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RESEARCH ARTICLE

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Calculation of the escape probabilities of Fe XVII resonance lines for the Voigt profile

Abstract Using the Voigt profile we obtained, we calculate the escape probabilities of Fe XVII resonance lines at 15.02, 13.28, 12.12, 11.13, 11.02 and 10.12 Å for optically thick plasma, both for slab and cylindrical geometry. The oscillator strength, the number density of the absorbing atoms in the ground state, and the optical depth in the line center are discussed in this calculation. Results show that the escape probabilities for the slab geometry are larger than that for the cylindrical geometry. This calculation is useful for the study of the Fe XVII resonance lines.

Keywords escape probabilities, Fe XVII resonance lines, plasma

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1 Introduction

In atom absorption measurement, the emergent intensity for an optically thick plasma is made up of the multiple escape probabilities and the emergent intensity for an optically thin plasma. Therefore, escape probability is an important parameter in atom absorption measurement.

Over the years, a number of calculations have been carried out to derive intensities of various X-ray and extreme-ultraviolet (EUV) lines in Fe XVII for comparison with observed spectra. The predicted intensities have not agreed with solar observations, particularly for the line at 15.02 Å. Resonance scattering has been suggested as the source for much of the disagreement.

Neon-like Fe XVII is present in solar flares and active regions in a broad temperature range $(2-10)\times10^6$ K because of the filled 2p shell. Strong resonance lines in the range $15-17$ Å have been observed from the Sun. The transitions of primary interest are $2s^22p^53d(^1P_1)\ \rightarrow 2s^22p^6(^1S_0)$ at 15.02 Å and $2s^22p^6$ $3p(^1P_1) \rightarrow 2s^22p^6\ (^1S_0)$ at 13.82 Å. Other lines of particular interest are those at 12.12, 11.13, 11.02, and 10.12 Å along with subordinate lines in the extreme-ultraviolet (EUV) region.

There have been a number of theoretical studies of the expected spectrum assuming optically thin conditions [1]. The observed intensity ratios of various Fe XVII soft X-ray lines from solar active regions do not agree with the optically thin calculated intensity ratios, and it was suggested that these lines suffer resonance scattering. Resonance scattering implies that the emitted photon is absorbed and reemitted, but not necessarily in the line of sight, so there can be an apparent loss or enhancement of flux, although the total flux integrated over 4π remains unchanged.

2 The escape probabilities of slab and cylindrical geometry

The emergent intensity for an optically thick plasma is given by

$$I_{ji} = N_j A_{ji} p_f = I_{ji}(\text{optically thin}) p_f \tag{1}$$

where N_j is the upper level population, andA_{ji} is the transition probability, and p_f is the escape probability.

The monodirectional single-flight or free-flight photon

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escape probability assuming a constant source function, is given by [2]

$$p_f = (\sqrt{\pi}\tau_0)^{-1} \int_{-\infty}^{\infty} \{1 - \exp[-\tau_0(-x^2)]\}\mathrm{d}x \quad (2)$$

where the dimensionless frequency variable $x = (\nu - \nu_0)/\Delta\nu_D$, and τ_0 is the optical depth at the line center. Here the Doppler profile $\Delta\nu_D$ is obtained when the turbulent (nonthermal) velocity V_{NT} is equal to zero in the general expression for the line profile,

$$\Delta\nu_D = \frac{10^8}{\lambda}\sqrt{\frac{1.663 \times 10^8 T}{M} + V_{NT}^2} \quad (3)$$

The escape probabilities, expressed as a logistic function, is given by [3]

$$p_f = \frac{1}{1 + \exp[b(\ln\tau_0 - c)]} \quad (4)$$

where b =2.410 527 and c =0.395 044 5 for the slab geometry and b =2.321 213 6 and c =0.223 355 45 for the cylindrical geometry. The present analysis is strictly valid for lines that are not self-reversed and is widely used.

The optical depth at the line center is an important parameter in plasma, which can be used to calculate many physical quantities, such as the escape probability, the resonance escape factor and the opacity. For an absorbing volume of width L, the optical depth at the line center is given by [4]

$$\tau_0 = \sigma NLP(0) \quad (5)$$

where $\sigma$is the Ladenburg cross-section given by $\sigma = (\pi e^2/mc) \cdot f_{ij}$, where $f_{ij}$is the oscillator strength of the resonance transition, and it is given by [5]

$$f_{ij} = \frac{\varepsilon_0 mc}{2\pi e^2}\frac{g_2}{g_1}\frac{\lambda_0^2}{\tau_{21}} \quad (6)$$

where ε_0 is permittivity of free space, and e is charge of electron, and m is mass of electron, and c is speed of light. Also, g is statistical weight, and τ_{21} is lifetime of excited state, and λ_0 is the resonance wavelength of the atom transition. In Eq. (5), $P(0)$ is the normalized line profile at $\Delta\nu = 0$, and N is the number density of the absorbing atoms in the ground state. The relation between N and the maximum absorption coefficient k_m for Voigt distribution is [6]

$$N = k_m \frac{(\Delta\nu_N + \Delta\nu_L)\Delta\nu_D}{4\lambda^2 f_{ij}}\sqrt{\frac{\pi}{\ln 2}}\frac{mc^3}{e^2} \quad (7)$$

where $\Delta\nu_N$, $\Delta\nu_L$ and $\Delta\nu_D$ are the natural, Lorentzian and Doppler half-widths, respectively, λ is the wave length of the central resonance line.

The Voigt function is defined as [7]:

$$f(\nu - \nu_0) = \frac{f' y}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-t^2)}{y^2 + (x-t)^2}\mathrm{d}t \quad (8)$$

where, $y = \dfrac{\Delta\nu_L}{\Delta\nu_D}(\ln 2)^{1/2}$, $x = \dfrac{\nu - \nu_0}{\Delta\nu_D}(\ln 2)^{1/2}$, $f' = \dfrac{1}{\Delta\nu_D}(\ln 2)^{1/2}$.

Through calculation, we have obtained an accurate Voigt profile as follows [8, 9]:

$$f(\nu - \nu_0) = \sqrt{\pi} f'[\exp(4\pi y^2 - z^2)\cos(4\pi yz) + \exp(4\pi y^2 - x^2)\cos(4\pi yx)] \quad (9)$$

where $z = \dfrac{\nu + \nu_0}{\Delta\nu_D}(\ln 2)^{1/2}$ is a new parameter we defined. Using this expression, we can finish the calculation successfully.

The half-widths can be calculated as follows [10].

The Doppler half-widths of an emission line is given by

$$\Delta\nu_D = \nu_0(7.1623 \times 10^{-7})(T/M)^{1/2} \quad (10)$$

where T is the absolute temperature of the gas, M is the mass of Fe atom.

The natural half-widths is given by

$$\Delta\nu_N = \frac{1}{2\pi\tau_{21}c} \quad (11)$$

where τ_{21} is the lifetime of excited state, c is speed of light.

The Lorentzian half-widths is given by

$$\Delta\nu_L = 2r_{\mathrm{air}}(296/T)^n P \quad (12)$$

where n is the temperature coefficient, and in general gas, $n = 0.75$, r_{air} is the widen coefficient in atmosphere, $r_{\mathrm{air}} = 3.34 \times 10^{-2}$cm/atm, and P is the pressure.

Table 1 The transition characteristics of six Fe XVII resonance lines.

λ/Å	15.02	13.82	12.12	11.13	11.02	10.12
g_i	1	1	1	1	1	1
g_j	3	3	3	3	3	3
τ_{21}/s	3.44×10^{-14}	2.99×10^{-13}	8.93×10^{-14}	1.61×10^{-13}	8.55×10^{-13}	9.09×10^{-13}

Table 2 The oscillator strengths, the Ladenburg cross-section and the number densities of the absorbing atoms in the ground state of the six Fe XVII resonance lines.

λ/Å	Oscillator strength	Ladenburg cross-section/m^2	Number densities/m^{-3}
15.02	2.9585	8.7060×10^{-16}	1.5515×10^{38}
13.82	0.2882	8.4809×10^{-17}	2.3524×10^{38}
12.12	0.7421	2.1838×10^{-16}	4.5348×10^{38}
11.13	0.3471	1.0214×10^{-16}	6.9442×10^{38}
11.02	0.0641	1.8863×10^{-17}	7.2951×10^{38}
10.12	0.0508	1.4949×10^{-17}	1.1179×10^{39}

3 The Fe XVII resonance lines

The Fe XVII level structure is distinguished in having all the excited levels much higher than the single ground level implying that even at high electron densities, most of $^1S_0$, the ion population is in the ground level. The six Fe XVII resonance lines have transition characteristics as stated in Table 1 [11].

4 Results and discussion

As mentioned in the abstract, we can assume the temperature to be 6×10^6K. For the atomic absorption spectrometer used in Ref. [3], the pressure is 10^{-4}Pa. Through calculation, the oscillator strengths, the Ladenburg cross-section and the number densities of the absorbing atoms in the ground state of the six Fe XVII resonance lines are shown in Table 2.

In our calculation, a single beam atomic absorption spectrometer was used, which is fitted with a 10-cm long rectangular burner that gives a wedge shaped flame with 10 cm length. The optical depth at the line center is shown in Fig. 1 when the width of the absorbing volume varies from 0 to 10 cm as in Eq. (5).

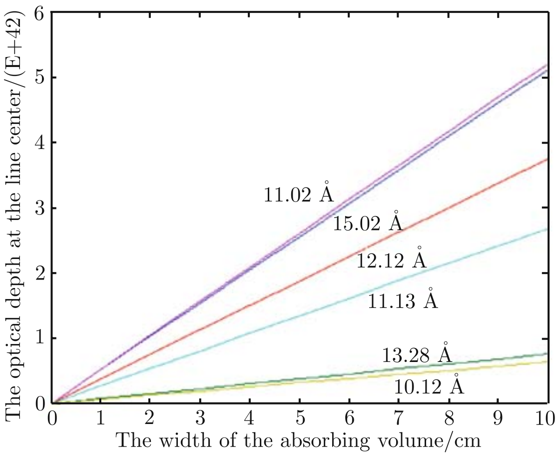

Fig. 1 The optical depth at the line center.

From Fig. 1 we can see that for the six Fe XVII resonance lines, when the width of the absorbing volume varies from 0 to 10 cm, the optical depth at the line center varies from $(0-5)\times10^{42}$, and for each resonance line the optical depth at the line center will increase when the width of the absorbing column increases. According to Eq. (2), the escape probabilities are shown in Fig. 2.

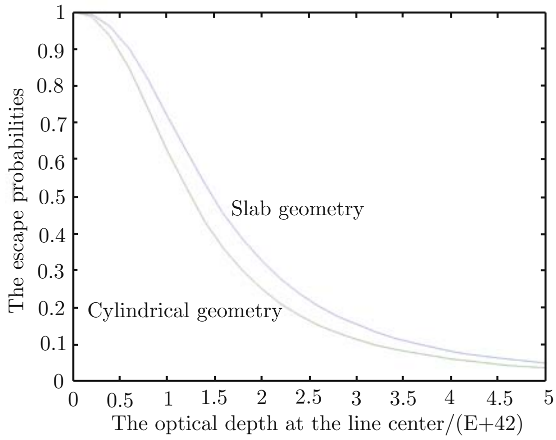

Fig. 2 The escape probabilities.

From Fig. 2, we can see that the escape probabilities for the slab geometry are larger than that for the cylindrical geometry. In our measurement, the emergent intensity for the slab geometry is larger than that for the cylindrical geometry, so this calculation is in agreement with the result that has been reported, in which the effects of opacity are somewhat less pronounced for cylindrical geometry than for slab geometry [12].

5 Conclusion

From the above discussion, for the six Fe XVII resonance lines, the following conclusions can be drawn:

(1) The Ladenburg cross-sections have a 10^{-16} order of magnitude;

(2) The number densities of the absorbing atoms in the ground state have a 10^{38} order of magnitude;

(3) The optical depth at the line center has a 10^{42} order of magnitude;

(4) The escape probabilities for the slab geometry are

larger than that for the cylindrical geometry, which is in agreement with the result that has been reported.

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References

1. H. R. Rugge and D. L. McKenzie, Astrophys. J., 1985, 297: 338
2. A. K. Bhatia and J. L. R. Saba, Astrophys. J., 2001, 563: 434
3. A. A. M. Habib and Z. El-Gohary, J. Quant. Spectrosc. Radiat. Transfer., 2002, 72: 341
4. H. W. Drawin, F. Emard, and F. Beiter, Plasma Physik, 1973,13: 143
5. F. E. Irons, J. Quant. Spectrosc. Radiat. Transfer., 1979, 22: 21
6. N. Barakat, M. Medhat, and Z. El-Gohary, J. Quant. Spectrosc. Radiat. Transfer., 1989, 42: 2
7. G. D. Rostn and F. S. Obaid, J. Quant. Spectrosc. Radiat. Transfer., 2005, 94:255
8. J. He and C. M. Zhang, J. Opt. A: Pure and Appl. Opt., 2005, 7: 613
9. J. He and Q. G. Zhang, J. Opt. A: Pure and Appl. Opt., 2007, 9: 565
10. J. He and Q. G. Zhang, Phys. Lett. A, 2006, 359: 256
11. D. A. Verner, E. M. Verner, and G. J. Ferland, Atomic Data and Nuclear Data Tables, 1996, 64: 1
12. A. K. Bhatia and S. O. Kastner, Astrophys. J., 1999, 516: 482