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RESEARCH ARTICLE

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What is the highest T_c for phonon-mediated superconductivity?

Abstract We suggest that the high-temperature superconductivity can be attributed to the director-roles of the van Hove singularity between an electron-electron interaction and an electron-phonon interaction. The difference between the critical temperature and the pairing temperature is presented, and the Fermi arc, the *d*-wave symmetry and the poor conductivity, etc., are discussed. In particular, the non-*s*-wave symmetry is predicted to have the highest T_c for superconductors.

Keywords superconductivity, phonon, van Hove singularity, Fermi surface

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1 Introduction

Many theories have been presented to enlighten us since high temperature superconductors were discovered two decades ago. However, the origin of the high-T_c has been a highly debated topic. This is because high temperature superconductors have many unusual properties when compared to the traditional superconductors that can be understood within the framework of the BCS theory. With the "director-roles" of the van Hove singularity on cuprate superconductors, we can understand most properties of the cuprate superconductors. For example, the explanations of the *T*-linear resistivity in very wide temperature sections, the predominantly *d*-wave symmetry, and the abnormal isotope effects, etc., have been explained in a paper at arXiv.org cond-mat\0609339. The key to understanding this topic may be in clarifying how the electron-electron interaction and the electron-phonon interaction work in superconductors. In this paper, we will discuss these topics as it relates directly to superconductivity, such as critical temperature, pairing temperature, Fermi arc, *d*-wave symmetry, and poor conductivity, etc. On these problems we further announce the origin of high-T_c superconductivity.

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2 Condensation and critical temperature

The superconducting phenomena appears as the temperature of materials go below the critical temperature under which some carriers are formed into the so-called pairs. Whatever the mediated excitations are, the pairs should have two types: the (nearly) localized pairs and the (nearly) free pairs. When the pairs are so free that each pair looks like a free-boson, the critical temperature can be determined by the well-known Bose-Einstein condensation temperature formula:

$$T_{\text{con.}} = \frac{\pi}{5.224^{2/3}} \frac{\hbar^2}{m_{\text{carr.}} k_{\text{B}}} x_{\text{pair}}^{2/3} \tag{1}$$

where x_{pair} is the number density of mobile pairs, $m_{\text{carr.}}$ the mass of each carrier. Because the carrier mass is larger than the electron mass, $m_{\text{carr.}} \geqslant m_{\text{e}}$, each atom radius $r \sim 10^{-9}$, all carriers are not paired, so $x_{\text{pair}}^{2/3} \ll 10^{18}$, hence we estimate $T_{\text{con.}} \ll 10^4$ K. Usually $T_{\text{cri.}} < T_{\text{con.}}$, that is, the highest critical temperature is in the room-temperature range. Even if there are existing free pairs, T_c the critical temperature might be zero as soon as $x_{\text{pair}}^{2/3} \ll 10^{14}$. Hence, one of our conclusions is that, enough number density of pairs is needed for the superconducting phenomena to occur.

3 Pairing and critical temperature

On the other hand, authors usually mix the pairing temperature with the critical temperature. The pairs can be formed at higher temperature, but the superconducting phenomenon may not appear. Superconductivity occurs when the pairs are not localized and the number density of pairs is large enough. If the pairing temperature is expressed as T^*, it is easy to value $T_{\text{cri.}} \ll T^*$ for nearly localized pairs, so $T_{\text{cri.}} < T^*$ for any pairs. What are the factors that decide the pairing temperature? Although the BCS-like gap equation may not be exact for high temperature superconductors, the problem about the pairing temperature may still be understood:

$$\Delta(\boldsymbol{k}) = -\sum_{\boldsymbol{q}} V(\boldsymbol{k}-\boldsymbol{q}) \frac{\Delta(\boldsymbol{q})}{2E(\boldsymbol{q})} \tanh \frac{E(\boldsymbol{q})}{2k_{\text{B}}T} \quad (2)$$

where $E(\boldsymbol{q})$ are the excitation energies relative to the Fermi surface, and $V(\boldsymbol{q})$ is the affective carrier-carrier interaction. If $T \to T^*$, the $E(\boldsymbol{q})$ does not depend on $\Delta(\boldsymbol{q})$. On expression (2) it is not difficult to find that the pairing temperature depends on two factors: the energy state density and the affective attraction around the Fermi surface. Their generalization can be shown by a series of successes on explaining the properties of cuprates. In a word, the critical temperature is lower than the Bose-Einstein condensation temperature and pairing temperature: $T_{\text{cri.}} < T_{\text{con.}}$, and $T_{\text{cri.}} < T^*$.

For simplification, we will explain the related problems with superconductivity below, and breakdown the Fermi surface into four segments: (carrier-carrier) attraction but non-singularity segment, non-attraction but (van Hove) singularity segment, both attraction and singularity segment, and neither attraction nor singularity segment. It will be shown that the changes of the attraction segment and/or singularity segment dominate the physics of the cuprate superconductors below.

4 *d*-wave symmetry

The predominant *d*-wave symmetry [1, 2] of a cuprate superconductor was shown in the experiments. Some authors attributed the *d*-wave symmetry to the strong correlation of electron systems. In fact, the *d*-wave symmetry shown in Fig. 2 can be simply understood on Fig. 1: when both the singularity and the attraction appear around the Fermi surface shown as the black scales about q_x (q_y) axis in Fig. 1, the *d*-wave symmetry occurs for high-T_c superconductivity or high-T^* localized pairs. This can be understood on expression (2) with Fig. 1. The gaps about k_x axis in Fig. 2 are from the attraction segments about the q_y axis in Fig. 1, and the gaps about k_y axis in Fig. 2 are from the attraction segments about the q_x axis in Fig. 1. In particular, as soon as the singularity segment or the attraction segment deviates from the Fermi surface, the *s*-wave symmetry will appear. Moreover, if the whole Fermi surface (curve) is the so-called attraction segment in the wave-vector space, the *s*-wave symmetry will dominate the superconductivity, and this may explain why the *s*-wave symmetry appears at overdoped cuprates, where the singularity segments may have deviated from the Fermi surface, or the singularity segments occupy most of the Fermi surface. At the same time, based on the same reason given above we make two predictions. First, the superconductivity of all 3D materials usually have *s*-wave symmetry; second, the non-*s*-wave symmetry has the highest T_c for superconductors. The latter will be explained below. The high-T_c needs the van Hove singularity, the van Hove singularity occurs at a wave-vector space meeting $\nabla_{\boldsymbol{k}} E(\boldsymbol{k}) = 0$ for the electron systems, but $\nabla_{\boldsymbol{k}} E(\boldsymbol{k}) = 0$ at most appear only at few segments of the Fermi surface, in the case of which non-*s*-wave symmetry appears.

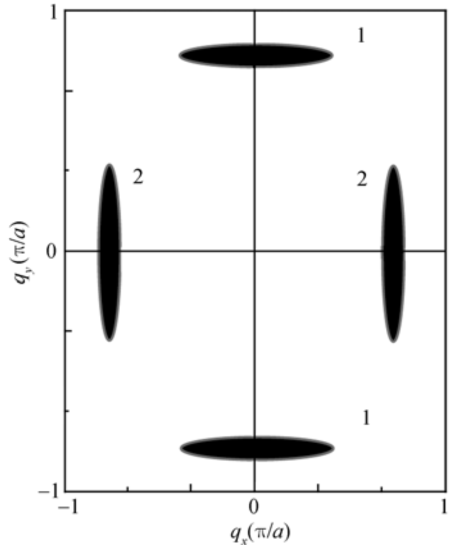

Fig. 1 Segments of the attraction or the van Hove singularity around Fermi surface about q_x (q_y) axis.

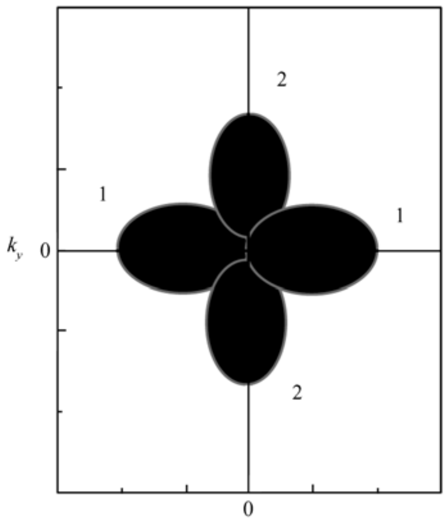

Fig. 2 *d*-Wave pairing gap where k_x (k_y) are wave vectors related to the Fermi surface.

5 Fermi arc

The Fermi arcs [4—7] observed in experiments, are in the nodal segments as shown in Fig. 3. The Fermi arcs should be in the so-called non-attraction but singularity segment, where the carriers behave with metallic properties from the lightly doped- to the optimally doped-region. We suggest that changes for strength and length of arcs are related to the carriers unpaired in the ***q***-space, and are controlled by the changes of locations of both the attraction segments and the singularity segments in ***q***-space. The length of the arcs usually becomes short with doped-holes, the *s*-wave symmetry appears for pairing gaps as discussed above. Since nodal metals are found, why are the cuprate materials the poor conductors in these regions?

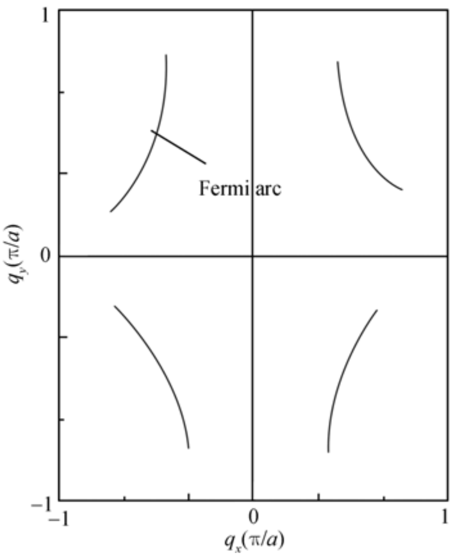

Fig. 3 Possible Fermi arcs are shown as the curve lines in diagonal directions in ***q***-space. It should be compared with Fig. 1.

6 Poor conductor

All cuprate compounds appear as the poor conductors from the low doped- to the optimally doped-region, and many authors attribute it to having fewer carriers in these materials. We present that even if there are fewer carriers, as soon as the carriers are closely concentrated around the Fermi surface duo to the van Hove singularity of the electron states, the material might still be "metal". An electron system behaves with metallic properties, because low energy exchanges between electrons dominate their properties. The van Hove singularity around the Fermi surface leads the carriers of cuprates to be so. This also explains why the carriers in the Fermi arcs behave with metallic properties. "Poor conductor" it is not because the total carriers introduced are few, but there are two causes. First, some of these carriers are localized through the forming of the so-called localized pairs (there are two types of pairs as discussed above) about q_x (q_y) axis. That is to say, the number of carriers in a CuO_2 plane is usually small than the number of holes introduced by doping when the localized pairs exist. Second, when the localized pairs are separated, the carriers about q_x (q_y) axis suffer from the strong scatter of nearly localized particles about q_x (q_y) axis.

One may find that main properties directly related to high temperature superconductivity are explained in this paper on a new phonon mechanism.

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