Front. Phys. China (2006) 4: 506－509
DOI 10.1007/s11467-006-0054-4

RESEARCH ARTICLE

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The evolution of opinions on scale-free networks

Abstract An opinion evolution model without "bounded confidence" is proposed in this paper. Computer simulation shows that our model can figure out the breakage of the co-existence of majority and minority after a period's evolution. With further analysis, our model shows that, without the influence of the external field, the opinions will finally die out to a limited small value no matter what the initial condition of the system is. On the other hand, we simulate the evolution of the opinions under the influence of an external field, and get some meaningful and instructional results.

Keywords opinion evolution, an external field, scale-free networks, power law relationship

PACS numbers 89.20.-a, 89.75.-k, 89.75.Fb, 89.75.Hc

1 Introduction

The last decade has witnessed an increasing interest in the study of complex networks. Since Watts and Strogatz's work [1] on small world networks in 1998, an explosion of works about complex networks has emerged, from the analysis of the topology of real networks [2－4] to the evolution dynamics of complex networks and using complex networks to model all kinds of complex systems [1,5] and different kinds of dynamic processes [6－8]. One of the interesting points is using complex networks to describe social problems [9－14]. In this paper we focus on the model of opinion evolution on complex networks.

Many models about opinion evolution dynamics have been proposed in recent years. At first, only binary opinions were considered [9,10,15,16], some of which used "social impact theory" [14,17,18] founded by Latané to describe the transition from private attitude to public opinion, which help us understand minority or majority consensus and the formation of the phenomenon of many agents sharing the same opinion [10]. Then, people extended the model to continuous opinions models [11, 12, 14]. However, they are all Bounded Confidence models, which means that they all set a parameter ε called bounded confidence as a threshold for the updating of opinions, i.e., two agents will interact with each other only when their opinions are close enough. Take the Deffuant *et al.* (D) model [12,13,19] and Hegselmann and Krause (HK) model [20] for example. They study how the damage spreading and how the fraction of perturbed agents vary with bounded confidence and time. In this paper, we propose a new opinion evolution model with the bounded confidence omitted because whether the opinions of two agents are close enough or not they will still interact with each other if they are connected to each other.

A social network is a set of agents or groups with relationships of different kinds among them [21,22], such as friendship, collaboration, business, sexual and other interactions. Now, think about the question how do you get or change your information about things that you care? Most probably, you exchange your opinion with your friends and get information from the TV, newspaper, even government policy and so on. In this paper, we propose a model to simulate this opinion evolution process. Computer simulation shows that, without the influence of the external field, the opinions will finally die out to a limited small value no matter what the initial condition of the system is. On the other hand, we simulate the opinion evolution under the external field and get the result that the external field has an important role in keeping the opinion $s(t)$ increasing and getting balanced after certain time steps, and that the time t_{equal} when the opinion $s(t)$ gets balanced and the power Q of the external field have a power law relationship.

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Received April 10, 2006

2 Our model

Our model is defined in the following way: we generate a social network, the nodes of which represent the agents in our society and the edges represent the relationship that allows the agents to exchange their information to change their opinions on something. The strength of agents i's opinion $s_i(t)$ (can show the degree of one's concern about a certain thing) at time t is a real number in the range of $[0,1)$, which varies under the influence of his nearest neighbors and the external field. We will define the quantities of our model in the following way. First, the quantity

$$Inf_j^{(i)}(t) = \frac{k_j}{\sum_{\langle i,l\rangle} k_l} s_j(t) \tag{1}$$

where $\sum_{\langle i,l\rangle}$ is the sum of all the nearest neighbors of agent i. $Inf_j^{(i)}(t)$ represents the influence of agent j on agent i at time t, which is related to j's opinion $s_j(t)$, j's degree k_j and the total degree of the nearest neighbors of agent i.

Second, every agent has a different role in the network, which can be described as something like the "charge" of agent i:

$$e_i = \frac{k_i}{\sum_j k_j} \tag{2}$$

where $\sum_j$ is the sum of all the agents.

Third, we define the average opinion of agent i's neighbors at time t:

$$\overline{s_i(t)} = \frac{\sum_{\langle i,j\rangle} k_j s_j(t)}{\sum_{\langle i,l\rangle} k_l} \tag{3}$$

$\overline{s_i(t)}$ can describe the opinions of the local community around agent i.

Finally, we define the "power" Q of the external field, which reflects the strength of the influence of the external field, Q acts on all the agents. Then the influence of the external field to agent i is;

$$Qe_i = Q\frac{k_i}{\sum_j k_j} \tag{4}$$

There are many real factors that influence people to change their opinions about things in our society, such as mass media and government policies. We consider these cases as the influence of an external field. With the influence of the external field, the agents update their opinions according to the following equations. When $s_i(t) = 0$,

$$s_i(t+1) = \begin{cases} \frac{1}{k_i}\sum_{\langle i,j\rangle} Inf_j^{(i)}(t) + Qe_i, & \text{with Prob. } 50\% \\ 0, & \text{with Prob. } 50\% \end{cases} \tag{5}$$

and when $s_i(t) \neq 0$,

$$s_i(t+1) = s_i(t) + \Theta(\overline{s_i(t)} - s_i(t))\frac{s_i(t)}{k_i}\sum_{\langle i,j\rangle} Inf_j^{(i)}(t) + \Theta(Q - s_i(t))Qe_i s_i(t) \tag{6}$$

where,

$$\Theta(x) = \begin{cases} 1, & x \geqslant 0 \\ -1, & x < 0 \end{cases} \tag{7}$$

is the step function. The step function means the neighbors of agent i (or the external field) can have negative influence on $s_i(t)$ if $\overline{s_i(t)}$ (or Q) is weaker than $s_i(t)$ as well as positive influence while $\overline{s_i(t)}$ (or Q) is stronger than $s_i(t)$ at time t. And the coupling of $s_i(t)$ and $s_j(t)$ (or Q) indicates i's intention to keep its opinion unchanged. This interaction process is similar to the interaction among people in real life: the agent often changes his opinion about things due to the influence of his surrounding friends and other external factors but intends to keep his opinion unchanged at the same time.

3 Results

We realize our model on fixed scale-free networks since scale-free topology is the most common topology in complex networks and the dynamics of opinion are the same on both growing and fixed scale-free networks [16, 23]. The scale-free network used in this paper is BA scale-free network with the number of agents $N = 400$, the average degree $\langle k\rangle = 14.725$, the clustering coefficient $C = 0.0747$ and the power law exponents of degree distribution $\gamma = 2.7$.

Figure 1 is the result of the computer simulation of the model when the external field $Q = 0$. Here we use $\theta_i(t) = +1$ to replace the opinion $s_i(t)$ if agent i's opinion $s_i(t) \geqslant 0.5$, while we use $\theta_i(t) = -1$ to replace the opinion $s_i(t)$ if agent i's opinion $s_i(t) < 0.5$ according to the binary models. Compared with the result in Schweitzer, Zimmermann and Muhlenbein (SZM)'s work [10], we can see from Fig. 1 that the stable coexistence between majority and minority breaks down after a period's evolution, and then almost all the agents share the same opinion. This has the same meaning as figured out in SZM's work but their work cannot figure out which opinion will come down. On the contrary, our model can figure out that the agents will share the same opinion $\theta_i(t) = -1$. We will give some explanations below.

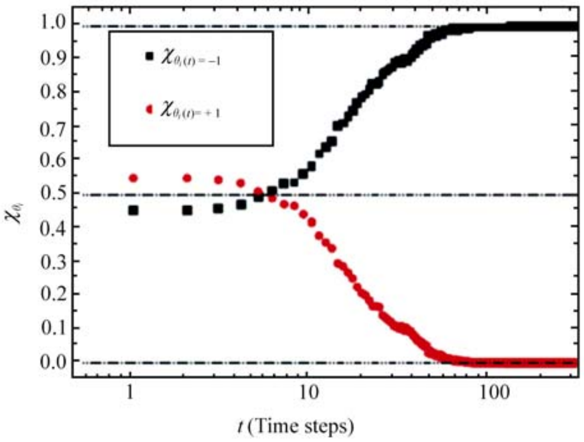

Fig. 1 Computer simulation of the relative minority/majority sizes vs. time t.

Figure 2 is the result of the evolution of the average opinion $s(t) = \frac{1}{N}\sum_i s_i(t)$ under the influence of different external field. All the curves in Fig. 2 are 20 times average and when $s_i(t) < 0$ it is automatically assumed to be zero. From Fig. 2 (a) we can see that the opinion is decreasing continuously when $Q = 0$. In fact, we do the simulation when $Q = 0$ with several different initial conditions and find that the evolution of the average opinion of the network has the same evolving trend and decreases as time elapses to a limited small value no matter how strong the average opinion of the network at the beginning is. Take one famous film star in real life for example. If he keeps on playing roles in films, more and more people will know him by seeing his films. But once he stops playing roles in films any more, most people will forget him sooner or later no matter how famous he used to be. This is the same case as the result that we get in Fig. 1.

When $Q \neq 0$, the evolution also has the same trend under different initial conditions [see Figs. 2 (b)—(d)] and they all reach equilibrium at high opinion values that are closest to the power Q of the external field at last. What's more, in the relationship between the time t_{equal} when the average opinion $s(t)$ gets balanced and the power Q of the external field (see Fig. 3), we can see that they show a power law relationship.

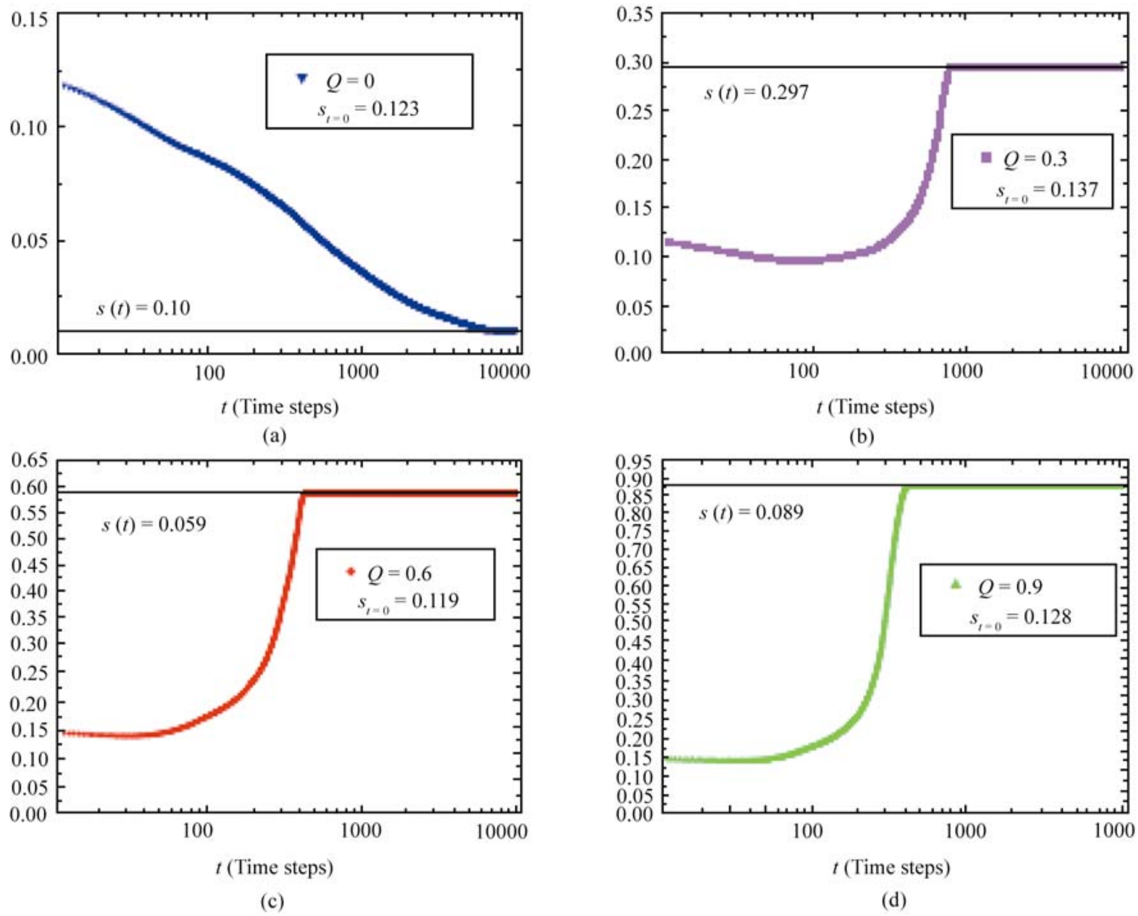

Fig. 2 The evolution of the average opinion of agents vs. time t under the influence of the external field (all the curves are 20 times average).

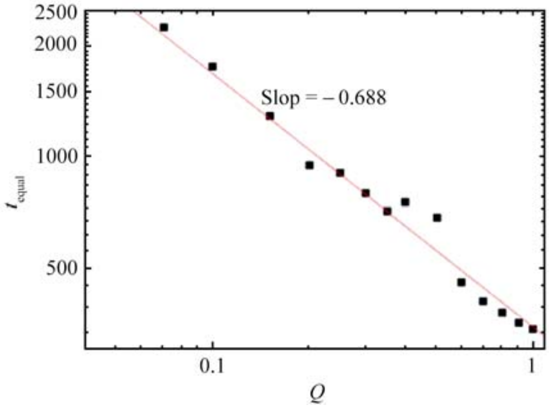

Fig. 3 The time when the average opinion get balanced vs. the power Q of the external field.

4 Conclusions

In this paper, we proposed a model of opinion evolution without bounded confidence on scale-free networks. It assumes that every agent has a continuum opinion on some issue and it is influenced by its nearest neighbors and an external field. Here, we only consider the evolution of all the agents' average opinion as time elapses because it is unnecessary to know the opinion of every agent at each time step. Through analyzing the computer simulation, we can get the same result as SZM's work [10] about the minority/majority consensus but our model can figure out which one is the winner at the end with only the agents' internal interaction. With the influence of the external field, more importantly, we can see that the role of the external field is of so much importance in our model that it can make the opinion on the scale-free network reach equilibrium at a high value instead of dying out. Finally, we find that there is a power law relationship between the time when our model reaches equilibrium and the power of the external field. This is an interesting result for us and we will try to figure out the reason of its formation in our future work.

What's more, during the work of this paper, we found that the evolution process of the opinion is very complex since it has strong relationship with the topology of the network, and we will try to go on our study to find more information about the opinion evolution on complex networks.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant Nos. 70571027 and 70401020).

References

1. Watts D. J. and Strogatz S. H., Nature, 1998, 393: 440
2. Pastor-Satorras R., Vazquez S., and Vespignani A., Phys. Rev. Lett., 2001, 87: 258701
3. Ebel H., Mielsch L. -I., and Bornhlodt S., Phys. Rev. E., 2002, 66: 035103
4. Li W. and Cai X., Phys. Rev. E., 2004, 69: 046106
5. Barabási A. -L., Albert R., Science, 1999, 286: 509
6. Pastor-Satorras R. and Vespignani A., Phys. Rev. Lett., 2001, 86: 3200
7. Santos F. C. and Pacheco J. M., Phys. Rev. Lett., 2005, 95: 098104
8. Cohen R., Erez K., ben-Avraham D., and Havlin S., Phys. Rev. Lett., 2004, 86: 3682
9. Kaizoji T., arXiv: physics/0601106
10. Schweitzer F., Zimmermann J., and Muhlenbein H., Physica A, 2002, 303: 189
11. Fortunato S., arXiv: cond-mat/0408648
12. Weisbuch G., Eur. Phys. J. B, 2004, 38: 339
13. Geffuant G., Amblard F., Weisbuh G., and Faure T., Journal of Artifical Societies and Social Simulation, 2002, 4: paper 1
14. Lewensrein M., Nowak A., and Latané B., Phys. Rev. A, 1992, 45: 763
15. Fortunato S. and Stauffer D., arXiv: cond-mat/0501730
16. Wu F. and Huberman B. A., arXiv: cond-mat/0407252
17. Nowak A., Szamrej J., and Latané B., Psych. Rev., 1990, 97: 362
18. Latané B., Am. Psychol., 1981, 36: 343
19. G. Deffuant, D. Neau, F. Amblajd, and G. Weisbuch, Adv. Complex Systems, 2003, 3: 87
20. Hegselmann R. and Krause U., J. Arti. Soc. Social Simul., 2000, 5: 3
21. Wasserman S. and Faust K., Social Networks Analysis, Cambridge :Cambridge University Press, 1994
22. Scott J., Social Network Analysis: A Handbook, 2nd ed., London: Sage Publication, 2000
23. Sousa A. O., arXiv: cond-mat/0406766