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RESEARCH ARTICLE

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Electronic double refraction due to the Rashba effect: Analytical and numerical results

Abstract By analogy with the classic effect of the double refraction of light, we investigate the relevant effect of an electron entering from the Non-Rashba region to the Rashba region in two-dimensional systems. It is shown that the effect of electronic double refraction is determined by a combined parameter $\gamma = m^{*}\lambda_{\mathrm{F}}\,\alpha/2\pi\hbar^{2}$, rather than both the Rashba coefficient α and wavelength λ_{F} of a Fermi electron, separately. For the case of normal incidence, the analytical expressions for the wavefunction of the electron are presented; it is predicted that the Rashba spin-orbit coupling can induce a current perpendicular to the normal incident direction of the electron. Moreover, the general case of incident electron with any given momentum and spin state are studied numerically in detail, including the abrupt changes of spin direction and the two-step characters for reflection.

Keywords spin-orbit coupling interaction, spin polarized current, electronic double refraction

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1 Introduction

Since Datta and Das proposed, as a spintronic analogy of electro-optical modulators, the original idea of spin-field-effect transistor (s-FET) in 1990 [1], it has been widely believed that the Rashba effect [2, 3] may play an important role in producing and controlling the spin-polarized currents in nonmagnetic semiconductor devices. This belief had inspired not only the revival of interest about the physics of spin-orbit interaction in solids [4 – 14], but also the designs of device setups based upon the Rashba spin-precession [15–26], although none of them has been realized in the laboratory because of the difficulty of spin-polarized injection from ferromagnetic contacts to semiconductors [27–29].

Recently, an alternative way to generate spin-polarized currents in heterostructures by means of the spin-double refraction effect is explored [30–35]. It is suggested that a lateral interface connecting two regions with different strength of Rashba spin-orbit coupling interaction can be used as a spin polarizer for electrons in two-dimensional semiconductor heterostructures, and the incident wave of the electron can be split as two beams of electronic wave, each of which is completed spin polarized. The transmission and reflection waves strongly depend on the initial states of electrons, and can be manipulated by the strength of Rashba coupling.

In this work, we carry out, compared with the well-known effect of the double refraction of light [36], some investigations on the phenomena of electronic double refraction occurring at an interface between the Non-Rashba region and Rashba region of two dimensional electron gas. Section 2 is devoted to describe our model system and present some basic formulae. In Section 3, we present some results about the characters of electronic double refraction by analytical investigation and numerical simulations. Some further discussions and conclusions are presented in Section 4. For completeness, an appendix is added to discuss the problem of cross-term currents in the Rashba region.

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2 Model and formalism

We consider a two-dimensional system that contains an interface between the Non-Rashba region and Rashba region with spin-orbit coupling interaction, as depicted in Fig. 1. The Rashba effect describes the influence of a large local electric field arising from the structure inversion asymmetry (SIA), which can also be modulated by the applied voltage across the heterostructure [4, 5, 7]. Under the approximation of effective mass, the Hamiltonian of an electron moving in such a system is always taken as a simple form [9], i.e.

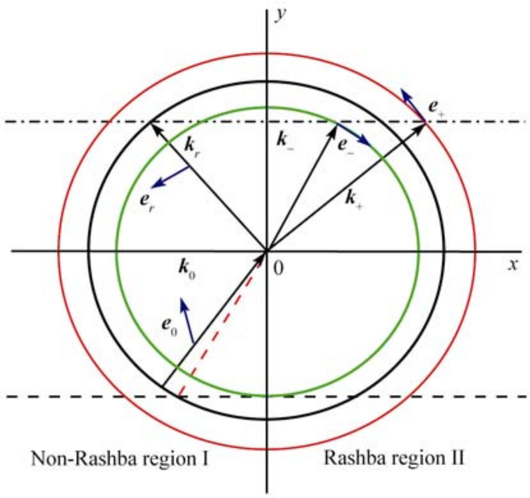

Fig. 1 (*Color online*) Schematic illustration of electronic double refraction. An electron with given momentum $\boldsymbol{k}_0$ and spin $|\boldsymbol{e}_0\rangle$ is reflected and doubly refracted at the interface $x = 0$ between the Non-Rashba region and the Rashba region.

$$\mathcal{H} = \frac{1}{2m^*}(\hat{p}_x^2 + \hat{p}_y^2) + \frac{\alpha}{\hbar}(\sigma_x \hat{p}_y - \sigma_y \hat{p}_x) \tag{1}$$

here $\hat{p}_x, \hat{p}_y$ and m^* represent the momentum operator and effective mass of the electron, respectively; $\hbar = h/2\pi$ denotes the Planck's constant. The symbol α stands for the Rashba coefficient measuring the strength of Rashba effect, which is assumed to be zero in the Non-Rashba region of $x <$ 0 and a positive constant in the Rashba region of $x > 0$; σ_x, σ_y and σ_z are the well-known Pauli matrices.

The Schrödinger equation for an electron described by the Hamiltonian of Eq. (1) can be written in a scaleless form as:

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = \left[\frac{\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}}{2} - \alpha' \boldsymbol{\sigma} \cdot (\boldsymbol{n} \times \boldsymbol{k})\right]\Psi \tag{2}$$

where $\hat{\boldsymbol{k}} \equiv -\mathrm{i}\nabla$, $\alpha' = m^* \ell \alpha / \hbar^2$, $\boldsymbol{\sigma}$ is the vector of Pauli matrices and $\boldsymbol{n}$ denotes the unit vector of the positive direction of z axis. The parameter ℓ, which is defined as the unit of length, can be chosen as any characteristic length of the system. Because only the electrons at the Fermi surface are taken into account in this work, we define the wavelength of Fermi electrons as length unit, i.e., $\ell = \lambda_\mathrm{F}$. The units of other relevant quantities, such as energy, momentum, wave vector and time should be $\tilde{E} = \hbar^2 / m^* \lambda_\mathrm{F}^2$, $\tilde{p} = \hbar / \lambda_\mathrm{F}$, $\tilde{k} = 1/\lambda_\mathrm{F}$, and $\tilde{T} = \hbar / \tilde{E}$, respectively. In any uniform region with constant Rashba coefficient, an explicit expression for the density of probability current can be derived from Eq. (2), which is

$$\boldsymbol{J} = \frac{\mathrm{i}}{2}(\nabla \Psi^\dagger)\Psi - \frac{\mathrm{i}}{2}\Psi^\dagger(\nabla \Psi) + \alpha' \, \Psi^\dagger (\boldsymbol{n} \times \boldsymbol{\sigma})\Psi \tag{3}$$

For an incident electron with any given momentum $\boldsymbol{k}_0 = (k_{0x}, k_{0y})$ and spin state $|\boldsymbol{e}_0\rangle$, the energy $\omega = k_0^2$ and momentum component k_{0y} of the electron are conserved, the wavefunction of eigenstates can be assumed a general form as $\Psi = \mathrm{e}^{\mathrm{i}(k_{0y}y - \omega t)}\psi$. The wavefunction of the electron in the Non-Rashba region can always take the form of

$$\psi_\mathrm{I} = |\boldsymbol{e}_0\rangle \mathrm{e}^{\mathrm{i}k_{0x}x} + r|\boldsymbol{e}_r\rangle \mathrm{e}^{-\mathrm{i}k_{0x}x}, \quad \text{for } x < 0 \tag{4}$$

here $|\boldsymbol{e}_r\rangle$ describes the spin-state of reflected wave, and r denotes the reflection coefficient.

In a homogeneous region with constant Rashba coefficient, the eigenvalues of momentum $\boldsymbol{k} = (k_x, k_{0y})$ for state $\psi = \mathrm{e}^{\mathrm{i}k_x x}|\boldsymbol{e}\rangle$ can be drawn from the time-independent Schödinger equation $[k^2 - 2\alpha' \boldsymbol{\sigma} \cdot (\boldsymbol{n} \times \boldsymbol{k})]|\boldsymbol{e}\rangle = k_0^2|\boldsymbol{e}\rangle$, which yields two eigenvalues of k, i.e., $k_\pm^2 - k_0^2 = \pm 2\alpha' k_\pm$, and $k_\pm = (k_0^2 + \alpha'^2)^{1/2} \pm \alpha'$. In the presentation of σ_z, the corresponding spinors $|\boldsymbol{e}_\pm\rangle$ can be generally written out as:

$$|\boldsymbol{e}_\pm\rangle = \frac{1}{\mathcal{N}_\pm}\begin{pmatrix} \mp(k_{0y} + \mathrm{i}k_{\pm x}) \\ k_\pm \end{pmatrix} \tag{5}$$

here $k_{\pm x} = (k_\pm^2 - k_{0y}^2)^{1/2}$ and $\mathcal{N}_\pm$ denote the x components of wave-vectors of the two refracted waves and normalized constants, respectively. The other two modes with $k_{\pm x} = (k_\pm^2 - k_{0y}^2)^{1/2}$ are ignored because no reflected wave exists in the right region for the obvious reasons of physics. Then the function for diffracted waves can be assumed as:

$$\psi_\mathrm{II} = t_+|\boldsymbol{e}_+\rangle \mathrm{e}^{\mathrm{i}k_{+x}x} + t_-|\boldsymbol{e}_-\rangle \mathrm{e}^{\mathrm{i}k_{-x}x}, \quad \text{for } x > 0 \tag{6}$$

Noting that $k_{+x} = (k_+^2 - k_{0y}^2)^{1/2}$ stays real, the expression k_+ in Eq. (5) can be simplified as:

$$|\boldsymbol{e}_+\rangle = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}(\phi_+ + \pi/2)} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix} \tag{7}$$

here ϕ_+ denotes the refraction angle of k_+ wave, and $\cos \phi_+ = k_{+x}/k_+$. In contrast, there is a critical angle $\phi_c = \arcsin(k_-/k_0)$ for the k_- mode. If $-k_- \leqslant k_{0y} \leqslant k_-$, the incident angle is not greater than the critical value, then the relevant spin state is similar to that of k_+ mode, i.e.,

$$|\boldsymbol{e}_-\rangle = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}(\phi_- - \pi/2)} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix} \tag{8}$$

and ϕ_- denotes the refraction angle of k_- wave, and $\cos\phi_- = k_{-x}/k_-$. Otherwise $k_{-x} = \mathrm{i}\tilde{k}_{-x}$ and $\tilde{k}_{-x} \equiv (k_{0y}^2 - k_-^2)^{1/2}$, which means that the k_- mode becomes an evanescent wave, and the corresponding refraction angle is defined as $\phi_- = \pi/2$. The spin state defined in Eq. (8) should be modified as:

$$|\boldsymbol{e}_-\rangle = \frac{1}{\mathcal{N}} \begin{pmatrix} k_{0y} - \tilde{k}_{-x} \\ k_- \end{pmatrix} \tag{9}$$

with normalization constant $\mathcal{N} = [2k_{0y}(k_{0y} - \tilde{k}_{-x})]^{1/2}$. The spin directions $\boldsymbol{e}_+$ and $\boldsymbol{e}_-$ of double refracted waves are always perpendicular to the direction of the momentum $\boldsymbol{k}_\pm = (k_{\pm x}, k_{0y})$, i.e., $\varphi_+ = \phi_+ + \pi/2$ and $\varphi_- = \phi_- + \pi/2$, as shown in Fig.1.

According to the formula of Eq. (3), the density of probability current of the incident, reflected, and refracted waves defined in Eqs. (4), (6) can be found as $\boldsymbol{J}_0 = \boldsymbol{k}_0$, $\boldsymbol{J}_r = |r|^2 \boldsymbol{k}_r$, and $\boldsymbol{J}_\pm = |t_\pm|^2 (1 \mp \alpha'/k_\pm)\boldsymbol{k}_\pm$ (if k_{-x} becomes a pure imaginary number, then $J_{-x} = 0$). Therefore, the reflecting coefficient $\mathcal{R} \equiv J_{rx}/J_{0x}$ and the two refracting coefficients $\mathcal{T}_\pm \equiv J_{\pm x}/J_{0x}$ can be expressed as

$$\mathcal{R} = |r|^2, \quad \mathcal{T}_\pm = (1+\gamma^2)^{1/2} |t_\pm|^2 \frac{\cos\phi_\pm}{\cos\phi_0} \tag{10}$$

here we introduce a new parameter $\gamma \equiv a'/k_0$. In appendix A, we will deal with the problem of cross-term currents between the incident wave and reflected wave, and between the two refracted waves. It will be shown exactly that $\mathcal{R} + \mathcal{T}_+ + \mathcal{T}_- = 1$.

To solve the Schrödinger equation completely, we still need another two boundary conditions:

$$\Psi|_{0^+} = \Psi|_{0^-}, \quad \left.\frac{\partial \Psi}{\partial x}\right|_{0^+} = \left.\frac{\partial \Psi}{\partial x}\right|_{0^-} + \mathrm{i}\alpha' \sigma_y \Psi|_0 \tag{11}$$

The latter can guarantee the conservation of probability current [9]. By inserting the wavefunctions of Eqs. (4), (6) into Eq. (11), we can derive a group of equations to find the amplitudes t_+, t_- and the reflected wave as below:

$$\left(\frac{n_+ k_{+x}}{k_+} + \cos\phi_0 - \gamma\sigma_y\right) t_+ |\boldsymbol{e}_+\rangle + \left(\frac{n_- k_{-x}}{k_-} + \cos\phi_0 - \gamma\sigma_y\right) t_- |\boldsymbol{e}_-\rangle = 2\cos\phi_0 |\boldsymbol{e}_0\rangle \tag{12}$$

$$r|\boldsymbol{e}_r\rangle = t_+|\boldsymbol{e}_+\rangle + t_-|\boldsymbol{e}_-\rangle - |\boldsymbol{e}_0\rangle \tag{13}$$

where $n_\pm \equiv k_\pm/k_0 = (1+\gamma^2)^{1/2} \pm \gamma$, φ_0 and θ_0 denote the spin direction of the incident electron. If k_{-x} is real, then $k_{\pm x}/k_\pm = \cos\phi_\pm$, and $n_\pm \sin\phi_\pm = \sin\phi_0$, which means that the Snell's law of refraction holds for double refraction, and that the parameters n_+ and n_- play the role of refraction index for the electron.

It is worthwhile to emphasize, based upon the discussion above, that the effect of electronic double refraction is determined by the parameter $\gamma = m^* \lambda_{\mathrm{F}} \alpha / 2\pi\hbar^2$, a combination of the Rashba coefficient α and wavelength λ_{F} of the incident electron, rather than both of them separately.

3 Reflection and double refraction

A beam of light passing through some uniaxial crystals such as calcite or quartz, is split into two rays. The two waves are each plane polarized, and travel with different wavelengths, or have different indices of refraction [36].

Similarly, the matter wave of an electron entering the Rashba region from the Non-Rashba region can also be doubly refracted; one is the k_+ mode and the other is k_- mode. They are each spin polarized and described by different refraction indices n_+ and n_-.

The quantity of double refraction is depicted by the difference of two refraction indices. For a calcite crystal, the two indices of ordinary light and extraordinary light are n_o = 1.66 and n_e = 1.49, respectively; and then Δn = 0.17.

When taken into account, a typical semiconductor system, such as InAs system [10], the relevant parameters are chosen as typical values [26] α = 0.6 eV·Å, E_{F} = 99.54 meV, $m^* = 0.05\, m_0$, then $\lambda_{\mathrm{F}} = 174$ Å, $k_{\mathrm{F}} = 3.62\times10^{-2}$ Å, α' = 0.683 and $\gamma \approx 0.1$. The difference of refraction indices $\Delta n \equiv n_+ - n_- \approx 0.20$, which should be large enough for the occurrence of double refraction.

In principle, all information about the double refraction of an electron entering the Rashba region, as depicted in Fig.1, can be dug out from Eqs. (12) and (13). The next two subsections are devoted to study some typical cases with different incident angles.

3.1 Case of normal incidence

For the case of normal incidence, i.e., k_{0y} = 0, and $\boldsymbol{e}_+ = \boldsymbol{j}$, $\boldsymbol{e}_- = -\boldsymbol{j}$, the problem can be solved analytically. The reflection wave

$$r|\boldsymbol{e}_r\rangle = -\frac{(1+\gamma^2)^{1/2} - 1}{(1+\gamma^2)^{1/2} + 1} |\boldsymbol{e}_0\rangle \tag{14}$$

undergoes a phase shift of π radian, but its spin state doesn't change. The transmission wave

$$\psi_{\rm II} = \frac{2}{(1+\gamma^2)^{1/2}+1}|\chi_{\rm II}\rangle \tag{15}$$

with $|\chi_{\rm II}\rangle = {\rm e}^{{\rm i}k_+x}\langle \boldsymbol{j}|\boldsymbol{e}_0\rangle|\boldsymbol{j}\rangle + {\rm e}^{{\rm i}k_-x}\langle -\boldsymbol{j}|\boldsymbol{e}_0\rangle|-\boldsymbol{j}\rangle$ behaves as a spin precession around the $-y$ axes with a space period $L_x = \ell\pi/\alpha' = 2m'\alpha'/\hbar^2$.

According to Eq. (10), the reflection and refraction coefficients can be calculated analytically, i.e.,

$$\mathcal{R} = \left[\frac{(1+\gamma^2)^{1/2}-1}{(1+\gamma^2)^{1/2}+1}\right]^2 \tag{16}$$

$$\mathcal{T}_\pm = \frac{2(1+\gamma^2)^{1/2}}{[(1+\gamma^2)^{1/2}+1]^2}(1\pm\sin\theta_0\sin\varphi_0) \tag{17}$$

the reflection is independent of the spin state of the incident electron, but both of the two refraction coefficients $\mathcal{T}_+$ and $\mathcal{T}_-$ depend on it.

In addition, the spin-orbit interaction can induce a current perpendicular to the normal incident direction of the electron. The y component of current

$$J_y = J_{y0}\sin(2\alpha' x + \delta) \tag{18}$$

oscillates periodically in the Rashba region with amplitude

$$J_{y0} = \alpha'(1-\sin\theta_0^2\sin\varphi_0^2)^{1/2}\left[\frac{2}{1+(1+\gamma^2)^{1/2}}\right]^2 \tag{19}$$

and phase shift

$$\delta = \arcsin\left[\frac{\cos\varphi_0\sin\theta_0}{(1-\sin\theta_0^2\sin\varphi_0^2)^{1/2}}\right] \tag{20}$$

We predict that this kind of persistent current of charge can be observed in experiment.

3.2 Case of oblique incidence

The case of oblique incidence can only be studied numerically. For a practical system of InAs semiconductor [12], we find that $\gamma \approx 0.1$ and $\mathcal{R} \approx 10^{-4}$ following Eq. (10), which means a very weak reflection. To show more explicitly the effect, we should assume larger values of the Rashba constant α and/or lower energy values of $E_{\rm F}$ to reach large enough values of parameter γ, for example $\gamma = 0.2$, 0.5, 0.7, 1.0.

For the transportation of light wave in uniaxial crystals, a plane wave can always be treated as a superposition of a couple of orthogonal polarized plane waves [36], i.e., the *ordinary ray* (o-ray) and the *extraordinary ray* (e-ray). However, the spin eigenstates of an electron transporting in the Rashba region are not orthogonal, the two modes of k_+ and k_- are mixed together by the spin-orbit coupling interaction, and we cannot deal with an incident wave of electron as two independent polarized waves. As a prominent consequence, the spin state of the reflected wave undergoes an abrupt change at the interface $x = 0$ in a complex way on the initial spin states.

The inner production $\boldsymbol{e}_r\cdot\boldsymbol{e}_0$ is adopted to represent the change of spin direction of reflected electron, and the unit vector $\boldsymbol{e}_0$ and $\boldsymbol{e}_r$ of the spin direction of incident and reflected electron can be calculated as $\boldsymbol{e}_0 = \langle \boldsymbol{e}_0|\boldsymbol{\sigma}|\boldsymbol{e}_0\rangle$ and $\boldsymbol{e}_r = \langle \boldsymbol{e}_r|\boldsymbol{\sigma}|\boldsymbol{e}_r\rangle$, respectively. Although the spin direction remains unchanged after reflection for the *normal* incident electron with any spin state, it does alter dramatically for the *oblique* incident electron. For example, we do numerical calculations about the inner production $\boldsymbol{e}_r\cdot\boldsymbol{e}_0$ of an electron with incident angle $\phi_0 = \pi/6$, this function varies with the polar angle θ_0 and azimuth angle φ_0 of the spin direction of the incident electron as shown in Fig. 2. In contrast with $\boldsymbol{e}_r\cdot\boldsymbol{e}_0 \equiv 1$ for a normal incident electron, the function for an oblique incident electron can hold $\boldsymbol{e}_r\cdot\boldsymbol{e}_0 = 1$ only for two special initial spin states.

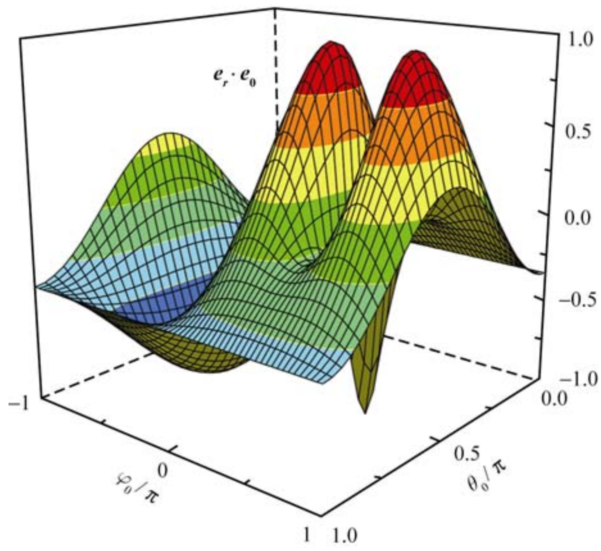

Fig. 2 (*Color online*) The inner production $\boldsymbol{e}_r\cdot\boldsymbol{e}_0$, for the spin direction of the reflected wave with the spin direction of the incident wave varies with the spin states of the incident electron for given incident angle $\phi_0 = \pi/6$ and parameter $\gamma = 0.7$.

To demonstrate more clearly the laws of reflection and refraction, we also calculate the coefficients of reflection and refraction for the incident electron with different incident angles and spin states. The three plots in Fig. 3 show the typical way of the coefficients depending on the initial state of an incident electron for a given incident angle $\phi_0 = \pi/6$ and system parameter $\gamma = 0.7$. Compared with Eq. (16) and Eq. (17), the formulae for normal incidence, the reflection coefficient $\mathcal{R}$ is no longer a constant, while the two

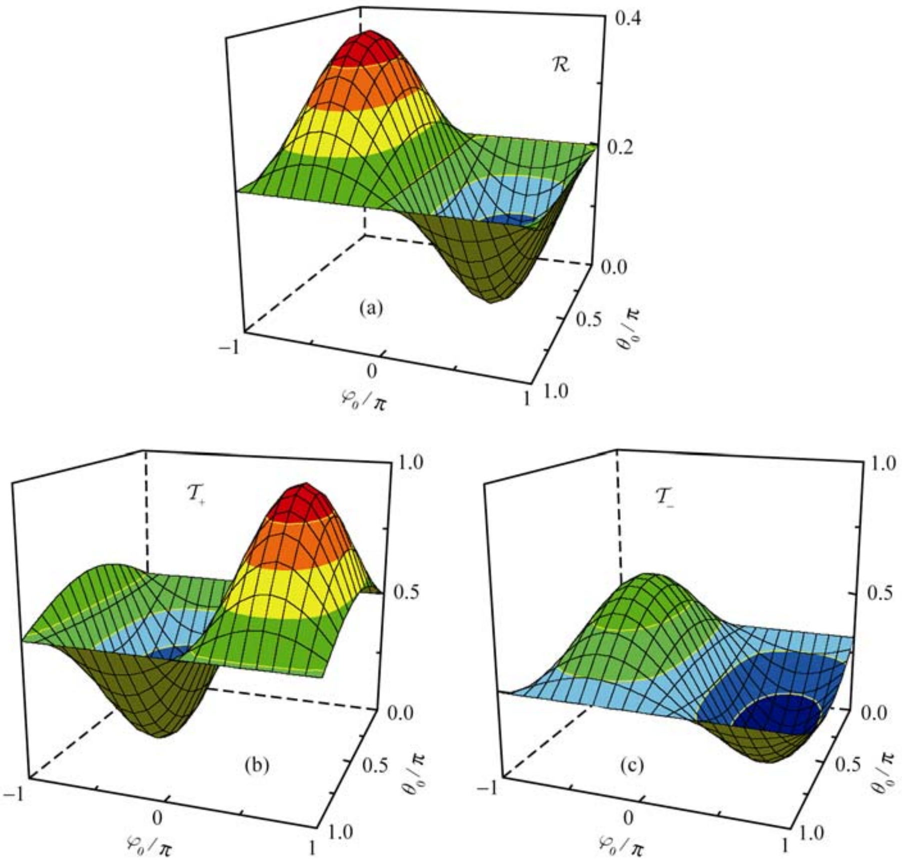

Fig. 3 (*Color online*) The reflection and refraction coefficients $\mathcal{R}$, $\mathcal{T}_+$ and $\mathcal{T}_-$ vary with the spin states of incident electron for $\phi_0 = \pi/6$ and $\gamma = 0.7$.

refraction coefficients $\mathcal{T}_+$ and $\mathcal{T}_-$ vary with θ_0 and φ_0 in a way similar to that described by Eq. (17). We have also confirmed the equality $\mathcal{R} + \mathcal{T}_+ + \mathcal{T}_- = 1$ by calculating the quantity $\mathcal{T}_+ + \mathcal{T}_-$, which compensates the variation of $\mathcal{R}$ exactly.

The refraction coefficients $\langle \mathcal{T}_+ \rangle$, $\langle \mathcal{T}_- \rangle$ and reflection coefficient $\langle \mathcal{R} \rangle$ averaged over all possible spin states with equal probability are some other interested quantities. As shown in Fig. 4, all of them are monotonic functions of the incident angle ϕ_0. Because there exists a critical angle $\phi_c = \arcsin n_-$ for the k_- mode, both of the reflection coefficient $\mathcal{R}$ and refraction coefficient $\mathcal{T}_-$ behave like two-step functions. According to the definition $n_- = (1+\gamma^2)^{1/2} - \gamma$ after Eq. (13), we obtain the corresponding critical angles $\phi_c/\pi = 0.31$, 0.21, 0.17 and 0.14 for different parameters $\gamma = 0.2$, 0.5, 0.7 and 1.0. It can be seen obviously from Fig. 4 that, the total reflection for k_- mode, and the abrupt changes of the curves of $\langle \mathcal{T}_+ \rangle$ and $\langle \mathcal{R} \rangle$, consistently occurs at these incident angles; however, the coefficient $\mathcal{T}_+$ smoothly decreased with the incident angle ϕ_0.

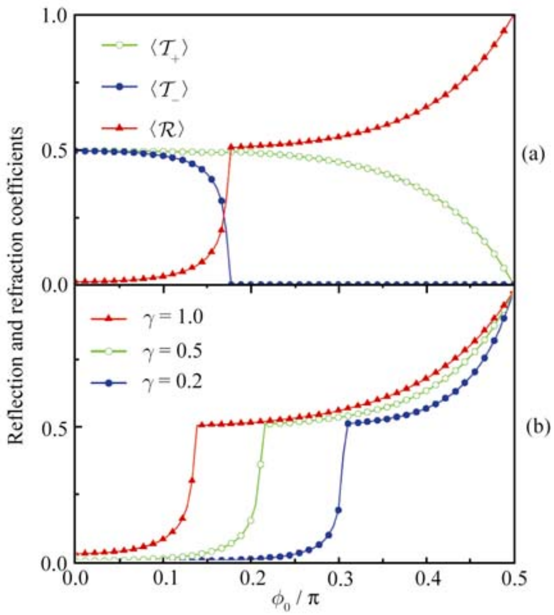

Fig. 4 (*Color online*) The refraction coefficients $\langle \mathcal{T}_+ \rangle$, $\langle \mathcal{T}_- \rangle$ and reflection coefficient $\langle \mathcal{R} \rangle$ averaged over all possible spin states vary with incident angle. **(a)** For given parameter $\gamma = 0.7$; **(b)** For typical values of parameter $\gamma = 0.2$, 0.5, 1.0.

4 Discussions and conclusions

In this work, we systematically study the effect of double refraction of an electron occurring at an intersurface between two half infinite regions, from the Non-Rashba (normal metal) region to the Rashba (semiconductor) region.

Similar to the well-known double refraction effect of light, an incident wave of electron with given momentum and spin state partially enters the Rashba region, and is divided into a couple of completed spin polarized currents. Both of the effects for light and electron share a common origin that, for any given incident wave, the two modes of refraction waves have different refraction indices, i.e., $n_e \neq n_o$ for light and $n_+ > n_-$ for electron. Otherwise, the laws of double refraction for electron due to Rashba spin-orbit coupling interaction re more complicated than that of the light effect. This distinction is simply attributed to the fact that the spin states of k_+ and k_- modes for electron in the Rashba region are not orthogonal, but the o-ray and e-ray in uniaxial crystals are always polarized perpendicularly.

We derive a set of basic equations, i.e., Eqs. (12), (13), to study the electronic double refraction at the single interface between the Non-Rashba region and Rashba region. It is shown explicitly that this effect is determined by a combined parameter $\gamma = m^* \lambda_{\mathrm{F}} \alpha / 2\pi\hbar^2$ instead of two independent parameters α and α_{F}. Therefore, we suggest a new way to enhance the effect of double refraction of electron due to Rashba spin-orbit interaction. Besides the conventional way by increasing the gate voltage [4], we can also control this effect by changing the Fermi energy of the incident electron.

The exact results for the case of normal incidence are derived analytically. It is found that the influence of the interface at $x = 0$ does not alter the spin direction of reflected or refracted electron for any initial spin state. It is predicted that the spin-orbit interaction can induce a current perpendicular to the normal incident direction of the electron.

The numerical calculations in the case of oblique incidence reveal more details about the law of electronic double reflection. We find that for an electron with nonzero incident angle, the spin state of the reflected wave often undergoes an abrupt change at the interface $x = 0$, see Fig. 2. As shown in Fig. 3, the coefficients of reflection and refraction depend on the incident angle and the initial spin state of the electron in a rather complex way. The origin of the two-step character of reflection and one refraction mode, as shown in Fig. 4, is simply attributed to the total reflection of this refraction mode.

The problem of probability current is re-studied in Section 2 and the appendix A. At first, we find that the identity $\mathcal{R} + \mathcal{T}_+ + \mathcal{T}_- = 1$ requires a subtle understanding about the relevant coefficients $\mathcal{T}_+$, $\mathcal{T}_-$ and $\mathcal{R}$; secondly, we prove exactly that the cross term of the current remains zero even taking into account the spin-orbit coupling interaction.

Finally, we expect that the electronic double refraction effect can also be used to design a new kind of spin-filter device similar to the most commonly used Nicol prism for the production of linearly polarized light [36].

Appendix A Cross-term currents

According to Eq. (3), the basic formula for density of probability current, the interference between the incident wave and reflected wave, or between the two refracted waves will produce cross-term currents.

In the Non-Rashba $x < 0$, the cross term can be written as

$$\tilde{\boldsymbol{J}}_{\mathrm{I}} = \frac{1}{2}[(\mathrm{i}\nabla \varPsi_r^+)\varPsi_0 - \varPsi_r^+(\mathrm{i}\nabla \varPsi_0) + c.c.] \tag{A.1}$$

here the symbol $c.c.$ means complex conjugate. By adopting the wavefunction presented in Eq. (4), i.e.,

$$\varPsi_0 = |\boldsymbol{e}_0\rangle \mathrm{e}^{\mathrm{i}(k_{0x}x + k_{0y}y)}, \quad \varPsi_r = r|\boldsymbol{e}_r\rangle \mathrm{e}^{\mathrm{i}(-k_{0x}x + k_{0y}y)} \tag{A.2}$$

we can prove that the y component of this cross term is y independent, and its x component is zero, i.e.,

$$\tilde{\boldsymbol{J}}_{\mathrm{I}y} = k_{0y}(r\langle \boldsymbol{e}_0 | \boldsymbol{e}_r\rangle \mathrm{e}^{-\mathrm{i}2k_{0x}x} + c.c.)\boldsymbol{j}, \quad \tilde{\boldsymbol{J}}_{\mathrm{I}x} = 0 \tag{A.3}$$

In the Rashba region $x > 0$, the expression for cross term

$$\begin{aligned}\tilde{\boldsymbol{J}}_{\mathrm{II}} = &\frac{1}{2}[(\mathrm{i}\nabla \boldsymbol{\varPsi}_r^+)\boldsymbol{\varPsi}_+ - \boldsymbol{\varPsi}_-^+(\mathrm{i}\nabla \boldsymbol{\varPsi}_+) + c.c.]\\ &+ \alpha' [\boldsymbol{\varPsi}_+^+(\boldsymbol{n}\times\boldsymbol{\sigma})\boldsymbol{\varPsi}_- + c.c.]\end{aligned} \tag{A.4}$$

is more complicated. By using the wavefunctions of Eq. (6), i.e.,

$$\varPsi_+ = t_+|\boldsymbol{e}_+\rangle \mathrm{e}^{\mathrm{i}(k_{+x}x + k_{0y}y)} \tag{A.5}$$

$$\varPsi_- = t_-|\boldsymbol{e}_-\rangle \mathrm{e}^{\mathrm{i}(k_{-x}x + k_{0y}y)} \tag{A.6}$$

we can obtain the expressions of the cross-term currents that

$$\tilde{\boldsymbol{J}}_{\mathrm{II}y} = -\mathrm{i}t_+t_-^* \frac{\mathrm{e}^{\mathrm{i}\phi_-} + \mathrm{e}^{-\mathrm{i}\phi_+}}{2}\left(\alpha' + k_{0y}\frac{\sin\dfrac{\phi_- - \phi_+}{2}}{\cos\dfrac{\phi_- + \phi_+}{2}}\right)\mathrm{e}^{\mathrm{i}(k_{+x} - k_{-x})x}\boldsymbol{j} + \boldsymbol{c.c.} \tag{A.7}$$

$$\tilde{\boldsymbol{J}}_{\mathrm{II}x} = t_+t_-^* \frac{\mathrm{e}^{\mathrm{i}\phi_-} - \mathrm{e}^{-\mathrm{i}\phi_+}}{2}\left(\alpha' - \frac{k_{+x} + k_{-x}}{2}\frac{\sin\dfrac{\phi_- - \phi_+}{2}}{\sin\dfrac{\phi_- + \phi_+}{2}}\right)\mathrm{e}^{\mathrm{i}(k_{+x} - k_{-x})x}\boldsymbol{i} + \boldsymbol{c.c.} \tag{A.8}$$

With the help of identity $k_+ - k_- = 2\alpha'$, it is easy to show that the x component of the cross term still stays zero, i.e., $\boldsymbol{J}_{\mathrm{II}x} = 0$, and its y component is y independent even in the Rashba region. This result is consistent with the law of conservation of probability currents. Based on the above results, the formula for the conservation of probability currents can be reduced to the form of x component, $J_{0x} + J_{rx} = J_{+x} + J_{-x}$, i.e., $\mathcal{R} + \mathcal{T}_{+} + \mathcal{T}_{-} = 1$. All our numerical calculations of this paper are checked by this identity.

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