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Lax connections and Solutions of the Hybrid Superstring on $AdS_2 \times S^2$

Abstract   In this paper, we show that the Lax connections can yield new classical solutions with a spectral parameter of the hybrid formulism for the Type IIB superstring in an $AdS_2 \times S^2$ background with Ramond-Ramond flux. This series of classical solutions have the same infinite set of classically conserved charges.

1   Introduction

The AdS/CFT correspondence [1, 2] between the classical theory of supergravity on AdS in the bulk and the quantum conformal supersymmetric Yang-Mills theory on the boundary has stimulated much interest in the study of the corresponding string theory with worldsheet methods.

The classical/quantum integrabilities in string theories are expected to play an important role in the study of the AdS/CFT correspondence, which have attracted much attention. In recent years, the existence of integrable structures on the superstring has been discussed in papers [3−7]. The classical integrability and non-local charges in the type IIB string

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theory on $AdS_5 \times S^5$ was suggested in an early work [8] by regarding the S^5 part as a nonlinear σ model. For the non-symmetric coset space $\frac{PSU(2,2\,|\,4)}{SO(1,4) \times SO(5)}$ [9], Bena, Polchinski, and Roiban [10] have discovered the one-spectrum parameter family of flat connections, which can be used to construct an infinite number of non-local conserved charges, such that it may be exactly solvable. Then, by twistly transforming the vierbein, the Lax connections with a spectral parameter were equivalently given by Hou [11].

The pure spinor superstring on $AdS_5 \times S^5$ was given by Berkovits [12], and the hybrid description was given in $AdS_2 \times S^2$ [13, 14], $AdS_3 \times S^3$ [14–16]. In Ref. [17] the corresponding flat currents were derived. In Ref. [18] it was proved that the nonlocal charges in the string theory are BRST-invariant and physical, and in Ref. [19] it was shown that there exists an infinite set of nonlocal BRST-invariant currents at the quantum level. Recently, the Lax operator for the superstring in the γ-deformed background generated by TsT transformations was obtained [20–23].

Recently, for the Green-Schwarz superstring on $AdS_5 \times S^5$, in the paper [24] we found that the Cartan 1-forms associated with the flat connections given by Bena, Polchinski and Roiban also satisfy equations of motion and the Virasoro constraint of the Green-Schwarz Superstring on $AdS_5 \times S^5$. Thus, one can generate a series of classical solutions with a spectrum parameter from the existing one, and these solutions have the same infinite set of classically conserved quantities. In the present work, we will try to prove that the worldsheet components of Lax connections also satisfy the equations of motion in the hybrid superstring on $AdS_2 \times S^2$ with R-R flux.

The paper is organized as follows. In Section 2, we first recall the necessary facts about the Lie superalgebra $psu(1,1|2)$, andthen review the hybrid action for the Type

IIB superstring in an $AdS_2 \times S^2$ background with R-R flux. In Section 3, we construct the currents with a spectral parameter which satisfies the Maurer-Cartan equations. In Section 4 we present the calculations showing that these currents also satisfy the equation of motion and the Virasoro constraint. In Section 5 we make some further discussions.

2   The hybrid superstring in an $AdS_2 \times S^2$ background

Superstring propagating in the $AdS_2 \times S^2$ space-time can be described as the non-linear sigma model with a Wess-Zumino-Witten (WZW) term whose target space is the following coset [13, 25]:

$$\frac{PSU(1,1|2)}{U(1) \times U(1)}$$

Here, the supergroup $PSU(1,1|2)$ with the Lie superalgebra $psu(1,1|2)$ is the isometry group of the $AdS_2 \times S^2$ superspace.

The Lie superalgebra $su(1,1|2)$ are the algebra of 4×4 supermatrices with bosonic diagonal blocks and fermionic off-diagonal blocks:

$$G = \begin{pmatrix} A & X \\ Y & B \end{pmatrix} \qquad (1)$$

These supermatrices are required to have vanishing supertrace $\mathrm{str} M = \mathrm{tr} A - \mathrm{tr} B = 0$ and to satisfy the following reality condition:

$$HG + G^\dagger H = 0 \qquad (2)$$

For our further purpose, it is convenient to pick up the hermitian matrix H to be

$$H = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\mathbb{I} \end{pmatrix} \qquad (3)$$

where σ_3 is the pauli matrix and $\mathbb{I}$ denotes the identity matrix of the corresponding two dimensions. The matrices A and B are even, while X, Y are odd (linearly depend on fermionic variables). The condition (2) implies that [13]

$$A = -\sigma_3 A^\dagger \sigma_3 , \;\; B = -B^\dagger , \;\; X = \mathrm{i}\sigma_3 Y^\dagger \qquad (4)$$

The Lie superalgebra $psu(1,1|2)$ is defined as the algebra $su(1,1|2)$ over the $u(1)$ factor, it has no realization in terms of 4×4 matrix.

The essential feature of the Lie superalgebra $su(1,1|2)$ is that it admits a $\mathbb{Z}_4$ automorphism, so that the condition $\mathbb{Z}_4(H) = H$ determines the maximal subgroup to be $u(1) \times u(1)$ that leads to the definition of the coset for the

sigma model. The $\mathbb{Z}_4$ automorphism Ω takes an element of $su(1,1|2)$ to another $G \to \Omega(G)$ as

$$\Omega(G) = \begin{pmatrix} JA^t J & -JY^t J \\ JX^t J & JB^t J \end{pmatrix}, \quad \text{where } J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (5)$$

Since $\Omega^4 = 1$, the eigenvalues of Ω are i^p ($p = 0, 1, 2, 3$). Therefore, one can decompose the superalgebra $\mathcal{G}$ as

$$\mathcal{G} = \mathcal{H}^0 \oplus \mathcal{H}^1 \oplus \mathcal{H}^2 \oplus \mathcal{H}^3 \qquad (6)$$

where $\mathcal{H}^p$ denotes the eigenspace of Ω such that if $H^p \in \mathcal{H}^p$ then

$$\Omega(H^p) = \mathrm{i}^p H^p \qquad (7)$$

It is worth pointing out that $\Omega(\mathcal{H}^0) = \mathcal{H}^0$ determines the $\mathcal{H}^0 = u(1) \times u(1)$ subalgebra, and $\mathcal{H}^2$ represents the remaining bosonic generators of the superalgebra $\mathcal{G}$, while $\mathcal{H}^1$ and $\mathcal{H}^3$ consist of the fermionic generators of the superalgebra. The automorphism also implies an important relation:

$$[H^p, H^q] \in \mathcal{H}^{p+q} \,(\mathrm{mod}\,4) \qquad (8)$$

The hybrid action for the Type IIB superstring in an $AdS_2 \times S^2$ background with Ramond-Ramond flux takes the form [7, 13]:

$$S_{AdS} = \int \mathrm{d}^2 z \Big[\frac{1}{2}\mathrm{str}(J_+^2 J_-^2) + \frac{1}{4}\mathrm{str}(J_-^3 J_+^1)$$
$$+ \frac{3}{4}\mathrm{str}(J_+^3, J_-^1) + S_{\mathrm{com}} + S_{\mathrm{ghost}} \qquad (9)$$

where the currents $J_\pm^p$ ($p = 0, 1, 2, 3$) are given by

$$J_\pm^0 + J_\pm^1 + J_\pm^2 + J_\pm^3 = J_\pm = G^{-1}\partial_\pm G$$
$$J_\pm^0 = J_\pm^{cd} T_{cd}, \quad J_\pm^1 = J_\pm^\alpha T_\alpha$$
$$J_\pm^2 = J_\pm^c T_c, \quad J_\pm^3 = J_\pm^{\hat\alpha} T_{\hat\alpha} \qquad (10)$$

where G is the group element, $A = cd, \alpha, c, \hat\alpha$ and $J_\pm^\alpha$, $J_\pm^{\hat\alpha}$ are Grassmann odd functions while $J_\pm^{cd}$, $J_\pm^c$ are Grassmann even functions. T_A are the $psu(1,1|2)$ Lie superalgebra generators.

For the action (9), S_{com} is the action for compactification variables and S_{ghost} is an action for the chiral and anti-chiral bosonic ghosts, which we will not write explicitly. However, it is emphasized that the hybrid action is different from the pure spinor superstring action in an $AdS_5 \times S^5$ background [12, 17], since in the $D = 4$ and $D = 6$ actions the worldsheet ghosts are Lorentz scalars in stead of the Lorentz ghost currents on $AdS_5 \times S^5$.

The action is global $psu(1,1|2)$-invariant, and it is also gauge-invariant under a local $U(1) \times U(1)$ transformation parametrized by $\Omega \in \mathcal{H}^0$,

$$\delta J^i_+ = [J^i_+, \Omega], \quad \delta J^i_- = [J^i_-, \Omega]$$
$$\delta J^0_+ = \partial_+ \Omega + [J^0_+, \Omega], \quad \delta J^0_- = \partial_- \Omega + [J^0_-, \Omega]$$
$$\delta(S_{\text{com}} + S_{\text{ghost}}) = 0 \tag{11}$$

The form of the action (9) is very useful since from which now we can easily derive the equations of motion. If the variation of the group element g as $\delta g = g\delta X$ is taken into account, the variation of the current $J = g^{-1}dg$ is equal to

$$\delta J_\pm = -g^{-1}\delta g g^{-1}\partial_\pm g + g^{-1}\partial_\pm \delta g = \partial_\pm \delta X + [J_\pm, \delta X] \tag{12}$$

To find the variations of the currents $J^p_\pm (p = 0,1,2,3)$, the relation (8) is used with decomposing δX as $\delta X = \delta X^{(0)} + \delta X^{(1)} + \delta X^{(2)} + \delta X^{(3)}$ where $\delta X^p \in \mathcal{H}^p$. Then one finds

$$\begin{aligned}
\delta J^0_\pm &= \partial_\pm \delta X^0 + [J^0_\pm, \delta X^0] + [J^1_\pm, \delta X^3] \\
&\quad + [J^2_\pm, \delta X^2] + [J^3_\pm, \delta X^1] \\
\delta J^1_\pm &= \partial_\pm \delta X^1 + [J^0_\pm, \delta X^1] + [J^1_\pm, \delta X^0] \\
&\quad + [J^2_\pm, \delta X^3] + [J^3_\pm, \delta X^2] \\
\delta J^2_\pm &= \partial_\pm \delta X^2 + [J^0_\pm, \delta X^2] + [J^1_\pm, \delta X^1] \\
&\quad + [J^2_\pm, \delta X^0] + [J^3_\pm, \delta X^3] \\
\delta J^3_\pm &= \partial_\pm \delta X^3 + [J^0_\pm, \delta X^3] + [J^1_\pm, \delta X^2] \\
&\quad + [J^2_\pm, \delta X^1] + [J^3_\pm, \delta X^0]
\end{aligned} \tag{13}$$

Based on these results, we perform the variation of the action (9) and have the equations of motion:

$$\frac{1}{2}D_+ J^3_- + \frac{3}{2}D_- J^3_+ = -\frac{1}{2}[J^2_+, J^1_-] - \frac{1}{2}[J^1_+, J^2_-] \tag{14}$$

$$\frac{3}{2}D_+ J^1_- + \frac{1}{2}D_- J^1_+ = \frac{1}{2}[J^2_+, J^3_-] + \frac{1}{2}[J^3_+, J^2_-] \tag{15}$$

$$D_+ J^2_- + D_- J^2_+ = -[J^1_+, J^1_-] + [J^3_+, J^3_-] \tag{16}$$

where $D_\pm = \partial_\pm + [J^0_\pm,]$. And following from the variation of the action over the metric g_{ij}, in the light-cone gauge, the $J^2_\pm$ sector should satisfy the Virasoro constraint:

$$\text{str}(J^2_+ J^2_+) = \text{str}(J^2_- J^2_-) = 0$$
$$\text{str}(J^2_+ J^2_-) = \text{str}(J^2_- J^2_+) = c(c \text{ is constant}) \tag{17}$$

In terms of the $\mathbb{Z}_4$ grading of the superalgebra as well as the relation (8), the Maurer-Cartan equations $\partial_+ J_- - \partial_- J_+ + [J_+, J_-] = 0$ can be rewritten as

$$\begin{aligned}
\partial_+ J^0_- - \partial_- J^0_+ &= -[J^0_+, J^0_-] - [J^1_+, J^3_-] \\
&\quad - [J^2_+, J^2_-] - [J^3_+, J^1_-]
\end{aligned} \tag{18}$$
$$D_+ J^3_- - D_- J^3_+ = -[J^1_+, J^2_-] - [J^2_+, J^1_-] \tag{19}$$
$$D_+ J^1_- - D_- J^1_+ = -[J^2_+, J^3_-] - [J^3_+, J^2_-] \tag{20}$$
$$D_+ J^2_- - D_- J^2_+ = -[J^1_+, J^1_-] - [J^3_+, J^3_-] \tag{21}$$

From the equations of motion and the Maurer-Cartan equations, we can have the following relations:

$$\begin{aligned}
\partial_+ J^0_- - \partial_- J^0_+ &= -[J^0_+, J^0_-] - [J^1_+, J^3_-] \\
&\quad - [J^2_+, J^2_-] - [J^3_+, J^1_-] \\
D_+ J^1_- &= 0, \quad D_- J^1_+ = [J^2_+, J^3_-] + [J^3_+, J^2_-] \\
D_+ J^2_- &= -[J^1_+, J^1_-], \quad D_- J^2_+ = [J^3_+, J^3_-] \\
D_+ J^3_- &= -[J^1_+, J^2_-] - [J^2_+, J^1_-], \quad D_- J^3_+ = 0
\end{aligned} \tag{22}$$

In the next two sections, just by these relations, we will prove that the worldsheet components of the Lax connections satisfy not only the Maurer-Cartan equations but also the equations of motion.

3 Lax connections in the hybrid superstring

Introducing the currents $\mathcal{J}^p_\pm(\lambda)$ with the spectral parameter λ [6] which is equivalent to the pure spinor superstring Lax connection by setting the pure spinor ghosts to zero [16],

$$\begin{aligned}
\mathcal{J}^0_\pm(\lambda) &= J^0_\pm \\
\mathcal{J}^2_\pm(\lambda) &= \lambda^{\pm 2} J^2_\pm \\
\mathcal{J}^1_+(\lambda) &= \lambda J^1_+, \quad \mathcal{J}^1_-(\lambda) = \lambda^{-3} J^1_- \\
\mathcal{J}^3_+(\lambda) &= \lambda^3 J^3_+, \quad \mathcal{J}^3_-(\lambda) = \lambda^{-1} J^3_-
\end{aligned} \tag{23}$$

We will prove that $\mathcal{J}^p_\pm(\lambda)$ satisfies all the Maurer-Cartan equations, one can have

$$\begin{aligned}
&\partial_+ \mathcal{J}^0_-(\lambda) - \partial_- \mathcal{J}^0_+(\lambda) + [\mathcal{J}^0_+(\lambda), \mathcal{J}^0_-(\lambda)] \\
&\quad + [\mathcal{J}^1_+(\lambda), \mathcal{J}^3_-(\lambda)] + [\mathcal{J}^2_+(\lambda), \mathcal{J}^2_-] + [\mathcal{J}^3_+(\lambda), \mathcal{J}^1_-(\lambda)] \\
&= \partial_+ J^0_- - \partial_- J^0_+ + [J^0_+, J^0_-] + [J^1_+, J^3_-] \\
&\quad + [J^2_+, J^2_-] + [J^3_+, J^1_-] = 0
\end{aligned} \tag{24}$$

$$\begin{aligned}
&D_+ \mathcal{J}^3_-(\lambda) - D_- \mathcal{J}^3_+(\lambda) + [\mathcal{J}^1_+(\lambda), \mathcal{J}^2_-(\lambda)] \\
&\quad + [\mathcal{J}^2_+(\lambda), \mathcal{J}^1_-(\lambda)] \\
&= \lambda^{-1}(-[J^1_+, J^2_-] - [J^2_+, J^1_-]) + \lambda^{-1}[J^1_+, J^2_-] \\
&\quad + \lambda^{-1}[J^2_+, J^1_-] = 0
\end{aligned} \tag{25}$$

$$\begin{aligned}
&D_+ \mathcal{J}^1_-(\lambda) - D_- \mathcal{J}^1_+(\lambda) + [\mathcal{J}^2_+(\lambda), \mathcal{J}^3_-(\lambda)] \\
&\quad + [\mathcal{J}^3_+(\lambda), \mathcal{J}^2_-(\lambda)] \\
&= -\lambda([J^2_+, J^3_-] + [J^3_+, J^2_-]) + \lambda[J^2_+, J^3_-] \\
&\quad + \lambda[J^3_+, J^2_-] = 0
\end{aligned} \tag{26}$$

$$\begin{aligned}
&D_+ \mathcal{J}^2_-(\lambda) - D_- \mathcal{J}^2_+(\lambda) + [\mathcal{J}^1_+(\lambda), \mathcal{J}^1_-(\lambda)] \\
&\quad + [\mathcal{J}^3_+(\lambda), \mathcal{J}^3_-(\lambda)] \\
&= -\lambda^{-2}[J^1_+, J^1_-] - \lambda^2[J^3_+, J^3_-] + \lambda^{-2}[J^1_+, J^1_-]
\end{aligned}$$

$$+\lambda^2[J_+^3, J_-^3] = 0 \tag{27}$$

Hence it is proved that $\mathcal{J}_\pm^0(\lambda)$, $\mathcal{J}_\pm^2(\lambda)$, $\mathcal{J}_\pm^1(\lambda)$ and $\mathcal{J}_\pm^3(\lambda)$ all satisfy the Maurer-Cartan equations (18), (19), (20), and (21).

Now, the Lax connections $\mathcal{J}_\pm(\lambda)$ with the spectral parameter λ are constructed directly from the currents (23):

$$\begin{aligned}
\mathcal{J}_+(\lambda) &= G(\lambda)^{-1}\partial_+ G(\lambda) \\
&= J_+^0 + \lambda^{\,2}J_+^2 + \lambda J_+^1 + \lambda^{\,3}J_+^3 \\
\mathcal{J}_-(\lambda) &= G(\lambda)^{-1}\partial_- G(\lambda) \\
&= J_-^0 + \lambda^{-2}J_-^2 + \lambda^{-3}J_-^1 + \lambda^{-1}J_-^3
\end{aligned} \tag{28}$$

It is obvious that the Lax connections $\mathcal{J}_\pm(\lambda)$ (28) satisfy the compatibility condition for the integrable systems (the zero curvature condition):

$$\partial_+\mathcal{J}_-(\lambda) - \partial_-\mathcal{J}_+(\lambda) + [\mathcal{J}_+(\lambda), \mathcal{J}_-(\lambda)] = 0 \tag{29}$$

By using the Lax connections (28), and one can construct the corresponding monodromy matrix $T(\lambda)$:

$$T(\lambda) = \mathcal{P}\exp\left\{-\int_0^{2\pi}\mathcal{J}_\sigma(\lambda)\mathrm{d}\sigma\right\} \tag{30}$$

The Lax connections and the monodromy matrix of the superstring would naturally lead to an infinite number of non-local conserved charges which are intimately connected to the classical integrability of the theories. In the next section, it will be shown that the currents also satisfy the equations of motion and the Virasoro constraint.

4 Lax connections and solutions in the hybrid superstring

In this section, we will show that the Lax connections can yield the new classical solutions with the spectral parameter of the Hybrid Superstring on $AdS_2 \times S^2$.

From the equations of motion and the Maurer-Cartan equations, one has

$$\begin{aligned}
&\frac{1}{2}D_+\mathcal{J}_-^3(\lambda) + \frac{3}{2}D_-\mathcal{J}_+^3(\lambda) + \frac{1}{2}[\mathcal{J}_+^2(\lambda), \mathcal{J}_-^1(\lambda)] \\
&+\frac{1}{2}[\mathcal{J}_+^1(\lambda), \mathcal{J}_-^2(\lambda)] \\
&=\frac{1}{2}\lambda^{-1}(-[J_+^1, J_-^2] - [J_+^2, J_-^1]) + \frac{1}{2}\lambda^{-1}[J_+^1, J_-^2] \\
&+\frac{1}{2}\lambda^{-1}[J_+^2, J_-^1] = 0
\end{aligned} \tag{31}$$

$$\begin{aligned}
&\frac{3}{2}D_+\mathcal{J}_-^1(\lambda) + \frac{1}{2}D_-\mathcal{J}_+^1(\lambda) - \frac{1}{2}[\mathcal{J}_+^2(\lambda), \mathcal{J}_-^3(\lambda)] \\
&-\frac{1}{2}[\mathcal{J}_+^3(\lambda), \mathcal{J}_-^2(\lambda)]
\end{aligned}$$

$$\begin{aligned}
&=\frac{1}{2}\lambda([J_+^2, J_-^3] + [J_+^3, J_-^2]) - \frac{1}{2}\lambda[J_+^2, J_-^3] \\
&-\frac{1}{2}\lambda[J_+^3, J_-^2] = 0
\end{aligned} \tag{32}$$

$$\begin{aligned}
&D_+\mathcal{J}_-^2(\lambda) + D_-J_+^2(\lambda) + [\mathcal{J}_+^1(\lambda), \mathcal{J}_-^1(\lambda)] \\
&-[\mathcal{J}_+^3(\lambda), \mathcal{J}_-^3(\lambda)] \\
&=-\lambda^{-2}[J_+^1, J_-^1] + \lambda^2[J_+^3, J_-^3] + \lambda^{-2}[J_+^1, J_-^1] \\
&-\lambda^2[J_+^3, J_-^3] = 0
\end{aligned} \tag{33}$$

Therefore, the currents $\mathcal{J}_\pm^p(\lambda)$ satisfy all the equations of motion. Finally, we should consider the Virasoro constraint, one finds

$$\begin{aligned}
\mathrm{str}(\mathcal{J}_+^2(\lambda)\mathcal{J}_+^2(\lambda)) &= \lambda^4\mathrm{str}(J_+^2 J_+^2) = 0 \\
\mathrm{str}(\mathcal{J}_+^2(\lambda)\mathcal{J}_-^2(\lambda)) &= \mathrm{str}(J_+^2 J_-^2) = c \\
\mathrm{str}(\mathcal{J}_-^2(\lambda)\mathcal{J}_+^2(\lambda)) &= \mathrm{str}(J_-^2 J_+^2) = c \\
\mathrm{str}(\mathcal{J}_-^2(\lambda)\mathcal{J}_-^2(\lambda)) &= \lambda^{-4}\mathrm{str}(J_-^2 J_-^2) = 0
\end{aligned} \tag{34}$$

So it is proven that the currents also satisfy the Virasoro constraint. Hence $G(\lambda)$ corresponding to $\mathcal{J}_\pm(\lambda)$ are the new classical solutions with the spectral parameter. When $\lambda = 1$, we recover the existing solution. That is to say, the series of new classical solutions $G(\lambda)$ can thus be generated from the existing one G.

In summary, it has been shown that, for the hybrid superstring on $AdS_2 \times S^2$, the currents with the spectral parameter satisfies not only the Maurer-Cartan equations but also the equations of motion, as well as the Virasoro constraint. Hence, the corresponding Lax connections can yield the new classical solutions with the spectral parameter.

5 Discussions

Now, the Lax connections $\mathcal{J}_\pm(\lambda, \mu)$ with the spectral parameter μ are constructed directly from the new solutions $G(\lambda)$:

$$\begin{aligned}
\mathcal{J}_+(\lambda, \mu) &= J_+^0 + \mu^2\lambda^2 J_+^2 + \mu\lambda J_+^1 + \mu^3\lambda^3 J_+^3 \\
\mathcal{J}_-(\lambda, \mu) &= J_-^0 + \mu^{-2}\lambda^{-2}J_-^2 + \mu^{-3}\lambda^{-3}J_-^1 + \mu^{-1}\lambda^{-1}J_-^3
\end{aligned} \tag{35}$$

Here, we make $\mu\lambda \to \lambda$, it is obviously that Lax connections (36) from the new solutions $G(\lambda)$ are consistent with them (28) from the original solution G. Therefore, the series of classical solutions have the same infinite set of classically conserved charges, undoubtedly.

It is interesting to note that the Lax connections of the hybrid superstring on $AdS_3 \times S^3$ which take the similar form with the hybrid superstring on $AdS_2 \times S^2$; furthermore, the

Lax connections can also yield the classical solutions with the spectral parameter. However, for the pure spinor superstring on $AdS_5 \times S^5$, because of the existence of the ghost currents $N_{\pm}$, the currents with the spectral parameter don't satisfy the equations of motion.

For this hybrid superstring, we have also checked that the $G(\lambda)$ and the original G give the same action S. However, we don't know the physical implications about this yet.

References

1. J. M. Maldacena, Adv. Theor. Math. Phys., 1998, 2: 231
2. J. M. Maldacena, Int. J. Theor. Phys., 1999, 38: 1113, arXiv:hep-th/9711200
3. B. Chen, Y. L. He, P. Zhang, and X. C. Song, Phys. Rev. D, 2005, 71: 086007, arXiv:hep-th/0503089
4. C. A. S. Young, Phys. Lett. B, 2006, 632: 559, arXiv:hep-th/0503008
5. I. Adam, A. Dekel, L. Mazzucato, and Y. Oz, Journal of High Energy Physics, 2007, 06: 085, arXiv:hep-th/0702083
6. X. H. Wang, Z. Y. Wang, X. L. Cai, P. Song, B. Y. Hou, and K. J. Shi, Commun. Theor. Phys., 2006, 45: 663
7. X. N. Xie and R. H. Yue, Commun. Theor. Phys., 2007, 48: 680
8. G. Mandal, N. V. Suryanarayana, and S. R. Wadia, Phys. Lett. B, 2002, 543: 81, hep-th/0206103
9. R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B, 1998, 533: 109, arXiv:hep-th/9805028
10. I. Bena, J. Polchinski, and R. Roiban, Phys. Rev. D, 2004, 69: 046002, arXiv:hep-th/0305116
11. B. Y. Hou, D. T. Peng, C. H. Xiong, and R. H. Yue, 2004, arXiv:hep-th/0406239
12. N. Berkovits, Journal of High Energy Physics, 2000, 0004: 018, arXiv:hep-th/0001035
13. N. Berkovits, M. Bershadsky, T. Hauer, S. Zhukov, and B. Zwiebach, Nucl. Phys. B, 2000, 567: 61, arXiv:hep-th/9907200
14. N. Berkovits, Class. Quant. Grav., 2000, 17: 971, arXiv:hep-th/9910251
15. N. Berkovits, C. Vafa, and E. Witten, Journal of High Energy Physics, 1999, 9903: 018, arXiv:hep-th/9902098
16. N. Berkovits, Nucl. Phys. B, 2000, 565: 333, arXiv:hep-th/9908041
17. B. C. Vallilo, Journal of High Energy Physics, 2004, 0403: 037, arXiv:hep-th/0307018
18. N. Berkovits, Journal of High Energy Physics, 2005, 0502: 060, arXiv:hep-th/0409159
19. N. Berkovits, Journal of High Energy Physics, 2005, 0503: 041, arXiv:hep-th/0411170
20. S. Frolov, Journal of High Energy Physics, 2005, 0505: 069, arXiv:hep-th/0503201
21. T. McLoughlina and I. Swanson, Journal of High Energy Physics, 2006, 0608: 084, arXiv:hep-th/0605018
22. L. F. Alday, G. Arutyunov, and S. Frolov, Journal of High Energy Physics, 2006, 0606: 018, arXiv:hep-th/0512253
23. P. A. Grassi and J. Kluson, arXiv:hep-th/0611151
24. L. X. Zhang, S. Wu, X. N. Xie, C. H. Xiong, R. H. Yue, and K. J. Shi, Chin. Phys. Lett., 2007, 24: 3092
25. J. G. Zhou, Nucl. Phys. B, 1999, 559: 92, arXiv:hep-th/9906013