REVIEW ARTICLE

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Quark mass and the masses of Goldstone bosons

Abstract   Based on the Dyson-Schwinger Equations (DSEs) of QCD in the "rainbow" approximation, the fully dressed quark propagator $S_f(p)$ is investigated, and then an algebraic parametrization form of the propagator is obtained as a solution of the equations. The dressed quark amplitudes A_f and B_f which built up the fully dressed quark propagator, and the dynamical running masses M_f, which is defined by A_f and B_f for light quarks u, d and s, are calculated, respectively. Using the predicted current masses m_f, quark local vacuum condensates, and our predicted value of pion decay constant, the masses of Goldstone bosons K, π and η and their in-medium values are also evaluated. Our predictions fit to data and to many other different calculations quite well. The numerical results show that the mass of quark is dependent of its momentum p^2. The fully dressed quark amplitudes A_f and B_f have correct behaviors and can be used for many purposes in our future researches on non-perturbative QCD.

Keywords   DSEs, quark propagator, quark mass, Goldstone boson mass

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1   Introduction

Calculations of quark masses are one of the most important subject in the investigation of non-perturbative QCD. An ac-

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curate determination of these parameters is in fact extremely important, for both phenomenological and theoretical applications. The charm and bottom masses, for instance, enter through the heavy quark expansion, the theoretical expressions of several cross sections and decay rates. From a theoretical point of view, an accurate determination of quark masses may give insights on the physics of flavor, revealing relations between masses and mixing angles, or specific textures in the quark mass matrix, which may originate from still uncovered flavor symmetries.

The values of quark masses cannot be directly measured in experiments because quarks are confined inside hadrons. On the other hand, quark masses are fundamental (free) parameters of the QCD theory and, as such, they cannot be directly computed on the basis of purely theoretical considerations. The values of quark masses can only be determined by comparing the theoretical evaluation of a given physical quantity, which depends on quark masses, with the corresponding experimental value. Typically, in lattice QCD the pion, kaon and ϕ meson masses are used to compute the values of light quark masses, whereas the b-quark mass is determined by computing the mass of the B or the Υ meson. Different choices are all equivalent in principle, and the differences in the results, obtained by using different hadron masses as input parameters, give an estimate of the systematic error. As all other parameters of the Standard Model Lagrangian, quark masses can be defined as effective couplings, which are both renormalization scheme and scale dependent. A scheme commonly adopted for quark masses is the $\overline{MS}$ scheme, with a renormalization scale chosen in the short − distance region in order to make this quantity accessible to perturbative calculations. It is a common practice to quote the values of light quark masses at the renormalization scale $\mu = 2.0$ GeV, where the heavy quark masses are usually given at the scale of the quark mass itself, e.g. $\bar{m}_b$.

In this work we introduce new phenomenological functions for fully dressed quark propagator needed to characterize the space-time structure of the non-local quark condensates. Both the forms of these functions and the parameters are found by fits to experiment as well as considerations of analyticity, e.g. in a study of parton distribution functions, the space-time scale of a non-local quark condensates was determined by a fit of a monopole form in space-time to experimental data.

In principle, the quark propagator can be obtained from the DSEs. However, solving DSEs requires the knowledge of the dressed quark-gluon vertex Γ^ν and dressed gluon propagator $G_{\mu\nu}$ which are still unknown, so that in effect one must once again consider modeling. The advantage of the DSEs approach is that one can use its self-consistency to search for the forms of the propagator, which is the characteristics used in this paper.

In the present work, we use the DSEs approach in the "rainbow " approximation ($\Gamma^\nu = \gamma^\nu$) to investigate the dynamical running masses of quarks. The quark mass is defined from fully dressed quark propagator. The organization of this paper is as follows. In Section 2, the DSEs is briefly introduced and its solution is presented. In Section 3, the dynamical running mass of quark (light quarks), $M_f(p^2)$, is defined by the solution and p^2— dependence of the mass functions $M_f(p^2)$ are presented. A transition from the current quark mass to the mass of the constituent quark is found out dynamically. At the same time, the masses of Goldstone bosons, π, K and η and their in-medium counterparts are also reproduced by use of our predicted dynamical running quark masses, vacuum condensates with predicted pion decay constant f_π in Section 4. Finally, Section 5 is reserved for summary and concluding remarks stemmed from this study.

2   Dyson-Schwinger Equation and its solution

2.1   Dyson-Schwinger Equation in QCD

The Dyson-Schwinger Equations (DSEs) [1, 2] are a tower of coupling integral equations that relate the Green's functions of QCD to one another, which are non-perturbative in nature. solving these equations provides the solution of QCD. This tower of the equations includes the Bethe-Salpeter (BS) equation [3−7], the solution of which is the quark-antiquark bound state amplitude. The mathematical expression of the DSEs is given by

$$i[S_f(p)]^{-1} = i[S_f^0(p)])^{-1}$$

$$+ \frac{4}{3}g^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \gamma^\mu S_f(k)\Gamma^\nu(k,p)G_{\mu\nu}(q) \qquad (1)$$

In Eq.(1), g is the strong coupling constant of QCD which relates to QCD running coupling constant $\alpha_s(Q)$ by the equation of $\alpha_s(Q) = \dfrac{g^2}{4\pi}$. $G_{\mu\nu}(q)$ denotes fully dressed gluon propagator and has the form of

$$G_{\mu\nu}(q) = \frac{1}{q^2}\left[\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{1}{1-\Pi(q)} + \xi\frac{q_\mu q_\nu}{q^2}\right] \quad (2)$$

with ξ being a gauge parameter ($\xi = 0$ is the Landau gauge and $\xi = 1$ is for Feynman gauge). $\delta_{\mu\nu} = \mathrm{diag}(1,1,1,1)$ is the Euclidean metric. $\Pi(q)$ can be expressed by the following identity [8−11]:

$$\Pi_{\mu\nu}(q) = i\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)q^2\Pi(q)$$

$$= -g^2 \int \mathrm{d}^4 x\, \mathrm{e}^{iqx}\langle 0|T[J^\mu(x)J^\nu(0)]|0\rangle_{\mathrm{pp}} \qquad (3)$$

where the subscript "pp" indicates the proper part, i.e., only one vector-meson-irreducible graphs contribute to $\Pi_{\mu\nu}(q)$.

$\Gamma^\nu(k,p)$ in Eq.(1) is Bethe-Salpeter amplitude describing the dressed gluon-quark coupling vertex. Gauge covariance requires that the BS amplitude, $\Gamma^\nu(k,p)$, must satisfy the Ward-Takahashi identity [12−15]:

$$(k-p)_\mu i\Gamma_\mu(k,p) = S_f^{-1}(k) - S_f^{-1}(p) \qquad (4)$$

In the quantum field theory, the BS amplitude for a two-body quark-antiquark bound state is obtained as a solution of the homogeneous BS equation:

$$\Gamma_M^{rs}(k,p) = -\int \frac{\mathrm{d}^4 k'}{(2\pi)^4} K_{ab,cd}^{rs,tu}(k',k,p)$$

$$\cdot \left[S_f\left(k' - \frac{1}{2}p\right)\Gamma_M\left(k' + \frac{1}{2}p\right)\right]_{cd}^{tu} \qquad (5)$$

where Γ_M is the proper meson-quark vertex and $K_{ab,cd}^{rs,tu}$ is the kernel. A commonly used approximation for the kernel, $K_{ab,cd}^{rs,tu}$, is the generalized ladder approximation, in which

$$K_{ab,cd}^{rs,tu}(k',k,p)$$

$$= g^2 G^{\mu\nu}(k'-p)\left(\frac{\lambda^a}{2}\right)_{ac}\left(\frac{\lambda^a}{2}\right)_{db}(\gamma^\mu)_{rt}(\gamma^\nu)_{us} \qquad (6)$$

Solving Eq.(5) for Γ_M yields so-called dressed quark core of the bound state.

As it is impossible to solve the complete set of DSEs, one has to truncate this infinite tower in a physically acceptable way to reduce them to something that is soluble. To begin with, we make a further simplification by replacing the full

vertex $\Gamma^\nu(k,p)$ in Eq.(1) in terms of its bare one γ^ν and the full gluon propagator $G_{\mu\nu}(q)$ by its bare counterpart $G^0_{\mu\nu}(q)$ in which $\Pi = 0$ and then $G_{\mu\nu}$ has a form:

$$G^0_{\mu\nu}(q) = \frac{1}{q^2}\left[\delta_{\mu\nu} + (\xi - 1)\frac{q_\mu q_\nu}{q^2}\right] \tag{7}$$

which becomes $G^0_{\mu\nu}(q) = \delta_{\mu\nu}/q^2$ in Feynman gauge $\xi = 1$. Thus, Eq.(1) then becomes to

$$\mathrm{i}(S_f(p))^{-1} = \mathrm{i}[S^0_f(p)]^{-1}$$
$$+\frac{4}{3}g^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\gamma^\mu S_f(k)\gamma^\nu G^0_{\mu\nu}(q) \tag{8}$$

This procedure is called as "Rainbow" approximation of DSEs and used widely.

2.2 Solution of Dyson-Schwinger Equations

The DSEs provide a valuable non-perturbative tool for studying field theories. Phenomena such as confinement and dynamical chiral symmetry breaking, which cannot be explained by perturbative treatments, can be understood in terms of the behavior of particle propagators obtained by solving nonlinear integral equations of DSEs. However, the full set of DSEs for any particular field theory contains an infinite tower of equations and is thus intractable. A common approach for dealing with gauge field theories is to approximate the fermion— gauge— boson vertex by a suitable ansatz depending only on the dressed single particle propagator. The problem is then reduced to that of solving a finite set of coupled equations for the fermion and gauge boson propagators.

Ideally, of course , one would solve the DSEs for vertex itself, however, this equation involves the kernel of the fermion — antifermion Bethe-Salpeter equation which cannot be expressed in a closed form, i.e., the Skeleton expansion [16, 17] of this kernel involves infinitely number of terms, an approximation or truncation of the system must therefore be made at a very early stage. An effective way to do this is to make an ansatz for the vertex satisfying certain criteria with which the solution of the vertex equation must itself satisfy. At the present time, this later approach is the most effective manner in which to proceed since it allows for a study of the relative importance of particular vertex characteristics while avoiding the technical difficulties associated with solving the vertex equation directly.

The DSEs for fully dressed quark propagator in Eq. (1) has been intensively studied. The numerical solution obtained using a gluon propagator with an integrable singularity in the infrared region and a quark-gluon vertex of the Ref. [18] can be well represented by the following algebraic parametrization:

$$S_f(p) = -\mathrm{i}\gamma p \cdot \sigma^f_V(p^2) + \sigma^f_s(p^2) \tag{9}$$

Studies of two-point quark DSEs, employing a model gluon propagator $G_{\mu\nu}(q)$, suggest that the qualitative features of the confined quark propagator can be well described by the algebraic form:

$$\sigma^f_s = \frac{\bar{\sigma}^f_s}{\Lambda} \tag{10}$$

$$\sigma^f_V = \frac{\bar{\sigma}^f_V}{\Lambda^2} \tag{11}$$

where $\bar{\sigma}^f_{s,V}$ are given, respectively, by

$$\bar{\sigma}^f_s(x) = \frac{1-\exp(-b^f_1 x)}{b^f_1 x}\frac{1-\exp(-b^f_3 x)}{b^f_3 x}$$
$$\cdot\left[b^f_0 + b^f_2\frac{1-\exp(-\Lambda' x)}{\Lambda' x}\right]$$
$$+\bar{m}_f\frac{1-\exp[-2(x+\bar{m}_f)]}{x+\bar{m}^2_f} \tag{12}$$

and

$$\bar{\sigma}^{\mathrm{f}}_V(x) = \frac{2(x+\bar{m}^2_f)-1+\exp[-2(x+\bar{m}^2_f)]}{2(x+\bar{m}^2_f)^2} \tag{13}$$

with $\bar{m}_f = \dfrac{m_f}{\Lambda}$, $x = \dfrac{p^2}{\Lambda^2}$, $\Lambda' = 10^{-4}$ GeV and $\Lambda = 0.566$ GeV. The parameters b^f_i (i=0,1,2,3) and m_f (f=u, d, s, $\cdots$) are given in Table 1 [18−20].

Table 1 The parameters of confined quark propagators in Eqs.(12), (13). m_f is the current quark mass with flavor f in QCD Lagrangian.

Flavour(f)	b^f_0	b^f_1	b^f_2	b^f_3	m_f/MeV
u	0.131	2.90	0.603	0.185	5.1
d	0.131	2.90	0.603	0.185	5.1
s	0.105	2.90	0.740	0.185	127.5

The algebraic forms for $\sigma^f_s(p^2)$ and $\sigma^f_V(p^2)$ are analytic everywhere in finite complex p^2 plane. This guarantees that the quark propagator $S_f(p)$ has no Lehmann representation and hence there are no quark-production thresholds in the calculation of observable. The absence of such thresholds admits the interpretation that $S_f(p)$ describes the propagation of a confined quark.

3 Dynamically running mass of quark

3.1 Definition of quark mass in QCD

Non-perturbative definitions of quark masses are provided by

the chiral Ward identities of QCD [21]. These definitions allow us to express the renormalized quark mass, in a given scheme and at a given renormalization scale, in terms of the renormalization constants and bare quantities. Three independent definitions for quark mass (vector Ward identity definition, axial Ward identity definition, and definition from the quark propagator) have been proposed so far , which are all equivalent in principle, in the sense that they lead to the same value of the renormalized mass. In this paper, we use the definition from the quark propagator.

As free quarks do not exist, the definition of their masses is somewhat problematic and has ambiguities. One could, for example, take the quark mass as the average energy of the quark bound in a hadron in its ground state, or as the probable mass it would have were it to be free. This "free mass" is the mass that appears in the expressions describing quark currents. All these mass values are indirectly determined in experiment in one way or another. At present no one knows how to calculate quark masses exactly from first principles.

In order to introduce necessary formalism of the quark mass, we briefly discuss the fully dressed renormalized quark propagator denoted by $S_f(p)$ in the momentum space. The general form of the inverse of the fully dressed quark propagator in a covariant gauge is given [22−24] by

$$S_f^{-1}(p) = \not{p} - m_f - \Sigma(p) \equiv Z^{-1}(p^2)[\not{p} - M_f(p^2)]$$
$$\equiv A_f(p^2)\,\not{p} - B_f(p^2) \tag{14}$$

In Eq.(14), $Z^{-1}(p^2) \equiv A_f(p^2)$ is the momentum dependence of the quark wave function renormalization. $\not{p} \equiv i\gamma^\mu p_\mu$, m_f is the renormalized quark mass, $\Sigma(p)$ is the quark self-energy. From Eq.(14) we see that the $M_f(p^2)$ must have the form as

$$M_f(p^2) \equiv \frac{B_f(p^2)}{A_f(p^2)} \tag{15}$$

Note that a subscript μ (to indicate renormalization-point dependence) is to be understood for all renormalized quantities. The Eq.(15) is the definition of our dynamical running mass of quark with flavor f from its fully dressed propagator $S_f(p)$.

However, sometimes one defines an effective quark mass by the following identity

$$M_f \equiv M_f(p^2 = 0) \tag{16}$$

with $M_f(p)$ defining differently by

$$\{\gamma_5, S_f^{-1}(p)\} = -2\gamma_5 M_f(p^2) \tag{17}$$

where $M_f(p^2)$ itself is not a gauge invariant quantity, but it is useful to think of it as physically real. Then, the quark propagator can be replace by the form

$$S_f^{-1}(p) = \not{p} - M_f(p^2) \tag{18}$$

which has three properties of the correct propagator: (1) $S_f^{-1}(p)$ behave like $(\not{p} - m_f)$ at large p since $\Sigma = 0$ at $p \to \infty$; (2) $S_f(0)$ is , by definition, $-M_f(0)^{-1}$ since $\not{p}$ in Eq.(18) is nil; (3) The anti-commutator, Eq.(17), by definition also, is $-M_f^{-1}(0)$. In this work we take the first definition of quark mass, Eq.(15), which is determined by quark propagator, and then we calculate its value by use of the phenomenological solution of the DSEs in " rainbow " approximation, Eq. (9).

Combining Eq.(9) and Eq. (14), we can obtain expressions of $A_f(p^2)$, $B_f(p^2)$ and the dynamical running quark mass $M_f(p^2)$ in terms of $\sigma_V^f(p^2)$ and $\sigma_s^f(p^2)$ given by Eqs. (10)−(13), with parameters given in Table 1. Obviously, inverting Eq. (14) and then comparing the resulting formulae with Eq.(9), one arrives at

$$\sigma_V^f = \frac{A_f}{p^2 A_f^2 + B_f^2} \tag{19}$$

and

$$\sigma_s^f = \frac{B_f}{p^2 A_f^2 + B_f^2} \tag{20}$$

Clearly, Eqs. (19), (20) lead us to the fact that A_f and B_f in Eqs.(19), (20) can be expressed by σ_V^f and σ_s^f as follows:

$$A_f(p^2) = \frac{\sigma_V^f}{(\sigma_s^f)^2 \left[p^2 \left(\dfrac{\sigma_V^f}{\sigma_s^f} \right)^2 + 1 \right]} \tag{21}$$

and

$$B_f(p^2) = \frac{1}{\sigma_s^f \left[p^2 \left(\dfrac{\sigma_V^f}{\sigma_s^f} \right)^2 + 1 \right]} \tag{22}$$

Therefore, now $A_f(p^2)$ and $B_f(p^2)$ can be evaluated from σ_V^f and σ_s^f in terms of Eqs. (10)−(13), and $M_f(p^2)$ can be obtained from Eq.(15).

Studying A_f and B_f in Eqs.(21), (22), we realize that A_f and B_f have the following features of

(1) In the limit that the quark momentum p^2 becomes large and space-like, the functions A_f and B_f approach an asymptotic limit:

$$\lim_{p^2 \to \infty} A_f(p^2) = 1.0 \tag{23}$$

$$\lim_{p^2 \to \infty} B_f(p^2) = m_f \tag{24}$$

Hence, the confined quark propagator, $S_f(p)$, reduces to a free quark propagator at large momentum:

$$\lim_{p^2 \to \infty} S_f^{-1}(p) = [S_f^0(p \to \infty)]^{-1} = i\gamma p - m_f \tag{25}$$

where $S_f^0(p)$ is the propagator of a free quark in QCD.

(2) For small quark momentum, the quark propagator $S_f(p)$ becomes quite different comparing with the free quark propagator $S_f^0(p)$. In this momentum region, there is a strong non-perturbative enhancement of the mass function $B_f(p^2)$. This enhancement is a manifestation of dynamical chiral symmetry breaking and confinement. One can identify the $M_f(p)$ in Eq.(15) as an effective quark mass of confined quark, and consequently, one can calculate the effective masses of up, down and strange quarks. The theoretical results of A_f, B_f and $M_f(p^2)$ are shown in Fig. 1−4 for u-, d- and s-quarks.
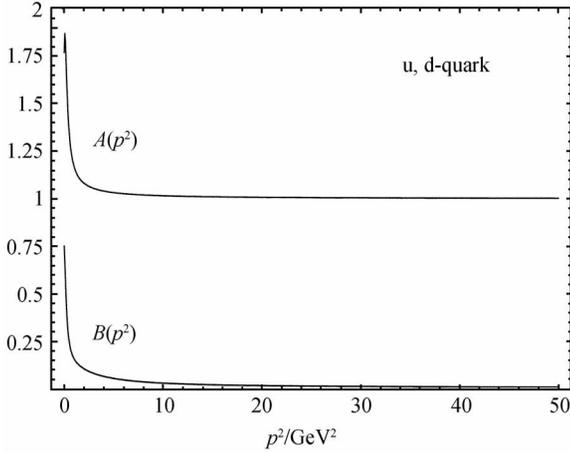

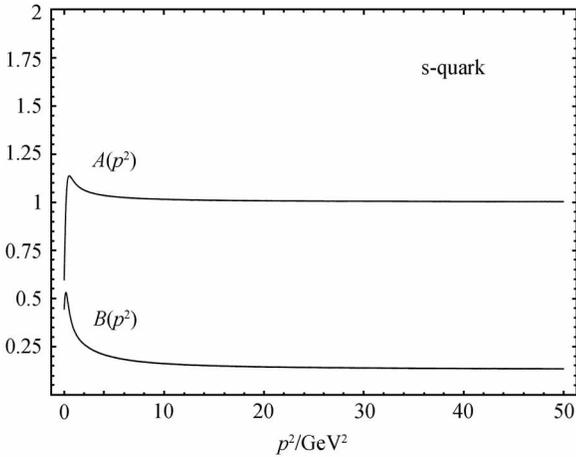**Fig. 1** p^2-dependence of self-energy functions $A_{u,d}$ and $B_{u,d}$ for up and down quarks.

Fig. 2 p^2-dependence of self-energy functions A_s and B_s for the strange quark.

3.2 Predictions of quark masses

Based on the above discussions, the quark current mass and effective mass can be obtained. The results are given in the following.

3.2.1 Prediction of current quark mass

The current quark masses of light quarks u,d and s can be produced by use of the Gell-Mann- Oaked-Renner relation [25]:

$$(m_u + m_d)\langle 0 \mid: \bar{q}(0)q(0) :\mid 0\rangle = f_\pi^2 M_\pi (1 - \delta_\pi) \qquad (26)$$

with $M_\pi = 139$ MeV being the mass of pion meson which has been determined by experimental measurements, and $\delta_\pi = 0.05 \pm 0.05$ [25, 26]. f_π is the pion decay constant given by

$$f_\pi^2 = \frac{3}{4\pi^2} \int s\mathrm{d}s \left\{ \left[\frac{B_f^2(s)}{sA_f^2(s) + B_f^2(s)} \right]^2 \right.$$
$$\cdot [A_f^2(s) + sA_f(s)A_f'(s)s^2(A_f'(s))^2 + s(B_f'(s))^2]$$
$$\left. - \frac{B_f(s)B_f'(s)\frac{s}{2}\left[(B_f'(s))^2 + B_f(s)B_f''(s) \right]}{sA_f^2(s) + B_f^2(s)} \right\} \qquad (27)$$

and the $\langle 0 \mid: \bar{q}(0)q(0) :\mid 0\rangle$ is the local quark vacuum condensate which is determined theoretically by

$$\langle 0 \mid \bar{q}(0)q(0) \mid 0\rangle = -\frac{3}{4\pi^2} \int_0^{\mu^2} s\mathrm{d}s \frac{B(s)}{sA^2 + B^2(s)} \qquad (28)$$

in the "rainbow" approximation of DSEs with Feynman gauge parameter of $\xi = 1$.

Our calculations at the cut-off parameter $\mu^2 = 1$ GeV^2 produce that the $f_\pi = 93$ MeV which fits to experimental measurement of 93 MeV successfully. The numerical results of quark local vacuum condensates at the same point of $\mu^2 = 1$ GeV^2 are

$$\langle 0 \mid: \bar{q}(0)q(0) :\mid 0\rangle_{u,d} = -(251 \text{ MeV})^3 \qquad (29)$$

for u, d quarks and

$$\langle 0 \mid: \bar{q}(0)q(0) :\mid 0\rangle_s = -(292 \text{ MeV})^3 \qquad (30)$$

for s quarks.

Evidently, from Gell-Mann-Oaked-Renner relation, Eq. (26), and using our predictions of the quark vacuum condensates given by Eqs.(29), (30), pion decay constant $f_\pi = 93$ MeV and the mass of $M_\pi = 139$ MeV, we get

$$m_u + m_d = 9.437 \text{ MeV} \qquad (31)$$

On the other hand, the chiral perturbative theory [27−29] provides rather precise information on the ratios of the light quark masses. The two particular ratios [30, 31] are

$$\frac{m_s}{\bar{m}} = 24.4 \pm 1.5, \quad \frac{m_s^2 - \bar{m}^2}{m_d^2 - m_u^2} = (22.7 \pm 0.8)^2 \qquad (32)$$

with $\bar{m}=(m_u+m_d)/2$. From Eqs.(31), (32), we further arrive at

$$\frac{m_{\mathrm{u}}}{m_{\mathrm{d}}} = 0.535 \pm 0.049, \quad \frac{m_{\mathrm{s}}}{m_{\mathrm{d}}} = 19.180 \pm 1.30 \tag{33}$$

Comparing Eq.(31) with Eq.(33) leads the current masses of the light quarks, u, d and s to

$$m_{\mathrm{u}} = 3.29 \,\mathrm{MeV}, \quad m_{\mathrm{d}} = 6.15 \,\mathrm{MeV}$$

$$m_{\mathrm{s}} = 117.96 \,\mathrm{MeV} \tag{34}$$

These values are consistent with those given by QCD sum rules predictions [32−34], and many other theoretical calculations.

3.2.2　*Prediction of effective quark mass*

Now let us turn to the study of quark effective mass and investigate the dynamical transition from current mass of quark given by Eq.(34) into the constituent quark mass used widely in literature empirically and defined by Eq.(15). The dynamical masses of the light quarks are predicted in terms of formulism Eq. (15) with A_f and B_f given by Eqs. (21), (22). The results are shown in Fig. 3 and Fig. 4 for u, d and s
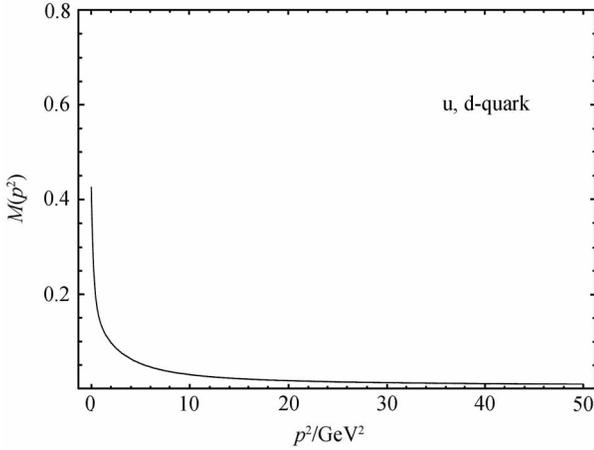

Fig. 3　p^2-dependence of the dynamical running mass $M_{\mathrm{u,d}}(p^2)$ for up and down quarks.
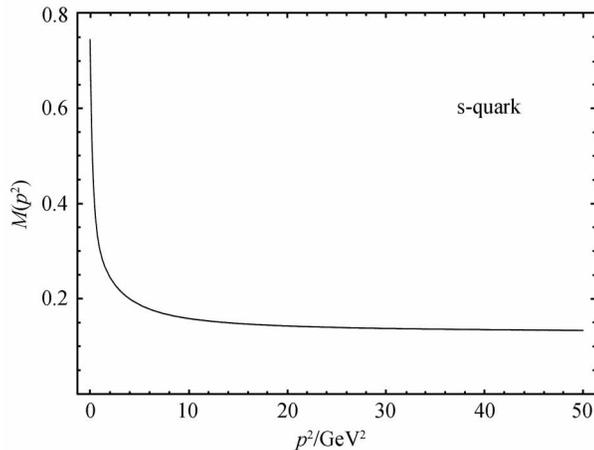

Fig. 4　p^2-dependence of the dynamical running mass $M_s(p^2)$ for the strange quark.

quark, respectively. As is seen from the Figs. 3 and 4, when $p^2 \to \infty$ the $M_f \to m_f$, the current quark mass. Since m_f is a negligible quantity so that the chiral symmetry is restored in this case. On the other hand, as $p^2 \to 0$ the M_f approaches to a finite value in units of MeV which has been dubbed constituent quark mass. Our predicted effective quark masses for light quarks from Eq.(15) are shown in Table 2.

Table 2　The masses of light quarks u, d and s predicted by Eqs.(15), (21), (22) with input parameters given in Table 1.

Different flavor of quark (f)	u	d	s
Const. quark mass/MeV	328	328	492

These masses are typical values used commonly in the constituent quark models and other theoretical calculations. Figure 3 show us how quark mass transits from its current mass to the corresponding constituent quark mass dynamically. This is an important issue for understanding dynamical transition of quark mass and QCD.

4　The masses of Goldstone bosons π, *K*, η and η'

In order to shed more lights on the validity of our theory presented in previous sections, we calculate the masses of Goldstone bosons π, K, η and η' in terms of the dynamical running mass m_f of quarks. The masses of the Goldstone bosons can expressed as follows:

$$M_\pi^2 = -\frac{\langle 0| : \bar{q}(0)q(0) : |0\rangle}{f_\pi^2} m_{\mathrm{u(d)}} \tag{35}$$

$$M_K^2 = -\frac{\langle 0| : \bar{q}(0)q(0) : |0\rangle}{f_\pi^2} \frac{1}{2}(m_{\mathrm{u}} + m_{\mathrm{s}}) \tag{36}$$

$$M_\eta^2 = -\frac{\langle 0| : \bar{q}(0)q(0) : |0\rangle}{f_\pi^2} \frac{1}{3}(m_{\mathrm{u}} + 2m_{\mathrm{s}}) \tag{37}$$

$$M_{\eta'}^2 = -\frac{\langle 0| : \bar{q}(0)q(0) : |0\rangle}{f_\pi^2} \frac{1}{3}(2m_{\mathrm{u}} + m_{\mathrm{s}}) \tag{38}$$

with $f_\pi = 93$ MeV also being pion decay constant which is produced by Eq.(27), $\langle 0 \mid: \bar{q}(0)q(0) :\mid 0\rangle$ is quark local vacuum condensate given by Eqs.(29), (30), and m_f is quark current mass given in Eq.(34). Using these predicted values of f_π, $\langle 0 \mid: \bar{q}(0)q(0) :\mid 0\rangle$ and m_f, we obtain the masses of the Goldstone bosons. The resultant of M_π, M_K, M_η and $M_{\eta'}$ masses are given in Table 3. From Table 3, we can see that our calculations fit experimental data of Goldstone bosons very well.

With all of above theoretical results we can also predict the masses of Goldstone bosons in nuclear medium. The in-medium masses of Goldstone bosons can be explicitly ex-

Table 3 The masses of Goldstone bosons M_π, M_K, M_η and $M_{\eta'}$ predicted by the dynamical running masses of quarks via Eqs. (27), (29), (30), (34).

Mass/MeV	M_π	M_K	M_η	$M_{\eta'}$
This theory	138	494	545	955
Experimental data [35]	139	495	548	958

pressed [16] as:

$$m_\pi^* = \frac{\langle 0 \,|\, \bar{q}(0)q(0) \,:|\, 0 \rangle_{\mu^2}^\rho}{f_\pi^2} m_{u,d}^\rho \tag{39}$$

$$m_K^* = \frac{\langle 0 \,|\, \bar{q}(0)q(0) \,:|\, 0 \rangle_{\mu^2}^\rho}{f_\pi^2} \frac{1}{2}[m_{u,d}^\rho + m_s^\rho] \tag{40}$$

$$m_\eta^* = \frac{\langle 0 \,|\, \bar{q}(0)q(0) \,:|\, 0 \rangle_{\mu^2}^\rho}{f_\pi^2} \frac{1}{3}[m_u^\rho + 2m_s^\rho] \tag{41}$$

$$m_{\eta'}^* = \frac{\langle 0 \,|\, \bar{q}(0)q(0) \,:|\, 0 \rangle_{\mu^2}^\rho}{f_\pi^2} \frac{1}{3}[2m_u^\rho + m_s^\rho] \tag{42}$$

where $\langle 0 \,|\, \bar{q}(0)q(0) \,:|\, 0 \rangle_{\mu^2}^\rho$ is in-medium quark local vacuum condensate, m_f^ρ the quark current mass in nuclear medium. The in-medium condensate is given [36, 37] by

$$\frac{\langle 0 \,|\, \bar{q}(0)q(0) \,:|\, 0 \rangle_{\mu^2}^\rho}{\langle 0 \,|\, \bar{q}(0)q(0) \,:|\, 0 \rangle_{\mu^2}}$$

$$= 1 - \frac{\rho}{f_\pi^2 m_\pi^2}\left\{ \Sigma_\pi + m_f \frac{d}{dm_f}\left[\frac{E(\rho)}{A} \right] \right\} \tag{43}$$

with $\Sigma_\pi = 45 \pm 10$ MeV being the pion- nucleon sigma term which determined by experiment. $E(\rho)/A$ is the nuclear binding energy per nucleon. This term contributes to the ratio in a negligible way. Therefore, we can claim that the dependence of the in-medium quark local vacuum condensate in Eq.(43) on the nuclear medium density ρ behaves lineally.

Assuming the mass of quark in nuclear medium is the same as the current mass in QCD Lagrangian, we then obtain the ratio of in-medium Goldstone boson mass to Goldstone boson mass in free space. Under this simplified assumption, the variation rule of in-medium Goldstone boson mass as the nuclear density ρ/ρ_0 (ρ_0 is normal nuclear density and is taken to be 0.17) increasing is the same for all bosons since the in-medium quark local vacuum condensates change in a same way. Finally, the conclusion is that the in-medium Goldstone boson mass decreases lineally as nuclear density ρ increasing.

5   Summary and concluding remarks

We study fully dressed quark propagator by use of the DSEs in the "rainbow" approximation and get a solution of the

DSEs which has been parameterized as an algebraic form. The parameterized quark propagator $S_f(p)$ is analytic everywhere in finite complex p^2 plane. This parameterized quark propagator has no Lehmann representation and hence there are no quark production thresholds in any calculations of observable. The absence of such thresholds admits the interpretation that $S_f(p)$ describes the propagator of a confined quark.

Based on the fully dressed quark propagator $S_f(p)$, we define the dynamical running mass of quark in QCD, by equation of $M_f(p^2) = B_f(p^2)/A_f(p^2)$, and then we obtain the transition from the current quark mass to the constituent quark mass. The masses commonly used in literature are quite well reproduced by our calculations. For example, $M_{u,d} = 0.328$ GeV at $p^2 = 0.107$ GeV^2 for u, d quarks and $M_s = 0.492$ GeV at $p^2 = 0.242$ GeV^2 for s quark. Therefore, the empirical values of constituent quark masses M_f are dynamically produced theoretically by our calculations at $p^2 \approx 0.107 - 0.242$ GeV^2. It should be emphasized that the quark mass is not a constant but it depends on the momentum p^2, namely, it is a dynamical running quantity.

Using Gellmann-Oaked-Renner mass formula, our predicted quark local vacuum condensates, on shell pion decay constant (93 MeV) and experimental data of pion mass (139 MeV), we also calculate the current masses of light quarks. The results are shown in Eq.(34) and fit the predictions given by QCD sum rules and by many other theoretical calculations. They are so small that the chiral symmetry must be nearly remained.

we also calculate the masses of Goldstone bosons, π, K, η and η' and their in-medium counterparts in terms of quark current masses, in-medium quark local vacuum condensates and pion decay constant, all are predicted in this work. The theoretical results of Goldstone bosons masses satisfy Gellmann-oaked- Renner mass formula and consistent with the experimental data if we take the up and down quark vacuum condensate to be $\langle 0| : \bar{q}q : |0 \rangle_{u,d}^{\mu^2=1\ \text{GeV}^2} = -(0.251\ \text{GeV})^3$, $\langle 0| : \bar{q}q : |0 \rangle_s^{\mu^2=1\ \text{GeV}^2} = -(0.292\ \text{GeV})^3$, and our predicted pion decay constant $f_\pi = 93$ MeV which is the on-shell quantity ($p^2 = M_\pi^2$) of pion decay constant, are used as inputs.

The dynamical amplitudes $A_f(p^2)$ and $B_f(p^2)$ are also analyzed from the algebraic expressions of σ_s^f and σ_V^f. These analysis can be used in our coming investigation of strong interaction processes such as structure of non-local quark vacuum condensates and diffractive processes, e.g., vector meson photo- and electro-productions in QCD.

Our concluding remark is that as quarks are confined in a color singlet hadron, the definition of their masses is somewhat problematic and has ambiguities. One could, for exam-

ple, take the quark mass as the average energy of the quark bound in the ground state of hadron, or as the probable mass it would have were it to be free. In this paper, we adopt $M_f(p^2) = B_f(p^2)/A_f(p^2)$ given by DSEs. Of course, this mass is quark effective mass. The transition from the current quark mass to the constituent quark mass is dynamically obtained in this work. The quark mass depends on the quark momentum p^2 and thus is a running quantity.

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