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RESEARCH ARTICLE

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Another model for a multiexcitonic quantum dot in an optical microcavity

Abstract Very recently, a multiexcitonic quantum dot in an optical microcavity have been theoretically studied [Herbert Vincka, Boris A. Rodriguez, and Augusto Gonzalez, Physica E, 2006, 35: 99−102]. However, due to the inevitable damping losses through the microcavity, in this work, we will present a more precise and sound model in the Lindblad form master equation to investigate the photonic properties of a single quantum dot (QD) in an optical microcavity system, in which the QD may confine the multiexcitons and be in resonant interaction with a single photonic mode of an optical microcavity. The excitation energies, and the properties of the emission photon from the QD microcavity are computed as functions of the exciton-photon coupling strength, detuning, and pump rate. We further compare our results with their results, and find that the calculated intensity of the emitted photon and the spectra crucially depend on the exciton-photon coupling strength g, the photon detuning, and the number of excitons in the QD. Finally, we will give a physical mechanism of the dressed-state picture for the strong coupling between the single mode of an optical microcavity and the QD emitters to explain the details of the emission photon spectra. Our study establishes useful guidelines for the experimental study of such multiexcitonic quantum dot in an optical microcavity system.

Keywords quantum dots, microcavity, optical properties, lindblad, exciton

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1 Introduction

In the past few years, semiconductor microcavities containing quantum dots (QDs) or quantum wells have gained considerable interests and have been extensively studied, both for the fundamental research on light-matter interaction [1−7] and for applications, such as low-threshold lasers [8−15] and single-photon sources [10−12, 25]. Strong coupling occurs when the emitter-photon interaction becomes larger than the combined atomic dipole decay rate and the cavity field decay rate. When this condition is fulfilled, the irreversible spontaneous emission process of the emitter is replaced by a coherent periodic energy exchange between the emitter and the photon in the form of Rabi oscillations with a frequency Ω for time scales shorter than the fastest dissipation times of the system, e.g. the photon life time [1, 2, 16]. The regime of strong coupling between a single exciton confined in a quantum dot (QD) and a single photon has been achieved in microcavity systems based on the pillar microcavities [6], in photonic crystal nanocavities [7] and in microdiscs [5, 8]. The strong exciton-photon coupling results in the formation of mixed exciton-photon eigenstates. In the strong coupling regime, the light-matter coupling is stronger so that the spontaneous emission becomes reversible. The photons emitted by the single emitter in an optical microcavity mode are reabsorbed, reemitted, etc., giving rise to the Rabi oscillations. In most of the studies, the cavity quantum electrodynamics (CQED) effects of a low- dimensional electron systems interacting with the optical modes, like the Purcell effect or the Rabi splitting, have been observed [4−12, 16, 21]. The strong-coupling regime for a single GaAs QD inserted in a microdisk cavity has been observed by E. Peter and coworkers [4]. In 2004, two other groups have demonstrated the strong coupling regime for a single quantum dot inserted in an optical microcavity [6, 7]. In Ref. [6], larger oscillator strength $f = 50$ InGaAs natural QD are inserted in a micropillar with a larger effective volume of 0.3 μm^3. In Ref. [7], InAs QDs with small oscillator strength $f = 10$ are inserted in a small effective volume $V = 0.04$ μm^3 photonic band gap microcavity. Furthermore, in Ref. [21], high-Q AlAs/GaAs micropillar cavities with large $In_{0.3}Ga_{0.7}As$ QDs as two-level emitters have been realized based distributed Bragg reflectors. In this system Q-factors of up to 43000 (for micropillars with a diameter of 4 μm) were achieved due to an optimized fabrication technology. The use of large $In_{0.3}Ga_{0.7}$ As quantum dots together with a small mode volume in high finesse micropillar cavities allowed us to overcome the threshold for strong coupling and to observe a vacuum Rabi splitting of 140 μeV due to the coherent interaction of a single QD emitter with a single photon. The control of spontaneous emission with QDs inserted in microcavities has been observed [4–10, 21] and applied to the realization of efficient single photon sources [10–12].

Furthermore, we have proposed and examined theoretically a new QD microcavity model based on a single semiconductor QD gain emitter in an optical microcavity [8], where the lasing occurs through discrete conduction states. The single QD as a two-level emitter is selectively placed in a high quality cavity, called a microdisk, which is resonant with an intersublevel QD transition. The quantitative results have shown that, when adjusting the QD-cavity coupling parameters to be the appropriate values, the optical microcavity coupling mode would lead to a very high intensity of the single QD microcavity laser with a low-threshold.

However, in the present work, we shall focus on the issue of the coupling of a single QD, which may confine multiexcitons (about 10), and a single photonic mode of an optical microcavity, as Vinck and coworkers have studied [22]. And we work out the multiexcitonic effects of a single QD in an optical microcavity, using a fully quantum- mechanical treatment method [8, 11−18]. The QDs can be embedded far apart within the semiconductor microcavity so that we can neglect the interactions, and thus the QD can be simply treated as a two-level system (vacuum-exciton), which is interacting with the cavity field. We assume that interaction between the QD and the single mode of an optical microcavity arises from the resonance of a single mode of the microcavity with the transition between levels, e. g., We took parameters typical of experimentally studied systems, as Herbert Vinck and coworkers have used [22].

2 Model

In the interaction picture, the resonant interaction between the QD and the single mode of an optical microcavity in the dipole approximation and the rotating-wave approximation and is described by the Hamiltonian

$$\hat{H}_I = \sum_i \mathrm{i}\hbar g\left(a^+\sigma_i^- - a\sigma_i^+\right) \tag{1}$$

where the photon creation and annihilation operators for the electromagnetic mode of the optical microcavity are a^+ and a, respectively. The coupling constant g describes the strength of the QD-cavity interaction. The coupling constant of the exciton-photon interaction is thus given by $g = \left(\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{e^2 f}{mV}\right)^{1/2}$ [5–8, 14, 24], where f is the exciton oscillator strength, V the effective modal volume, m the free electron mass, e the electron charge, and $\varepsilon_0\varepsilon_r$ the dielectric constant. The i-th exciton creation and annihilation operators of QD are defined as:

$$\sigma_i^+ = |e\rangle\langle g|, \quad \sigma_i^- = |g\rangle\langle e| \tag{2}$$

For this problem in an interaction picture, furthermore, the resulting Hamiltonian of the present single QD inside a microcavity is given by

$$\hat{H} = \sum_i \hbar\omega_i\sigma_i^+\sigma_i^- + \hbar\omega_c a^+ a + \sum_i \mathrm{i}\hbar g\left(a^+\sigma_i^- - \sigma_i^+ a\right) \tag{3}$$

Here, the energy of the photon mode is written as $E_{\text{gap}} + \hbar\omega$, where E_{gap} is the QD effective band gap, and $\hbar\omega$ is a magnitude of the order of a few meV. g is the pair-photon coupling strength. The Hamiltonian (3) preserves the polariton number

$$N_{\text{pol}} = N_{\text{ph}} + N_{\text{excitons}} \tag{4}$$

where $N_{\text{ph}} = a^+ a$, and $N_{\text{excitons}} = \sum_i \sigma_i^+\sigma_i^-$. Thus, we can obtain the same results as Herbert Vinck and coworkers have obtained [22]. The inclusion of photons is easily understood in the weak coupling regime, $g \to 0$. Both N_{ph} and N_{excitons} are good quantum numbers in this regime [22]. The total energy is [22]:

$$\begin{aligned} E_T &= N_{\text{ph}}\left(E_{\text{gap}} + \hbar\omega\right) + E\left(N_{\text{excitons}}\right) \\ &\approx N_{\text{pol}}\left(E_{\text{gap}} + \hbar\omega\right) + \left(25\text{ meV} - \hbar\omega\right)N_{\text{excitons}} \end{aligned} \tag{5}$$

when $\hbar\omega < 25$ meV (below resonance) the microcavity is filled with only photons and no excitons; when $\hbar\omega >$ 25 meV (above resonance), the number of excitons in the QD is at maximum [22].

Next, we turn to compute the position and the intensity of the photon emission from the microcavity. Due to the inevitable damping losses through the microcavity, we shall use the formulations of fully quantum-mechanical treatment and thus propose the density matrix master equation in the Lindblad form for a single QD inside an optical microcavity [8–12, 14, 17–20] instead to investigate the dynamical properties and the intensity of the emitted light by a single QD with the coupled radiation reservoir of a microcavity included. The dynamics of density matrix is governed by the master equation in the Lindblad form:

$$\frac{\partial}{\partial t}\rho = \frac{1}{\mathrm{i}\hbar}\left[\hat{H},\rho\right] + L_{\mathrm{relax}}\rho + L_{\mathrm{pump}}\rho = L\rho \tag{6}$$

The relaxations and pumping lead to a non-unitary evolution given by:

$$\begin{aligned} L_{\mathrm{relax}}\rho = & -\frac{\gamma}{2}\sum_i\left(\sigma_i^+\sigma_i^-\rho + \rho\sigma_i^+\sigma_i^- - 2\sigma_i^-\rho\sigma_i^+\right) \\ & -\kappa\left(a^+a\rho + \rho a^+a - 2a\rho a^+\right) \end{aligned} \tag{7}$$

$$L_{\mathrm{pump}}\rho = -\frac{\Gamma_{\mathrm{L}}}{2}\sum_i\left(\sigma_i^-\sigma_i^+\rho + \rho\sigma_i^-\sigma_i^+ - 2\sigma_i^+\rho\sigma_i^-\right) \tag{8}$$

where, the photon emissions of the QD with the decay rate γ from the upper level to the lower level as well as the cavity decay with the rate κ are included. We solved the density matrix differential equation (6) using a quantum trajectory algorithm [8, 17–20] to investigate the resonant interaction of the radiation field of a microcavity with the single QD.

We begin by examining the photoemission intensity versus the time evolution. Figure 1 (a)—(b) exhibit photon emission intensity versus the time evolution with various incoherent pump Γ_{L} and a coupling strength g. A plot of the average photon number $\langle n\rangle$ in an optical microcavity versus the incoherent pump Γ_{L} is shown in Fig. 1 with various coupling strength g (g =10 and 500 μeV, respectively). Here, we see that $\langle n\rangle$ increases as the pump is increased, and then declines dramatically as the pump further increases for a wide range of strong coupling strength g. This property behaves similarly to the one-atom laser [8, 9, 17–20]. We also find that it is a manifestation of the pump dependence in the language of microlasers, and a reduction of the spontaneous emission into the lasing mode. The result shows that the photon number in the cavity is always non-zero due to the spontaneous emission, unlike the semi-classical result, in which there is a laser threshold. Besides, the population inversion is always present. The solution of the master equation shows that the net stimulated emission dominates spontaneous emission well above the semi-classical threshold. In Fig. 2 (a)—(c), we have shown that the photon output spectra as an overview for various values of the coupling strength g and the incoherent pump Γ_{L}. Figure 2 (a)—(c) show the positions and intensity of the photon emission, which are in agreement with Vinck and Coworker's explicit computations and their conclusion that only the transition to the ground state of the $N_{\mathrm{pol}}-1$ system gives a significantly nonzero intensity [22]. In Fig. 2 (a), no Rabi-splitting is observed on the resonance. Such behavior is described in terms of the Purcell-effect and is characteristic for the weak coupling region, i.e., $g < \gamma/4$, [1, 14]. However, the enhancement of the emission on the resonance is rather low in this case, which indicates that the threshold for the strong exciton-photon coupling can be almost reached. However, in Fig. 2 (b) and (c), we find that the spectra contain two narrow peaks at $\Delta = \pm g$ with a small incoherent pump for the strong coupling strength g regime. This corresponds to the vacuum Rabi splitting. Two peaks will exist and become more and more pronounced with positions at $\Delta = \pm g\left(\sqrt{2}-1\right)$. Moreover, there are smaller peaks at $\Delta = \pm g\left(\sqrt{2}+1\right)$. It is obvious that the combination of the high Q and the small mode volume can lead to a strong coupling between the microcavity and the QD emitters that can be easily fabricated [5–8]. The origin of the vacuum Rabi splitting and these difference- and sum-frequency peaks can be understood in a dressed-state picture. If there is no QD-cavity coupling, the states $\left|g_i,n\right\rangle$ and $\left|e_i,n-1\right\rangle$ are degenerate. However, with the QD-cavity coupling present, the degeneracy is removed, leading to an energy difference of $2\hbar g\sqrt{n}$ between the dressed states $\left|\pm_i,n\right\rangle = (\left|g_i,n\right\rangle \pm \left|e_i,n-1\right\rangle)/\sqrt{2}$. For a very small pump rate, the mean photon number is smaller than 1. Therefore, only the lowest transitions between the dressed states are possible, namely, $\left|-_i,0\right\rangle \rightarrow \left|g_i,0\right\rangle$ and $\left|+_i,0\right\rangle \rightarrow \left|e_i,0\right\rangle$ with frequencies $\omega_0 \pm g$, xleading to a vacuum Rabi splitting. Furthermore, in order to illustrate the physical mechanism of the peaks at the sum and difference frequencies $\Delta = g\left(\sqrt{n+1} \pm \sqrt{n}\right)$, it is necessary to consider the dressed-state picture of the QD-cavity interaction system. There are four transitions between the states $\left|\pm_i,n\right\rangle$ and $\left|\pm_i,n-1\right\rangle$ with n = 1, which are corresponding to the frequencies with $\omega_0 \pm g\left(\sqrt{2}\pm 1\right)$. An increase of the coupling g increases the mixing between the excitonic and photon modes. The dressed-state picture of the QD-cavity interaction system made above explains the positions of the peaks in the emitted photon spectra, which are agreement with the work by Vinck *et al.* [22]. Furthermore, compared to their work [22], our model here has shown more details on the emitted photon spectra. With a non-vanishing pump rate, the peaks are not only broadened, but also shifted. For larger pump, a Mollow spectrum arises [19] with a dip at the center frequency and the sidebands are pushed to the larger frequencies by increasing incoherent pump, as shown in Fig. 2 (b) and (c) for stronger coupling strength g. If the incoherent pump is further increased, the dip in the Mollow spectrum vanishes and eventually a narrow coherent peak arises on top of the Mollow spectrum. In this region, the photon emission is strongest and this peak dominates the spectrum. This behavior is due to the fact that with increasing pump the number of photons in the microcavity mode increases.

3 Conclusions

In conclusion, due to the inevitable damping losses through the microcavity, in this work we have presented a more precise and sound model of the Lindblad form master equation using a fully quantum-mechanical treatment method to investigate the detailed photon emission properties for a multiexci-

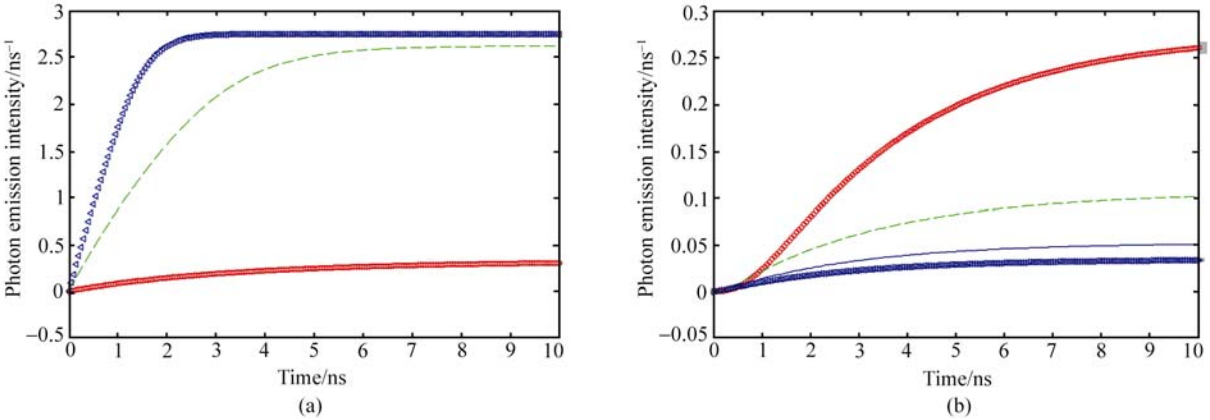

Fig. 1 Photon emission intensity in unit ns^{-1} versus time evolution for different pump and different coupling strength with the parameters κ = 0.2 ns^{-1}; γ = 0.5 ns^{-1}. **(a)** The result is obtained g = 500 μeV in the strong-coupling regime with different incoherent excitation pumps Γ ns^{-1}, Γ=1 ns^{-1} (*circle*), Γ=10 ns^{-1} (*dashed line*) and Γ= 20 ns^{-1}(*triangle*); **(b)** The result is obtained g =10 μeV in the weak-coupling regime with different incoherent excitation pumps Γ, Γ=1 ns^{-1} (*triangle*), Γ= 10 ns^{-1} (*solid line*), Γ= 20 ns^{-1} (*dashed line*), and Γ= 50 ns^{-1} (*circle*).

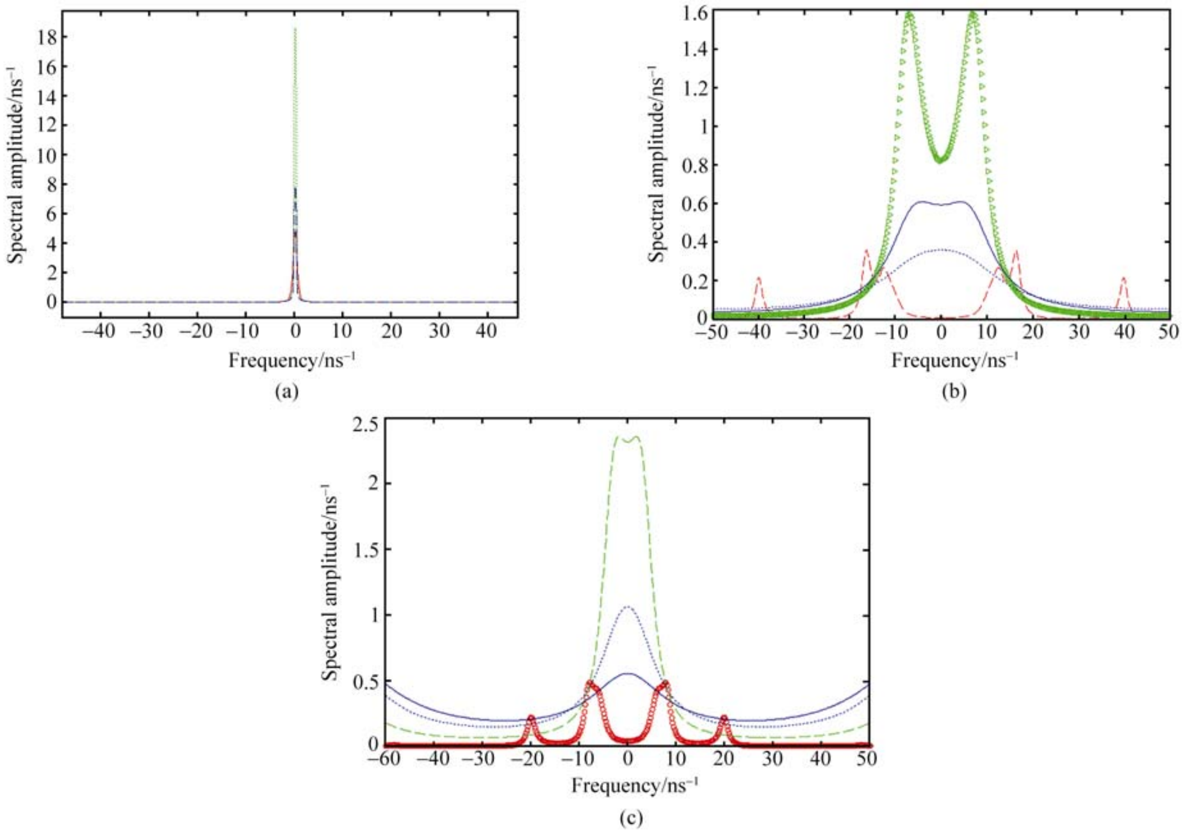

Fig. 2 The spectra of the photon emission versus detuning frequency for different coupling strength. **(a)** The result is obtained g = 10 μeV in the weak-coupling regime with different incoherent excitation pump Γ_L, Γ_L =1 ns^{-1} (*circle*), Γ_L = 20 ns^{-1} (*solid line*), and Γ_L = 30 ns^{-1} (*dashed line*); **(b)** The result is obtained g = 200 μeV in the strong-coupling regime with different incoherent pumps Γ_L, Γ_L =1 ns^{-1} (*dashed line*), Γ_L =10 ns^{-1} (*triangle*), Γ_L = 20 ns^{-1} (*solid line*), and Γ_L =30 ns^{-1} (*dotted line*); **(c)** The result is obtained g = 500 μeV in the strong-coupling regime with different incoherent pumps Γ, Γ=1 ns^{-1} (*circle*), Γ_L =10 ns^{-1} (*dashed line*), Γ_L = 20 ns^{-1} (*dotted line*), and Γ_L =30 ns^{-1} (*solid line*). The other parameters are the same as in Fig. 1.

tonic QD in an optical microcavity. The exciton-photon mixing and emission properties of the emitted photon were computed as functions of the exciton-photon coupling strength, the pump rate, and the photon detuning. Our results could have a relation to the very recent paper [22], where they theoretically studied the coupled modes of a medium-size QD, which may confine multiexcitons (about 10 electron-hole pairs), and a single photonic mode of an optical microcavity and the ground- state and excitation energies, the exciton-photon mixing in the wave functions and the emission of light from the microcavity are computed as functions of the pair-photon coupling strength, the photon detuning, and the polariton number. When we further compare our results with their results, we find that the calculated emitted photon intensity and spectra crucially depend on the exciton-photon coupling strength *g*, photon detuning, and the number of excitons in the QD. Finally, we give a physical mechanism for the strong coupling between the single mode of an optical microcavity and the QD emitters to explain the details of the emission photon spectra. Our study has established useful guidelines for the experimental study of such a multiexcitonic QD in an optical microcavity system. The new models and calculations in regard to this field are currently in progress.

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