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RESEARCH ARTICLE

**Chao-guang HUANG, Yu TIAN, Xiao-ning WU, Han-ying GUO**

# On torsion-free vacuum solutions of the model of de Sitter gauge theory of gravity



**Abstract** It is shown that all vacuum solutions of Einstein field equation with a positive cosmological constant are the solutions of a model of dS gauge theory of gravity. Therefore, the model is expected to pass the observational tests on the scale of solar systems and explain the indirect evidence of gravitational wave from the binary pulsars PSR1913+16.

**Keywords** torsion-free vacuum solutions, model of de Sitter gauge theory, gravity

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Astronomical observations show that our universe is probably an asymptotically de Sitter(dS) one [1−6]. This raises interest on dS gauge theories of gravity. A model of the dS gauge theory of gravity was first proposed in the 1970s [7−14]. The model can be stimulated from dS invariant special relativity [15−27] and the principle of localization [28−31], just like the Poincaré gauge theory of gravity which may be stimulated from the Einstein special relativity and the localization of Poincaré symmetry [32, 33]. The principle of localization is that the full symmetry of the special relativity as well as the laws of dynamics are both localized. The gravitational action of the model takes the Yang-Mills form of [7−14, 28−31]

$$S_{\rm GYM} = \frac{1}{4g^2}\int_{\mathcal{M}} {\rm d}^4 x e {\rm Tr}_{{\rm d}S}(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}) \tag{1}$$

where $e = \det(e^a_\mu)$ is the determinant of the tetrad $e^a_\mu$, $g$ is a dimensionless coupling constant introduced as usual in the gauge theory to describe the self-interaction of the gauge field,

$$\mathcal{F}_{\mu\nu} = (\mathcal{F}^{AB}_{\mu\nu}) = \begin{pmatrix} F^{ab}_{\mu\nu} + R^{-2}e^{ab}_{\mu\nu} & R^{-1}T^a_{\mu\nu} \\ -R^{-1}T^b{}_{\mu\nu} & 0 \end{pmatrix} \tag{2}$$

is the curvature of dS connection†

$$\mathcal{B}_\mu = (\mathcal{B}^{AB}{}_\mu) = \begin{pmatrix} B^{ab}{}_\mu & R^{-1}e^a_\mu \\ -R^{-1}e^b_\mu & 0 \end{pmatrix} \in so(1,4) \tag{3}$$

and ${\rm Tr_{dS}}$ is the trace for the $so(1,4)$ indices $A, B$. In Eq. (2), $F^{ab}{}_{\mu\nu}$ and $T^a{}_{\mu\nu}$ are the curvature and torsion tensors of the Lorentz connection $B^a{}_{b\mu} \in so(1,3)$, respectively, $R$ is the dS radius, and $e_{ab}{}^{\mu\nu} = e^\mu_a e^\nu_b - e^\nu_a e^\mu_b$. In terms of $F^{ab}{}_{\mu\nu}$ and $T^a{}_{\mu\nu}$, the gravitational action can be rewritten as:

$$S_{\rm GYM} = -\int_{\mathcal{M}} {\rm d}^4 x e\Big[\frac{1}{4g^2}F^{ab}{}_{\mu\nu}F_{ab}{}^{\mu\nu} -\chi(F-2\Lambda) - \frac{\chi}{2}T^a{}_{\mu\nu}T_a{}^{\mu\nu}\Big] \tag{4}$$

where $F = \frac{1}{2}F^{ab}{}_{\mu\nu}e_{ab}{}^{\mu\nu}$ the scalar curvature, the same as

† The same dS-connection with different dynamics has also been explored in Ref. [34−42].

Chao-guang HUANG[1, 5], Yu TIAN[2, 5] (✉), Xiao-ning WU[3], Han-ying GUO[4,5]

[1] Institute of Theoretical Physics, The Chinese Academy of Sciences, Beijing 100049, China
[2] Department of Physics, Beijing Institute of Technology, Beijing 100081, China
[3] Institute of Mathematics, Academy of Mathematics and System Science, The Chinese Academy of Sciences, Beijing 100080, China
[4] Institute of Theoretical Physics, The Chinese Academy of Sciences, Beijing 100080, China
[5] Kavli Institute for Theoretical Physics China at the Chinese Academy of Sciences, Beijing 100080, China
E-mail: huangcg@ihep.ac.cn; ytian@bit.edu.cn

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the action in the Einstein-Cartan theory, $\chi = 1/(16\pi G)$ is a dimensional coupling constant, $\Lambda = 3/R^2 = 3\chi g^2$ is the cosmological constant.

The gravitational field equations, obtained by the variation of the total action

$$S_{\rm T} = S_{\rm GYM} + S_{\rm M} \tag{5}$$

with respect to $e^a{}_\mu, B^{ab}{}_\mu$, are

$$T_a{}^{\mu\nu}{}_{||\nu} - F^\mu{}_a + \frac{1}{2}Fe^\mu_a - \Lambda e^\mu_a = 8\pi G(T_{{\rm M}a}{}^\mu + T_{{\rm G}a}{}^\mu) \tag{6}$$

$$F_{ab}{}^{\mu\nu}{}_{||\nu} = R^{-2}(16\pi G S^\mu_{{\rm M}ab} + S^\mu_{{\rm G}ab}) \tag{7}$$

Here, $S_{\rm M}$ is the action of the matter source with minimum coupling, "$||$" represents the covariant derivative using both Christoffel symbol $\{^\mu_{\nu\kappa}\}$ and connection $B^a{}_{b\mu}$, $F_a{}^\mu = -F_{ab}{}^{\mu\nu}e^b_\nu$,

$$T_{{\rm M}a}{}^\mu := -\frac{1}{e}\frac{\delta S_{\rm M}}{\delta e^a_\mu} \tag{8}$$

$$T_{{\rm G}a}{}^\mu := g^{-2}T_{{\rm F}a}{}^\mu + 2\chi T_{{\rm T}a}{}^\mu \tag{9}$$

are the tetrad form of the stress-energy tensor for matter and gravity, respectively, where

$$\begin{aligned} T_{{\rm F}a}{}^\mu &:= -\frac{1}{4e}\frac{\delta}{\delta e^a_\mu}\int {\rm d}^4x e {\rm Tr}(F_{\nu\kappa}F^{\nu\kappa}) \\ &= e^\kappa_a {\rm Tr}(F^{\mu\lambda}F_{\kappa\lambda}) - \frac{1}{4}e^\mu_a{\rm Tr}(F^{\lambda\sigma}F_{\lambda\sigma}) \end{aligned} \tag{10}$$

is the tetrad form of the stress-energy tensor for curvature and

$$\begin{aligned} T_{{\rm T}a}{}^\mu &:= -\frac{1}{4e}\frac{\delta}{\delta e^a_\mu}\int {\rm d}^4x e T^b{}_{\nu\kappa}T_b{}^{\nu\kappa} - \frac{1}{8\pi G}T^{\mu\nu}_a{}_{||\nu} \\ &= e^\kappa_a T_b{}^{\mu\lambda}T^b{}_{\kappa\lambda} - \frac{1}{4}e^\mu_a T_b{}^{\lambda\sigma}T^b{}_{\lambda\sigma} \end{aligned} \tag{11}$$

the tetrad form of the stress-energy tensor for torsion, and

$$S_{{\rm M}ab}{}^\mu = \frac{1}{2\sqrt{-g}}\frac{\delta S_{\rm M}}{\delta B^{ab}{}_\mu} \tag{12}$$

and $S_{{\rm G}ab}{}^\mu$ are spin currents for matter and gravity, respectively. Especially, the spin current for gravity can be divided into two parts:

$$S_{{\rm G}ab}{}^\mu = S_{{\rm F}ab}{}^\mu + 2S_{{\rm T}ab}{}^\mu \tag{13}$$

where

$$\begin{aligned} S_{{\rm F}ab}{}^\mu &:= \frac{1}{2\sqrt{-g}}\frac{\delta}{\delta B^{ab}{}_\mu}\int {\rm d}^4x\sqrt{-g}F = -e_{ab}{}^{\mu\nu}{}_{||\nu} \\ &= Y^\mu{}_{\lambda\nu}e_{ab}{}^{\lambda\nu} + Y^\nu{}_{\lambda\nu}e_{ab}{}^{\mu\lambda} \end{aligned} \tag{14}$$

$$\begin{aligned} S_{{\rm T}ab}{}^\mu &:= \frac{1}{2\sqrt{-g}}\frac{\delta}{\delta B^{ab}{}_\mu}\frac{1}{4}\int {\rm d}^4x\sqrt{-g}T^c{}_{\nu\lambda}T_c{}^{\nu\lambda} \\ &= T_{[a}{}^{\mu\lambda}e_{b]\lambda} \end{aligned} \tag{15}$$

are the spin current for curvature $F_{ab}{}^{\mu\nu}$ and torsion $T_a{}^{\mu\nu}$, respectively.

In Ref. [44], it is shown that all vacuum solutions of the Einstein field equation without cosmological constant are the solutions of Eq. (6) and Eq. (7) for the case of sourceless, torsion-free, and vanishing cosmological constant. However, a positive cosmological constant is vitally important for the dS gauge theories of gravity. Without a positive cosmological constant, the gravity should be a Poincaré or AdS one. Therefore, in order to see whether the model of dS gauge theory of gravity can pass the observational tests on the scale of a solar system, it should be important to explore if the vacuum solutions of the Einstein field equation with a positive cosmological constant do satisfy the equations of the model.

The purpose of the present Note is to show that it is just the case. That is, all vacuum solutions of the Einstein field equation with a positive cosmological constant are the solutions of the torsion-free vacuum equations of the model of dS gauge theory of gravity.

For the sourceless case, the torsion-free gravitational field equations of the model reduce to

$$\mathcal{R}^\mu{}_a - \frac{1}{2}\mathcal{R}e^\mu_a + \Lambda e^\mu_a = -8\pi G(T_{{\rm M}a}{}^\mu + T_{{\rm R}a}{}^\mu) \tag{16}$$

$$\mathcal{R}_{ab}{}^{\mu\nu}{}_{;\nu} = 0 \tag{17}$$

where $T_{{\rm R}a}{}^\mu = e^\nu_a T_{\rm R}{}^\mu{}_\nu$ the tetrad form of the stress-energy tensor of Riemann curvature $\mathcal{R}_{ab}{}^{\mu\nu}$, and a semicolon; is the covariant derivative using both the Christoffel and Ricci rotation coefficients. Eq. (16) is the Einstein-like equation, while Eq. (17) is the Yang equation [43]. It can be shown [44] that

$$\begin{aligned} T_{{\rm R}\mu}{}^\nu &= \mathcal{R}_{ab\mu\lambda}\mathcal{R}^{ab\nu\lambda} - \frac{1}{4}\delta^\nu_\mu(\mathcal{R}_{ab\lambda\kappa}\mathcal{R}^{ab\lambda\kappa}) \\ &= \frac{1}{2}(\mathcal{R}_{\kappa\sigma\mu\lambda}\mathcal{R}^{\kappa\sigma\nu\lambda} + \mathcal{R}^*{}_{\kappa\sigma\mu\lambda}\mathcal{R}^{*\kappa\sigma\nu\lambda}) \\ &= 2C_{\lambda\mu}{}^{\kappa\nu}\mathcal{R}^\lambda_\kappa + \frac{\mathcal{R}}{3}(\mathcal{R}^\nu_\mu - \frac{1}{4}\mathcal{R}\delta^\nu_\mu) \end{aligned} \tag{18}$$

where $\mathcal{R}_{\kappa\sigma\mu\lambda}$ is the Riemann curvature tensor, $\mathcal{R}^*_{\kappa\sigma\mu\lambda} = \frac{1}{2}\mathcal{R}_{\kappa\sigma\tau\rho}\epsilon^{\tau\rho}{}_{\mu\lambda}$ is the right dual of the Riemann curvature tensor, $C_{\lambda\mu\kappa\nu}$ is the Weyl tensor. In the last step in Eq. (18), the Géhéniau-Debever decomposition for the Riemann curvature,

$$\mathcal{R}_{\mu\nu\kappa\lambda} = C_{\mu\nu\kappa\lambda} + E_{\mu\nu\kappa\lambda} + G_{\mu\nu\kappa\lambda} \tag{19}$$

is used [45], where

$$E_{\mu\nu\kappa\lambda}=\frac{1}{2}(g_{\mu\kappa}S_{\nu\lambda}+g_{\nu\lambda}S_{\mu\kappa}-g_{\mu\lambda}S_{\nu\kappa}-g_{\nu\kappa}S_{\mu\lambda}) \quad (20)$$

$$G_{\mu\nu\kappa\lambda}=\frac{\mathcal{R}}{12}(g_{\mu\kappa}g_{\nu\lambda}-g_{\mu\lambda}g_{\nu\kappa}) \quad (21)$$

$$S_{\mu\nu}=\mathcal{R}_{\mu\nu}-\frac{1}{4}\mathcal{R}g_{\mu\nu} \quad (22)$$

On the other hand, the vacuum Einstein field equation with a (positive) cosmological constant reads

$$\mathcal{R}^{\mu}{}_{\nu}-\frac{1}{2}\mathcal{R}\delta^{\mu}_{\nu}+\Lambda\delta^{\mu}_{\nu}=0 \quad (23)$$

It results in

$$\mathcal{R}=4\Lambda, \quad \mathcal{R}^{\mu}{}_{\nu}=\Lambda\delta^{\mu}_{\nu} \quad (24)$$

and thus

$$S_{\mu\nu}=0 \quad (25)$$

Since the Weyl tensor is totally traceless, the stress-energy tenor for Riemann curvature vanishes, i.e.,

$$T_{\mathrm{R}\mu}{}^{\nu}=0 \quad (26)$$

Therefore, all vacuum solutions of Einstein field equation with a cosmological constant are solutions of Eq. (16). In addition, the Bianchi identity

$$\mathcal{R}^{\mu\nu}{}_{\lambda\sigma;\kappa}+\mathcal{R}^{\mu\nu}{}_{\kappa\lambda;\sigma}+\mathcal{R}^{\mu\nu}{}_{\sigma\kappa;\lambda}=0 \quad (27)$$

leads to

$$0=\mathcal{R}^{\mu\nu}{}_{\lambda\sigma;\nu}-\mathcal{R}^{\mu}{}_{\lambda;\sigma}+\mathcal{R}^{\mu}{}_{\sigma;\lambda}=\mathcal{R}_{\lambda\sigma}{}^{\mu\nu}{}_{;\nu}. \quad (28)$$

Namely, Yang equation (17) is also satisfied. [The last step of Eq. (28) is valid because of Eq. (24).]

Therefore, we come to the conclusion that all vacuum solutions of the Einstein field equation with a positive cosmological constant are the torsion-free vacuum solutions of the model of dS gauge theory of gravity. In particular, the dS, Schwarzschild-dS, and Kerr-de Sitter metrics satisfy the Eqs. (6) and (7). Note that the Birkhoff theorem has been proved for the gravitational theory (4) without a cosmological constant [46]. So, the model can pass the observational tests on the scale of a solar system. In addition, the model has the same metric waves as general relativity and thus can explain the indirect evidence of the existence of gravitational wave from the observation data on the binary pulsar PSR1913+16.

One might think that the above results are trivial because the Yang equation does not appear at all if the torsion-free condition is assumed in the action, in which case the tetrad and connection are not independent. However, the torsion-free manifold is just the specific situation of the the model. There is no reason to set the torsion to be zero before the variation.

In fact, it can be shown that all solutions of the vacuum Einstein field equation with a positive cosmological constant are also the vacuum, torsion-free solutions of the field equations when the terms

$$F_a{}^{\mu}F^a{}_{\mu},\ e^a_{\nu}e^b_{\mu}F_a{}^{\mu}F_b{}^{\nu},\ e^{ab}{}_{\lambda\sigma}e^{cd}{}_{\mu\nu}F_{ab}{}^{\mu\nu}F_{cd}{}^{\lambda\sigma},$$
$$e^b_{\sigma}e^c_{\mu}F_{ab}{}^{\mu}{}_{\nu}F_{ac}{}^{\nu\sigma},\ e^{\lambda}_a e^{\sigma}_b T^a{}_{\mu\lambda}T^{b\mu}{}_{\sigma},\ e^{\sigma}_a e^{\mu}_b T^a{}_{\mu\lambda}T^{b\lambda}{}_{\sigma}$$

are added in the gravitational Lagrangian. Obviously, the last two terms have no contribution to the vacuum, torsion-free field equations, while the middle two terms contribute the same as the term $F_{ab}{}^{\mu\nu}F^{ab}{}_{\mu\nu}$ does thus only alter the unimportant coefficients. The first two terms add the term $(R^{\mu}_{[a}e^{\nu}_{b]})_{;\nu}$ in Yang equation and the stress-energy tensor $R_{\mu\lambda}R^{\nu\lambda}-\frac{1}{4}\delta^{\nu}_{\mu}R_{\sigma\lambda}R^{\sigma\lambda}$ in Einstein equation. Both of them vanish for the solutions of the vacuum Einstein equation with a positive cosmological constant.

Obviously, the conclusion is still valid if the integral of the second Chern form of the dS connection over the manifold is added in the action. Finally, the similar discussions can be applied to the AdS case as well.

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