B^0) + p_B^1 \ln(p_B^1\rho_B^1) = a I_B^0 + b I_B^1 \tag{3}$$

and $I_{A_1B_1}$ is QID of an entangled states which is in the channel between A and B,

$$I_{A_1B_1} = I_{A_1}^0 \otimes I_{B_1}^1 + I_{A_1}^1 \otimes I_{B_1}^0 \tag{4}$$

here notice

$$\rho_A^0 \equiv |0\rangle_{AA}\langle 0|, \quad \rho_A^1 \equiv |1\rangle_{AA}\langle 1| \tag{5}$$

$$\rho_B^0 \equiv |0\rangle_{BB}\langle 0|, \quad \rho_B^1 \equiv |1\rangle_{BB}\langle 1| \tag{6}$$

Then, the realization of CNOT gate operation between A and B can be performed as follows:

(1) Performing a CNOT gate operation to $I_{A_1B_1}$, where QID in A_1 is control qubit and QID in B_1 is target qubit,

$$\begin{aligned}&\alpha I_A^0 \otimes I_{A_1B_1} \otimes I_B + \beta I_A^1 \otimes I_{A_1B_1} \otimes I_B\\ &\to \alpha I_A^0 \otimes \left(I_{A_1}^0 \otimes I_{B_1}^1 + I_{A_1}^1 \otimes I_{B_1}^1\right) \otimes I_B\\ &\quad + \beta I_A^1 \otimes \left(I_{A_1}^0 \otimes I_{B_1}^1 + I_{A_1}^1 \otimes I_{B_1}^1\right) \otimes I_B\end{aligned} \tag{7}$$

(2) Then the photons enter a cavity quantum electrodynamics system to realize CNOT gate operation between QID of the atom in node A, which is as control qubit, and the photon in B_1,

$$\begin{aligned}&\alpha I_A^0 \otimes \left(I_{A_1}^0 \otimes I_{B_1}^1 + I_{A_1}^1 \otimes I_{B_1}^1\right) \otimes I_B\\ &\quad + \beta I_A^1 \otimes \left(I_{A_1}^0 \otimes I_{B_1}^1 + I_{A_1}^1 \otimes I_{B_1}^1\right) \otimes I_B\\ &\to \alpha I_A^0 \otimes \left(I_{A_1}^0 \otimes I_{B_1}^1 + I_{A_1}^1 \otimes I_{B_1}^1\right) \otimes I_B\\ &\quad + \beta I_A^1 \otimes \left(I_{A_1}^0 \otimes I_{B_1}^0 + I_{A_1}^1 \otimes I_{B_1}^0\right) \otimes I_B\\ &= \alpha I_A^0 \otimes I_{A_1} \otimes I_{B_1}^1 \otimes I_B + \beta I_A^1 \otimes I_{A_1} \otimes I_{B_1}^0 \otimes I_B\end{aligned} \tag{8}$$

(3) Performing a CNOT gate operation between QID of atom on node B and QID of photon B_1using the QID in B_1 as the control qubit, Bob can get

$$\begin{aligned}&\alpha I_A^0 \otimes I_{A_1} \otimes I_{B_1}^1 \otimes I_B + \beta I_A^1 \otimes I_{A_1} \otimes I_{B_1}^0 \otimes I_B\\ &\to \alpha I_A^0 \otimes I_{A_1} \otimes I_{B_1}^1 \left(a I_B^1 + b I_B^0\right) + \beta I_A^1 \otimes I_{A_1} \otimes I_{B_1}^0 \otimes I_B\end{aligned} \tag{9}$$

this allows one to finally construct a CNOT gate operation for the QID between two spatially separated nodes A and B by using a pair of entangled photons,

$$\begin{aligned}&\left(\alpha I_A^0 + \beta I_A^1\right)\left(a I_B^0 + b I_B^1\right)\\ &\to \alpha a I_A^0 \otimes I_B^1 + \alpha b I_{A_1}^0 \otimes I_B^0 + \beta a I_A^1 \otimes I_B^0 + \beta b I_A^1 \otimes I_B^1\end{aligned} \tag{10}$$

where notice that the CNOT gate can be: $\rho_A^0 \otimes \rho_B^1 + \rho_A^1 \otimes \rho_B^0 \to \rho_A^0 \otimes \rho_B^1 + \rho_A^1 \otimes \rho_B^1$ which gives

$$\begin{aligned}&I_A^0 \otimes I_B^1 + I_A^1 \otimes I_B^0\\ &= \rho_A^0 \ln\rho_A^0 \otimes \rho_B^1 \ln\rho_B^1 + \rho_A^1 \ln\rho_A^1 \otimes \rho_B^0 \ln\rho_B^0\\ &\to \rho_A^0 \ln\rho_A^0 \otimes \rho_B^1 \ln\rho_B^1 + \rho_A^1 \ln\rho_A^1 \otimes \rho_B^1 \ln\rho_B^1\\ &= I_A^0 \otimes I_B^1 + I_A^1 \otimes I_B^1\end{aligned} \tag{11}$$

3 QID teleportation in network

Consider a model system consisting of two QID states described by

$$I_j = \rho_j \ln\rho_j, \quad j = 1, 2 \tag{12}$$

The information density to be teleported is the mixing states

$$I^1 = I_1 + I_2 \tag{13}$$

The information density input into the network (in Liouville space) is

$$\begin{aligned}I^{1-23} = \frac{1}{2}[&I_1 \otimes (I_1 \otimes I_1 + I_2 \otimes I_2)\\ &+ I_2 \otimes (I_1 \otimes I_1 + I_2 \otimes I_2)]\end{aligned} \tag{14}$$

where the first two QID as the qubits belong to Alice, and the third QID as a qubit belongs to Bob (in Liouville space); Alice's second QID and Bob's QID, both display as two qubits, start out in an "EPR" information density "state",

$$\frac{1}{2}\left(I_1 \otimes I_1 + I_2 \otimes I_2\right) \tag{15}$$

Then Alice sends her qubits through a CNOT gate for QID, where the first QID as control qubit and the second qubit as target qubit, giving

$$\begin{aligned} I^{1-23} = \frac{1}{2}[&I_1 \otimes (I_1 \otimes I_1 + I_2 \otimes I_2) \\ &+ I_2 \otimes (I_2 \otimes I_1 + I_1 \otimes I_2)] \end{aligned} \tag{16}$$

She then sends the first QID through a Hadamard gate for QID, obtaining

$$\begin{aligned} I^{1-23} = \frac{1}{2}[&(I_1 + I_2)(I_1 \otimes I_1 + I_2 \otimes I_2) \\ &+ (I_1 - I_2)(I_2 \otimes I_1 + I_1 \otimes I_2)] \end{aligned} \tag{17}$$

where the terms in the above equation may be simply regrouped and rewritten as

$$\begin{aligned} I^{1-23} = \frac{1}{2}[&|I_1 \otimes I_1)(I_1 + I_2) + |I_1 \otimes I_2)(I_2 + I_1) \\ &+ I_2 \otimes I_1(I_1 - I_2) + I_2 \otimes I_2(I_2 - I_1)] \end{aligned} \tag{18}$$

This expression naturally breaks down into four terms:

$$\begin{aligned} &\text{Alice: } I_1 \otimes I_1 \longrightarrow \text{Bob: } I_1 + I_2 \\ &\text{Alice: } I_1 \otimes I_2 \longrightarrow \text{Bob: } I_2 + I_1 \\ &\text{Alice: } I_2 \otimes I_1 \longrightarrow \text{Bob: } I_1 - I_2 \\ &\text{Alice: } I_2 \otimes I_2 \longrightarrow \text{Bob: } I_2 - I_1 \end{aligned} \tag{19}$$

If Alice performs a measurement and obtains the result $I_1 \otimes I_1$ or $I_1 \otimes I_2$, then Bob's system will receive $I_1 + I_2$ which is the original QID I^1. In the other two cases, depending on Alice's measurement outcome, the received QID of Bob will end up in one of three possible information densities. Then, by using quantum logical gates, he can recover the original QID [14]. Notice that the Hadamard gate operation for the QID can be realized by the following procedure: first, $\rho_1 \rightarrow \rho_1 \pm \rho_2 \rightarrow 1 - (\rho_1 \pm \rho_2)$, then, using $\rho_1\rho_2 = 0$ and series expansion, one gets

$$\begin{aligned} &[1 - (\rho_1 \pm \rho_2)] \ln [1 - (\rho_1 \pm \rho_2)] \\ &= (1 \mp \rho_1) \ln (1 \mp \rho_1) + (1 \mp \rho_2) \ln (1 \mp \rho_2) \end{aligned} \tag{20}$$

finally, let $1 \mp \rho_1 \rightarrow \rho_1$ and $1 \mp \rho_2 \rightarrow \rho_2$, which allows

$$I_1 \xrightarrow{\text{Hadamard}} I_1 \pm I_2 \tag{21}$$

4 Quantum Gaussian channel

Between two nodes of the network, a quantum channel can also be established. One possible quantum channel constructed is a Gaussian channel. Although there is coupling between photons emitted from the atoms confined in the cavity, the encoded information carrier in the channel are photons which may interact with the environmental field at temperature T. This is a typical quantum Brownian motion problem whose Hamiltonian is described by

$$H = H_S \otimes 1_B + 1_S \otimes H_B + H_{\text{int}} \tag{22}$$

where defining $H_S = \omega a^+ a$, $H_B = \int_0^\infty \omega_k b_k^+ b_k \mathrm{d}k$, and $H_{\text{int}} = \lambda \int_0^\infty \left(g_k a b_k^+ + g_k^* a^+ b_k\right) \mathrm{d}k$; a (a^+) is the creation (annihilation) operator for the oscillator, b_k^+ (b_k) is the creation (annihilation) operator of the k continuum field mode, ω_k is the Lamour frequency of spin k due to the Zeeman interaction, g_k denotes the coupling between the oscillator and kth field mode, and λ is a coupling number.

From Appendix Eq. (A-11), a master equation for the functional of the reduced density operator $I_S(t) = I[\mathrm{Tr}_B \rho(t)]$ can be obtained [14]; moreover, in the coherent representation, performing an integration by parts, a quantum Fokker-Planck equation for $I_S(t)$ can be obtained from Eq. (A-11),

$$\begin{aligned} &\frac{\partial}{\partial t} f(\alpha, \alpha^*, t) \ln f(\alpha, \alpha^*, t) \\ &= \left[\left(\frac{r_0}{2} + \mathrm{i}\omega_0\right) \frac{\partial}{\partial \alpha} \alpha + \left(\frac{r_0}{2} - \mathrm{i}\omega_0\right) \frac{\partial}{\partial \alpha^*} \alpha^* + r_0 N \frac{\partial^2}{\partial \alpha \partial \alpha^*}\right] \\ &\quad \cdot f(\alpha, \alpha^*, t) \ln f(\alpha, \alpha^*, t) \end{aligned} \tag{23}$$

where $|\alpha\rangle$ is a coherent state, $I_S(t)$ is expanded by

$$I_S(t) = \int \mathrm{d}^2\alpha f(\alpha, \alpha^*, t) \ln f(\alpha, \alpha^*, t) |\alpha\rangle \langle\alpha| \tag{24}$$

with definitions:

$$\omega_0 \equiv \mathrm{i}\omega + \mathrm{i}\mathcal{P} \int_0^\infty \frac{\lambda^2 |\eta_{jk}|^2}{\omega - k} \mathrm{d}\sigma_{jk} \tag{25}$$

$$r_0 \equiv \frac{\mathrm{i}2}{(N+1)} \frac{\pi\lambda^2 |\eta_{jk}|^2}{\mathrm{e}^{\beta\omega} - 1} \mathrm{e}^{\beta\omega} = \mathrm{i}\frac{1}{N} \frac{\pi\lambda^2 |\eta_{jk}|^2}{\mathrm{e}^{\beta\omega} - 1}$$

The above Fokker-Planck equation (23) may describe transmission of the information density signals (encoded in harmonic oscillator) along a quantum Gaussian channel by extending the concept of classical Gaussian channel for information. Concretely, let us consider a harmonic oscillator as ebit encoded quantum information on their coherent states. This oscillator consisting of n_1 photons is like the Brownian particle transmission in an information channel described by Eq. (23), i.e. the channel can be described by evolution operator induced by Eq. (23) acting on initial input QID. When the oscillator consisting of n_1 photons are transmitted from the input system, the oscillators consisting of n_2 photons from the noise system (environment) add to the signal, then m_1 photons are lost to the system through the channel, and m_2 photons are detected in the output system, with $n_1+n_2=m_1+m_2$ [16]. Furthermore, if the spectral decomposition of the density operator for the mixed states of n_1 photons is given by

$$\rho' = \sum_{n_1} p_{n_1} \left|n_1\right\rangle \left\langle n_1\right| \tag{26}$$

then its QID $\rho' \ln \rho'$ transmission in the channel remains the same quantum entanglement (parallelism) described by the spectral decomposition of the functional:

$$\rho' \ln \rho' = \sum_{n_1} p_{n_1} \ln p_{n_1} \left|n_1\right\rangle \left\langle n_1\right| \tag{27}$$

where $p_{n_1} \to p_{n_1} \ln p_{n_1}$ defined as a quantum information. This makes the quantum Gaussian channel to have an (parallelism) advantage over classical Gaussian channel.

The solution of Eq. (23) is given by $f(\alpha, \alpha^*, t) \ln f(\alpha, \alpha^*, t)$ with [17]

$$\begin{aligned} & f\left(\alpha, \alpha^*, t\right)\big|_{\left(\alpha_0, \alpha_0^*, 0\right)} \\ & = \frac{1}{\pi N\left(1-\mathrm{e}^{-r_0 t}\right)} \exp \left\{-\frac{\left[\alpha-\alpha_0 \mathrm{e}^{-\left(\frac{r_0}{2}\right) t} \mathrm{e}^{-\mathrm{i} \omega_0 t}\right]^2}{N\left(1-\mathrm{e}^{-r_0 t}\right)}\right\} \end{aligned} \tag{28}$$

Indeed, by substituting

$$\zeta^2 \equiv \pi N\left(1-\mathrm{e}^{-r_0 t}\right)\left(\alpha-\alpha_0 \mathrm{e}^{-\frac{r_0}{2} t} \mathrm{e}^{-\mathrm{i} \omega_0 t}\right)^2 \tag{29}$$

and

$$\sigma_z(t) \equiv \frac{\sqrt{2 \pi}}{2} N\left(1-\mathrm{e}^{-r_0 t}\right) \tag{30}$$

into the Gaussian ansatz Eq. (28), we obtain the solution as

$$f\left(\alpha, \alpha^*, t\right)=f_z(\zeta)=\frac{1}{\sqrt{2 \pi} \sigma_z(t)} \exp \left[-\frac{1}{2} \frac{\zeta^2}{\sigma_z^2(t)}\right] \tag{31}$$

with

$$\sigma_z(t)=\sigma_{\text{in}}(t)+\sigma_{\text{no}}(t) \tag{32}$$

where $\sigma_{\text{in}}^2(t)$ and $\sigma_{\text{no}}^2(t)$ represent the mean power of input signal and noise, respectively, they are assumed to correspond to the Gaussian distributed random variables by [17]

$$f_x(\xi)=\frac{1}{\sqrt{2 \pi} \sigma_{\text{in}}(t)} \exp \left[-\frac{1}{2} \frac{\xi^2}{\sigma_{\text{in}}^2(t)}\right] \tag{33}$$

and

$$f_y(\eta)=\frac{1}{\sqrt{2 \pi} \sigma_{\text{no}}(t)} \exp \left[-\frac{1}{2} \frac{\eta^2}{\sigma_{\text{no}}^2(t)}\right] \tag{34}$$

Then the solution (31) gives a formulation of the Bayesian estimation, which derives the condition information as

$$\begin{aligned} \mathcal{I}(x|z) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}\xi \mathrm{d}\zeta f_{x,z}(\xi, \zeta) \ln f_{x|z}(\xi|\zeta) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}\xi \mathrm{d}\zeta f_{x,z}(\xi, \zeta) \ln f_{x,z}(\xi, \zeta) \\ &\quad - \int_{-\infty}^{\infty} \mathrm{d}\zeta f_z(\zeta) \ln f_z(\zeta) \\ &= \mathcal{I}(x, z) - \mathcal{I}(z) \end{aligned} \tag{35}$$

Using Gaussian integral properties we obtain [17]

$$\begin{aligned} \mathcal{I}(x|z) =& -\frac{\ln\left(2\pi\sigma_{\text{no}}^2\right)}{2} \int_{-\infty}^{\infty} \mathrm{d}\xi f_{\text{in}}(\xi) - \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}\xi f_{\text{in}}(\xi) \\ & + \int_{-\infty}^{\infty} \mathrm{d}\xi f_{\text{in}}(\xi) \ln f_{\text{in}}(\xi) + \frac{1}{2} \ln\left(2\pi\sigma_z^2\right) + \frac{1}{2} \\ =& \frac{1}{2} \ln\left[2\pi \frac{\sigma_{\text{in}}^2(t)\left(\sigma_{\text{in}}^2(t)+\sigma_{\text{no}}^2(t)\right)}{\sigma_{\text{no}}^2(t)}\right] + \frac{1}{2} \end{aligned} \tag{36}$$

and

$$\mathcal{I}(x) = \int_{-\infty}^{\infty} \mathrm{d}\xi f_{\text{in}}(\xi) \ln f_{\text{in}}(\xi) = \frac{1}{2} \ln[2\pi\sigma_{\text{in}}^2(t)] + \frac{1}{2} \tag{37}$$

Therefore, the quantum dynamical mutual information formula [15, 19] for the quantum Gaussian channel can be generally obtained in the coherent state representation Eq. (B-4),

$$\begin{aligned} \mathcal{I}(x; z) =& \mathcal{I}(x|z) - \mathcal{I}(x) \\ =& \frac{1}{2} \ln\left\{1 + \frac{\left[\frac{\sqrt{2\pi}}{2} N\left(1-\mathrm{e}^{-r_0\tau}\right) - \sigma_y(\tau)\right]^2}{\sigma_{\text{no}}^2(\tau)}\right\} \\ =& \mathcal{I}\left(A_0(0); B_\alpha(\tau)\right) \end{aligned} \tag{38}$$

where $A_0(0)$ and $B_\alpha(\tau)$ have the same definitions as that in Eqs. (B-1) and (B-2).

5 Control connectivity

Based on the ultimate analysis, it is clear that we can construct the quantum channel to transmit QID between two nodes as a link to form a quantum information network. Because of the "quantum" property, each link between a pair of nodes has a probability to open or close. Furthermore, we suppose that there are new nodes added into the network stochastically (by random users). Each node added to the system at time t_i with energy ϵ_i can be described by the number of links $k_i\left(t, t_i, \epsilon_i\right)$ at time t. This makes the quantum information network similar to the Bose network describing Bose-Einstein condensation [6, 7].

Then, it is interesting to consider controlling the connectivity distribution for the network to arrive at a certain expected level by using the driving method. Now we suppose that the quantum information network is driven by using a harmonious unifying hybrid preferential method (HUHPM), which is controlled by a hybrid ratio, d/r. The basic concept for the HUHPM can be expressed as the following [19, 20]:

$$\text{HUHPM}=\begin{array}{c}\text{Random preferential attachment (RPA)}\\+\\\text{Deterministic preferential attachment (DPA)}\end{array}$$

This means that the HUHPM can rest on any type of network's original growth way and RPA pattern by adding the DPA pattern according to a certain arrangement of the degree distribution to arrive at a driving function G which allows one to get the expected pattern of links. This implementation combines the random connection with the deterministic connection by using the hybrid ratio to request growth scale size of the networks. Hence, the unified hybrid ratio can be defined as

$$\frac{d}{r}=\frac{\text{Time intervals } d \text{ of DPA}}{\text{Time intervals } r \text{ of RPA}}$$

where d is a number of time intervals (step) for DPA, and r is the number of RPA. In the process of the network evolution, the hybrid ratio must maintain the same value by combining RPA and DPA. Actually, which kind of preferential attachments carried at first is flexible. This means that one can use different orders to make the two hybrids grow the network in turn, until the required scale size is achieved. The main mechanism and principle for implementation of the hybrid growth network are as follows:

(1) The growth way: first, use each growing rule of the model to carry on the growth. For example, the un-weighted BA model starts growth from less isolated nodes, the quantity is m_0, then it increases in each time-interval to reach a new node with $m \leqslant m_0$ edges and connects this new node to m different existing nodes. For the weighted BBV model, it allows to produce new edges among the old nodes, while for the TDE model, it grows a new node at each time step. The new node connects to the old nodes with m new edges, and then the network continues to grow this way.

(2) Growth connection way: Each step adopting the kind of connection mechanism must accord to the hybrid ratio d/r. Under the final d/r remaining invariance, there are three kinds of hybrid connection orders:

HPAS-1: First carrying on RPA, then arranging rank of the degree of nodes from the biggest to the smallest, selecting m biggest degree nodes to carry on DPA.

HPAS-2: First DPA then RPA.

HPAS-3: The random number decides whether to carry on RPA or DPA as the first connection order. When the random number is smaller than the hybrid ratio, adopt RPA connection. Otherwise, adopt DPA connection, so that randomness and determination continuously carry on in turn, until the required hybrid ratio is satisfied to get the expected scale size of the network.

(3) DPA way: After each attachment, rank of the degree of nodes is reordered again by a certain order, such as from the biggest to the smallest: $k_1 > k_2 > \cdots > k_m > \cdots > k_n$, then m nodes is attached preferentially. This is a general way for DPA, which is quite natural.

After the procedures above for each model, the rank of the vertices degrees is then rearranged in a certain order of $k_1, \cdots, k_n$. The DPA is to be conducted for d time steps for the HUHPM networks according to the new rank of the vertices degrees above when choosing the nodes connected to the new node. This procedure creates a network with $N = r + d + m_0$ nodes. Then steps (3) and (4) of the HUHPM algorithm are repeated again. In this HUHPM algorithm, RPA and DPA are applied in turns under a certain hybrid ratio d/r, but may be different HPAS ways, until finally reaching the desired size of the network field for controlling the final links pattern of the network in the expected level (by administrator).

Thus, the rate of $k_i\left(t, t_i, \epsilon_i\right)$ with respect to time may be expressed as [6]

$$\frac{\partial k_i\left(t, t_i, \epsilon_i\right)}{\partial t}=m\frac{\mathrm{e}^{-\beta\epsilon_i}k_i\left(t, t_i, \epsilon_i\right)}{\sum\limits_j \mathrm{e}^{-\beta\epsilon_j}k_j\left(t, t_j, \epsilon_j\right)}+g\left[k_i\left(t, t_i, \epsilon_i\right)\right] \tag{39}$$

where $g\left[k_i\left(t, t_i, \epsilon_i\right)\right]$ is a driving function introduced by

HUHPM, β is inverse temperature $\frac{1}{T}$, m is defined as a new node attached by m links to m of the N existing nodes of the network. Generally, choosing a different type of driving function can cause different types of evolution of $k_i(t, t_i, \epsilon_i)$. To study this driving property, we choose the following two different functions:

(1) The $g[k_i(t, t_i, \epsilon_i)]$ is chosen as the exponential function $\exp[-k_i(t, t_i, \epsilon_i)]$, then the solution of Eq. (39) satisfies

$$k_i(t, t_i, \epsilon_i) = m\left(\frac{t}{t_i}\right)^{f(\epsilon_i)} + \int \exp[-k_i(t, t_i, \epsilon_i)]\,\mathrm{d}t \tag{40}$$

where $f(\epsilon)$ is the dynamic exponent which depends on energy ϵ, the chemical potential μ and the inverse temperature β, and is given by

$$f(\epsilon) = \mathrm{e}^{-\beta(\epsilon-\mu)} \tag{41}$$

This results in a differential equation:

$$\frac{\partial k_i(t, t_i, \epsilon_i)}{\partial t} = \frac{mf(\epsilon_i)}{t_i}\left(\frac{t}{t_i}\right)^{f(\epsilon_i)-1} + \exp[-k_i(t, t_i, \epsilon_i)] \tag{42}$$

its solution can be obtained by

$$k_i(t, t_i, \epsilon_i) = m\left(\frac{t}{t_i}\right)^{f(\epsilon_i)} + \ln\left\{\int \mathrm{d}t \exp\left[-m\left(\frac{t}{t_i}\right)^{f(\epsilon_i)}\right]\right\} \tag{43}$$

This shows that the curve of $k_i(t, t_i, \epsilon_i)$ with respect to t depends on the value of $f(\epsilon_i)$. For example, if fixing $f(\epsilon_i) = 2$, then

$$k_i(t, t_i, \epsilon_i) = m\left(\frac{t}{t_i}\right)^2 + \ln\left\{\frac{1}{2}\frac{\sqrt{\pi}}{\sqrt{m}} t_i \mathrm{signum}(t_i)\,\mathrm{erf}\left[\sqrt{m}\frac{t}{t_i \mathrm{signum}(t_i)}\right]\right\} \tag{44}$$

which is complicated; if fix $f(\epsilon_i) = 1$ then

$$k_i(t, t_i, \epsilon_i) = m\left(\frac{t}{t_i}\right) + \ln\left(-\frac{t_i}{m}\mathrm{e}^{-m\frac{t}{t_i}}\right) = \ln\left(-\frac{t_i}{m}\right) \tag{45}$$

which is not related to t. In the second case, the connectivity distribution $p(\epsilon)$, which represents the sum of the probabilities $P(k|\,\epsilon)$ of the node with energy ϵ, has a time-independent connectivity k, which allows the total capacity of the information transmission in the Gaussian channels connected to a node i for the network to be given by Eq. (38)

$$\mathcal{I} = \ln\left(-\frac{t_i}{m}\right)\mathcal{I}(A_0(0); B_\alpha(\tau)) \tag{46}$$

(2) The $g[k_i(t, t_i, \epsilon_i)]$ is chosen as an elliptic umbilic from $\left(y = k_i^{-1},\ x = t\right)$ which is one of seven catastrophic polynomials [22]:

$$g[k_i(t, t_i, \epsilon_i)] = t^3 - 3tk_i^{-2}(t, t_i, \epsilon_i) + a\left(t^2 + k_i^{-2}(t, t_i, \epsilon_i)\right) + ck_i^{-1}(t, t_i, \epsilon_i) + gt \tag{47}$$

then Eq. (39) becomes

$$\frac{\partial k_i(t, t_i, \epsilon_i)}{\partial t} = \frac{mf(\epsilon_i)}{t_i}\left(\frac{t}{t_i}\right)^{f(\epsilon_i)-1} + t^3 - 3tk_i^{-2}(t, t_i, \epsilon_i) + a\left(t^2 + k_i^{-2}(t, t_i, \epsilon_i)\right) + ck_i^{-1}(t, t_i, \epsilon_i) + gt \tag{48}$$

We assume that the solution is

$$k_i(t, t_i, \epsilon_i) = k + s \tag{49}$$

with

$$\frac{\partial k}{\partial t} = t^3 - 3t(k+s)^{-2} + a(t^2 + (k+s)^{-2}) + c(k+s)^{-1} + gt \tag{50}$$

where $s \equiv m\left(\frac{t}{t_i}\right)^{f(\epsilon_i)}$. Thus $k_i(t, t_i, \epsilon_i)$ is given by integral from Eq. (50),

$$\int \frac{(k+s)^2}{\varsigma(k+s)^2 - 3t + a + c(k+s)}\mathrm{d}k = t \tag{51}$$

where $\varsigma \equiv \left(t^3 + gt + at^2\right)$. Notice the following three integrals can be used in the Eq. (51) for getting the solution $k_i(t, t_i, \epsilon_i)$:

$$\begin{aligned}&\int \frac{k^2}{\varsigma(k+s)^2 - 3t + a + c(k+s)}\mathrm{d}k \\ &= \frac{k}{\varsigma} - \frac{2\varsigma s + c}{2\varsigma^2}\ln\left[\varsigma(k+s)^2 - 3t + a + c(k+s)\right] \\ &\quad + \frac{(2\varsigma s + c)^2 - 2\varsigma\left(\varsigma s^2 - 3t + a + cs\right)}{2\varsigma^2} \\ &\cdot \int \frac{\mathrm{d}x}{(t^3 + gt + at^2)x^2 - 3t + a + cx} \\ &\cdot \int \frac{2ks}{\varsigma(k+s)^2 - 3t + a + c(k+s)}\mathrm{d}k \\ &= \frac{2ks}{\varsigma}(\ln\varsigma - 2\varsigma s - c) - s\frac{2\varsigma s + c}{\varsigma}\int \frac{\mathrm{d}x}{\varsigma x^2 - 3t + a + cx}\end{aligned} \tag{52}$$

and

$$\int \frac{s^2\mathrm{d}x}{\varsigma x^2-3t+a+cx}$$
$$=\begin{cases}\dfrac{s^2 2}{\sqrt{(2\varsigma s+c)^2-4\varsigma(\varsigma s^2-3t+a+cs)}}\ln\dfrac{2\varsigma k+2\varsigma s+c-\sqrt{(2\varsigma s+c)^2-4\varsigma(\varsigma s^2-3t+a+cs)}}{2\varsigma k+2\varsigma s+c+\sqrt{(2\varsigma s+c)^2-4\varsigma(\varsigma s^2-3t+a+cs)}}, & \text{for } (2\varsigma s+c)^2>4\varsigma(\varsigma s^2-3t+a+cs)\\ \dfrac{s^2 2}{\sqrt{4\varsigma(\varsigma s^2-3t+a+cs)-(2\varsigma s+c)^2}}\arctan\dfrac{2\varsigma k+2\varsigma s+c}{\sqrt{4\varsigma(\varsigma s^2-3t+a+cs)-(2\varsigma+c)^2}}, & \text{for } (2\varsigma s+c)^2<4\varsigma(\varsigma s^2-3t+a+cs)\end{cases} \tag{53}$$

This shows that $k_i(t,t_i,\epsilon_i)$ consists of some products of irrational polynomial and logarithm (or arctan). The complex characteristics of $k_i(t,t_i,\epsilon_i)$ may be shown by adjusting some parameters of f,a,c,g,m and t_i. For example, the term contained in the square root in Eq. (53)

$$4\varsigma\left[\varsigma m\left(\frac{t}{t_i}\right)^{2f}-3t+a+cm\left(\frac{t}{t_i}\right)^{f}\right]-2\varsigma m\left(\frac{t}{t_i}\right)^{f}-c \tag{54}$$

may appear in certain catastrophes, where $\varsigma\equiv(t^3+gt+at^2)$. For example, the critical points are given by an equation:

$$0=\left\{4\varsigma\left[cf\frac{m}{t_i}\left(\frac{t}{t_i}\right)^{f-1}+2fm\frac{\varsigma}{t_i}\left(\frac{t}{t_i}\right)^{2f-1}-3\right]-2fm\frac{\varsigma}{t_i}\left(\frac{t}{t_i}\right)^{f-1}\right\} \tag{55}$$

and the folds points are given by an equation:

$$0=4\varsigma\left[cf\frac{m}{t_i^2}(f-1)\left(\frac{t}{t_i}\right)^{f-2}+2fm\frac{\varsigma}{t_i^2}(2f-1)\cdot\left(\frac{t}{t_i}\right)^{2f-2}\right]-2fm\frac{\varsigma}{t_i^2}(f-1)\left(\frac{t}{t_i}\right)^{f-2} \tag{56}$$

One of the solution for Eq. (56) is:

$$c=-\frac{1}{4fm\varsigma}\frac{t_i^2}{f-1}\left(\frac{t}{t_i}\right)^{-f+2}\left[-2fm\frac{\varsigma}{t_i^2}(f-1)\cdot\left(\frac{t}{t_i}\right)^{f-2}+8fm\frac{\varsigma^2}{t_i^2}(2f-1)\left(\frac{t}{t_i}\right)^{2f-2}\right] \tag{57}$$

which gives a non-linear equation in the (ς,c)-plane for fixing t (t_i), m and f, where ς is given by Eqs. (55) and (57):

$$\varsigma=\frac{1}{4fm}\left(\frac{t}{t_i}\right)^{-2f+1}\left[6t_i+fm\left(\frac{t}{t_i}\right)^{f-1}-2cfm\left(\frac{t}{t_i}\right)^{f-1}\right] \tag{58}$$

The Eq. (57) shows that the curve of folds is a non-linear line, in which the number of solutions $k_i(t,t_i,\epsilon_i)$ can be changed. This may drive the network to some catastrophe status.

6 Connectivity distribution and dynamical property

The connectivity distribution $P(k)$ can be obtained by the sum of the probabilities $P(k|\,\epsilon)$ which is a node with energy ϵ that has connectivity k. Thus, we have to sum up all the nodes with energy distribution $p(\epsilon)$, that gives [6]

$$P(k)=\int \mathrm{d}\epsilon p(\epsilon)\frac{1}{\frac{\partial k(t,\epsilon)}{\partial t}} \tag{59}$$

This allows one to get complicated $P(k)$ which may have positive exponential. For example, in case (1), one gets

$$P(k_i)=\int \mathrm{d}\epsilon p(\epsilon_i)\frac{1}{\frac{mf(\epsilon_i)}{t_i}\left(\frac{t}{t_i}\right)^{f(\epsilon_i)-1}+\mathrm{e}^{k_i(t,t_i,\epsilon_i)}} \tag{60}$$

and in case (2), one gets

$$P(k_i)=\int \mathrm{d}\epsilon p(\epsilon_i)\frac{1}{\frac{mf(\epsilon_i)}{t_i}\left(\frac{t}{t_i}\right)^{f(\epsilon_i)-1}+t^3-3tk_i^{-2}(t,t_i,\epsilon_i)+a[t^2+k_i^{-2}(t,t_i,\epsilon_i)]+ck_i^{-1}(t,t_i,\epsilon_i)+gt} \tag{61}$$

By choosing $f(\epsilon_i)-1=0.08$ or -0.8, $\ln\dfrac{mf(\epsilon_i)}{t_i}=3$ or 1, Eq. (60) becomes

$$P(k_i) = \int \mathrm{d}\epsilon p(\epsilon_i) \frac{1}{\mathrm{e}^3 \left(\frac{t}{1.08}\right)^{0.08} + \exp k_i} \sim \frac{1}{\mathrm{e}^3 \left(\frac{t}{1.08}\right)^{0.08} + \mathrm{e}^{k_i}} \quad (62)$$

or

$$P(k_i) = \int \mathrm{d}\epsilon p(\epsilon_i) \frac{1}{\mathrm{e} \left(\frac{t}{1.08}\right)^{-0.8} + \mathrm{e}^{k_i}} \sim \frac{1}{\mathrm{e} \left(\frac{t}{1.08}\right)^{-0.8} + \mathrm{e}^{k_i}} \quad (63)$$

and Eq. (61) becomes

$$P(k_i) = \int \mathrm{d}\epsilon p(\epsilon_i) \frac{1}{\mathrm{e}^3 \left(\frac{t}{1.08}\right)^{0.08} + t^3 - 3tk_i^{-2} + a\left(t^2 + k_i^{-2}\right) + ck_i^{-1} + gt} \sim \frac{1}{\mathrm{e}^3 \left(\frac{t}{1.08}\right)^{0.08} + t^3 - 3tk_i^{-2} + a\left(t^2 + k_i^{-2}\right) + ck_i^{-1} + gt} \quad (64)$$

or

$$P(k_i) = \int \mathrm{d}\epsilon p(\epsilon_i) \frac{1}{\mathrm{e} \left(\frac{t}{1.08}\right)^{-0.8} + t^3 - 3tk_i^{-2} + a\left(t^2 + k_i^{-2}\right) + ck_i^{-1} + gt} \sim \frac{1}{\mathrm{e} \left(\frac{t}{1.08}\right)^{-0.8} + t^3 - 3tk_i^{-2} + a\left(t^2 + k_i^{-2}\right) + ck_i^{-1} + gt} \quad (65)$$

This shows that in addition to $P(k)$ being a function k, it is also a function of t. Moreover, through observation, this function is convergent while t increases. Thus $\lim_{t\to\infty} P(k)$ exists and $P(k)$ is stable when $t \to \infty$. This allows one to study the property of $P(k)$ by fixing t. For example, in case (1), when $t = 1.08$, one gets

$$P(k) \sim \frac{1}{\mathrm{e}^3 + \mathrm{e}^k} \quad (66)$$

which is shown in Fig. 1 in which $P(k)$ decreases while k increases; while in case (2), when $t = 1.08$, $g = c = 1$,

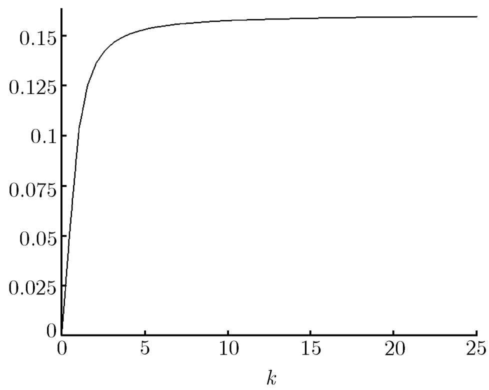

Fig. 1 The connectivity distribution $P(k) \sim \frac{1}{\mathrm{e}^3 + \mathrm{e}^k}$, where choosing $f(\epsilon_i) - 1 = 0.08$, $\ln mf(\epsilon_i)/t_i = 3$, and $t = 1.08$.

$a = 5$, one gets

$$P(k) \sim \frac{1}{\mathrm{e} + k^{-1} + 2.76k^{-2} + 3.5}$$

shown in Fig. 2 in which $P(k)$ increases while the pos-

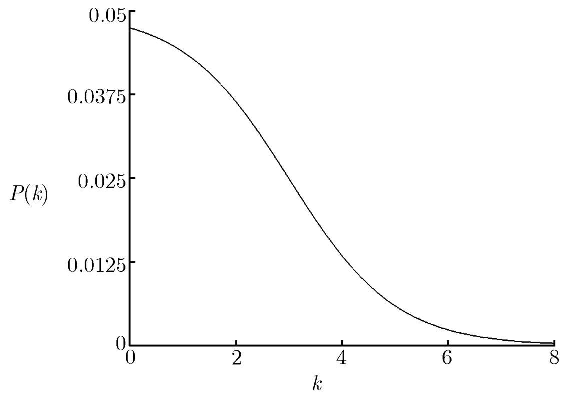

Fig. 2 The connectivity distribution $P(k) \sim \frac{1}{\mathrm{e}+k^{-1}+2.76k^{-2}+3.5}$, where $f(\epsilon_i) - 1 = -0.8$, $\ln mf(\epsilon_i)/t_i = 1$, $t = 1.08$, $g = c = 1$, $a = 5$.

itive power of k increase. This is quite interesting, in fact, the negative power of k can also be obtained by exchanging k_i^{-1} for k_i in the driving term, then, when $t = 1.08$, $g = c = 1$, $a = 5$, we have

$$P(k) \sim \frac{1}{\mathrm{e} + k + 2.76k^2 + 3.5} \quad (67)$$

shown in Fig. 3 in which $P(k)$ decreases with increase in the negative power of k.

The positive or negative power of k may influence the assortativity coefficient of the network r. Indeed, if r is a quantity corresponding to the correlation coefficient of the node degree at the ends of an edge of the network [25, 26],

$$r \propto \langle jk \rangle - \langle j \rangle \langle k \rangle \quad (68)$$

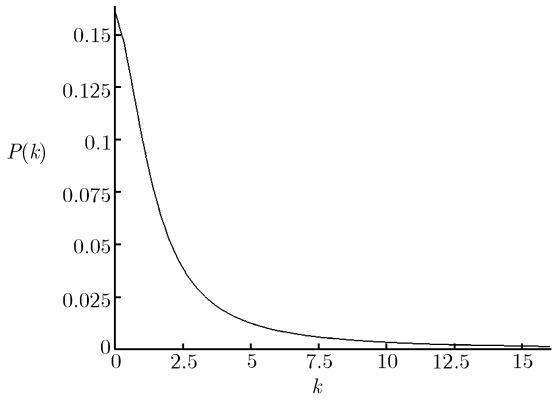

Fig. 3 The connectivity distribution $P(k) \sim \frac{1}{\mathrm{e}+k+2.76k^2+3.5}$, where $f(\epsilon_i) - 1 = -0.8$, $\ln mf(\epsilon_i)/t_i = 1$, $t = 1.08$, $g = c = 1$, $a = 5$.

then from

$$P(k) \sim k^{-\gamma} \to k^{\gamma} \tag{69}$$

we have

$$\langle jk\rangle - \langle j\rangle\langle k\rangle \sim \frac{jk^{-\gamma+3}}{(-\gamma+2)(-\gamma+3)} - \frac{\gamma jk^{-\gamma+2}}{(-\gamma+1)(-\gamma+2)} - \frac{j^{-\gamma+3}k^{-\gamma+3}}{(-\gamma+3)^2} \to \frac{jk^{\gamma+3}}{(\gamma+2)(\gamma+3)} - \frac{\gamma jk^{\gamma+2}}{(\gamma+1)(\gamma+2)} - \frac{j^{\gamma+3}k^{\gamma+3}}{(\gamma+3)^2} \tag{70}$$

which shows that the assortativity coefficient r is a function of k, j and γ, where the sign of γ can influence the r strongly.

Furthermore, it can be shown that the topological property change of the network can influence the dynamical property of the network. In fact, since $k_0(t) = m(t/t_0)^{f(\epsilon)}$ is driven to $k(t)$ that causes $P(k)$ to deviate from power law $\frac{1}{k}$, the initial energy ϵ_0 in a node

$$\epsilon_0 = \mu - \frac{1}{\beta}\ln\frac{\ln\frac{k_0}{m}}{\ln\frac{t}{t_0}} \tag{71}$$

can be transferred to the energy ϵ. For example, in case 1 ϵ_0 changes to

$$\epsilon = \mu - \frac{1}{\beta}\ln\frac{t\left[\frac{\mathrm{d}k(t)}{\mathrm{d}t} - \mathrm{e}^{-k(t)}\right]}{k_0} \tag{72}$$

and in case (2) ϵ_0 changes to

$$\epsilon = \mu - \frac{1}{\beta} \cdot \ln\frac{t\left\{\frac{\mathrm{d}k(t)}{\mathrm{d}t} - t^3 + 3tk^{-2}(t) - a[t^2+k^{-2}(t)] - ck^{-1}(t) - gt\right\}}{k_0} \tag{73}$$

where Eqs. (42) and (48) have been considered. These reveal that the topological property change of the network, such as $k(t)$ or $P(k)$ can influence the dynamical property of the network, such as energy ϵ, while the dynamical characteristics of the network are related to the structure of the topology of the network.

7 Conclusions and remarks

Using QID to perform quantum computing in a quantum network is possible. We have studied methods to use QID to realize quantum computing or quantum teleportation in the network. The photons entangled in the quantum channel are considered as flying ebit that transmit interaction between two separated nodes.

A particular Gaussian quantum channel is constructed. It is described by the quantum Fokker-Planck equation for QID, which permits photons encoded information to transmit quantum signals with certain quantum parallelism. Its formula of quantum dynamical mutual information, related to the dynamical evolution process, is discussed and studied.

All these allow one to study the evolution of the node connectivity for the quantum information network by using an external driving method such as HUHPM. For different types of driving function, different types of corresponding evolving links can be obtained. If the driving function is an exponential function, the evolving link depends on the choice of $f(\epsilon)$ and can be independent to time when $f(\epsilon) = 1$. In these cases, the driving function destroys the solution of the power-law. Finally, if the driving function is described by a polynomial, then the degree of the nodes can be expressed as a type of product of irrational polynomials and related logarithm, and the evolving links may show types of catastrophes.

The connectivity distribution can be given by the sum of the probabilities of a node with energy and connectivity. Thus, we have to sum up all the nodes with energy distribution to obtain the connectivity distribution. The connectivity distribution may be complicated and may increase with positive power of k, but the connectivity distribution with negative power of k can also be constructed by changing slightly the Elliptic umbilic driving function. The positive or negative power is decided by the architecture of the network and the conditional parameters, which also definitely influence the assortativity coefficient r. Finally, we show that the change of topological property of the network, such as the change of k_0 to $k(t)$ or $P(k_0)$ to $P(k)$, can influence the energy ϵ of the network, which reveals that the dynamical characteristics of the network can be related to the topological structure of the network.

We believe that the above result is interesting because many current technological networks (may include physical network) appear to possess the negative power of k, however the social networks actually appear to have positive power of k. It is not clear why networks behavior is like so; thus, we proposed an example of the quantum information network under specific conditions that produce the positive exponent scale. This may be helpful to clearly understand the different types of networks that have different topological characteristics with different mechanisms.

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Appendix A. The kinetic equations for QID

The Liouville equation for QID can be derived by directly starting from quantum Liouville equation,

$$\mathrm{i}\frac{\partial \rho}{\partial t} = [H, \rho] \tag{A-1}$$

such as

$$\begin{aligned}\mathrm{i}\frac{\partial \rho^2}{\partial t} &= \mathrm{i}\left(\frac{\partial \rho}{\partial t}\right)\rho + \mathrm{i}\rho\left(\frac{\partial \rho}{\partial t}\right)\\ &= [H,\rho]\,\rho + \rho\,[H,\rho]\\ &= \left[H,\rho^2\right]\end{aligned} \tag{A-2}$$

$$\begin{aligned}\mathrm{i}\frac{\partial \rho^n}{\partial t} &= \left[H,\rho^{n-1}\right]\rho + \rho^{n-1}\,[H,\rho]\\ &= [H,\rho^n]\,,\ \text{for any integer } n\end{aligned} \tag{A-3}$$

therefore the Eq. (A-4) can be obtained because QID, $I \equiv \rho \ln \rho$ can be expanded by a power series of ρ,

$$\mathrm{i}\frac{\partial \rho \ln \rho}{\partial t} = [H, \rho \ln \rho] \tag{A-4}$$

For an open system, a kinetic equation [23] can be given from a Schrödinger equation as

$$\mathrm{i}\frac{\partial \varphi_{\mathrm{proj}}}{\partial t} = \Theta \varphi_{\mathrm{proj}} \tag{A-5}$$

which results in

$$\mathrm{i}\frac{\partial \rho_{\mathrm{proj}}}{\partial t} = [\Theta, \rho_{\mathrm{proj}}] \equiv \Theta_L \rho_{\mathrm{proj}} \tag{A-6}$$

where P_n, $P_n + Q_n = 1$, is eigenprojector of the free Hamiltonian part of H, ρ_{proj} is defined by $\rho_{\mathrm{proj}} = |\varphi_{\mathrm{proj}}\rangle\langle\varphi_{\mathrm{proj}}|$, φ_{proj} is given by

$$\varphi_{\mathrm{proj}} = \Omega^{-1}\varphi \equiv \textstyle\sum_n \left(P_n + D_n C_n\right)^{-1}\left(P_n + D_n\right)\varphi \tag{A-7}$$

with

$$\begin{aligned}C_n &= D_n^{\dagger}\\ &= \frac{1}{z - Q_n H Q_n} Q_n H P_n,\ \text{for } z \in \text{ spectrum of } H\end{aligned} \tag{A-8}$$

and Θ is collision operator given by

$$\Theta = \sum_n \left(P_n H P_n + P_n H Q_n C_n P_n\right) \tag{A-9}$$

Now, using the same approach as Eqs. (A-1)–(A-3) again gives

$$\mathrm{i}\frac{\partial \rho_{\mathrm{proj}} \ln \rho_{\mathrm{proj}}}{\partial t} = [\Theta, \rho_{\mathrm{proj}} \ln \rho_{\mathrm{proj}}] = \Theta_L \rho_{\mathrm{proj}} \ln \rho_{\mathrm{proj}} \tag{A-10}$$

where $\rho_{\mathrm{proj}} \ln \rho_{\mathrm{proj}}$ is projected information density.

Under the second order approximation for coupling number, Eq. (A-10) can be simplified to the Master equation [24]. Indeed, by assuming that the projector $P = \mathrm{Tr}_R$, we have reduced the density operator for system S by $P\rho = \rho_S$, hence a general Markovian equation for the reduced information density, $I_S = \rho_S \ln \rho_S$ is also

$$\begin{aligned}&\mathrm{i}\frac{\partial}{\partial t} I_S\left(t\right)\\ &= \left(L_0 + P L_1 Q \frac{1}{Z^0 - Q L_0 Q \pm \mathrm{i}0} Q L_1 P\right) I_S\left(t\right)\end{aligned} \tag{A-11}$$

where Z^0 is an eigenvalue of L_0.

Appendix B. The dynamical mutual information of quantum channel

The mutual information between quantum channel (link) A and B can be extended from the formula of Shannon mutual information and calculated. Indeed, starting from the definition of the dynamical mutual information density [Eq. (38)] based on the classical dynamical mutual information theory proposed by Xing [19] we have:

$$I(A_0(0); B_l(\tau)) = I(A_0(0)) - I(A_0(0)|B_l(\tau)) \tag{B-1}$$

$$I(A_0(0)|B_l(\tau)) = I(Q_l(\tau)) + I(A_l(\tau)|B_l(\tau)) \tag{B-2}$$

where $A_l(\tau)$ is an input encoded state at time τ with special coordinate l (the channel length), $B_l(\tau)$ is an output ensemble encoded state at time τ with the coordinate l. $I(Q_l(\tau))$ is accumulated lost information density in the channel. When the transmission time of QID or signal through channel is long enough, the receiver gets the amount of information contained in the $B_l(\tau)$ from the output terminal $x = l$ at the time $t = \tau$ with respect to the $A_0(0)$, which is transmitted by the transmitter from the input terminal $x = 0$ at time $t = 0$. This is dynamical mutual information, which considers that the quantum channel has long length and noise in the transmission process, and is different from the usual "point" model of the channel (or zero transmitting time model) [19]. Thus, the $I(Q_l(\tau))$ is given by

$$\begin{aligned} I(Q_l(\tau)) &= I(A_0(0)) - I(A_l(\tau)) \\ &= [1 - \exp(-\mathrm{i}\mathcal{L}t)]I(A_0(0)) \end{aligned} \tag{B-3}$$

This allows the dynamical mutual QID to be obtained by

$$\begin{aligned} I(A_0(0); B_l(\tau)) &= I(A_0(0)) - I(A_0(0)|B_l(\tau)) \\ &= \exp(-\mathrm{i}\mathcal{L}t)I(A_0(0)) - I(A_l(\tau)|B_l(\tau)) \end{aligned} \tag{B-4}$$

notice here that the (generalized) evolution operator $\exp(-\mathrm{i}\mathcal{L}t)$ act on the initial QID $I(A_0(0))$ to obtain $B_l(\tau) = \exp(-\mathrm{i}\mathcal{L}t)\, I(A_0(0))$. This shows that the initial quantum signal (QID) will be transmitted from coordinate 0 to coordinate l during time τ in the quantum channel.

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