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RESEARCH ARTICLE

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Effective medium method of slightly compressible elastic media permeated with air-filled bubbles

Abstract An effective medium method is developed for the slightly compressible elastic media permeated with air-filled bubbles, according to the nonlinear oscillation of the bubble, which happens when compressional wave travels through the porous media. The effective Lame coefficients of the porous medium and the nonlinear elastic wave equation are deduced, based on the fact that the micro-unit of the effective medium should have the same stress and strain as the micro-unit of the porous media. The linearized properties obtained by this method are in good agreement with the results of Gaunaurd's classic theory [Gaunaurd G.C. and Überall H., J. Acoust. Soc. Am., 1978, 63: 1699—1711]. Furthermore, the nonlinear coefficient, which is an important property of the porous media, can also be acquired by this method.

Keywords effective medium, bubbles, compressional wave, nonlinear coefficient

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1 Introduction

The study of the propagation of elastic waves through the elastic and viscoelastic materials permeated with air-filled bubbles is particularly important because it is useful in a large variety of situations, such as underwater noise reduction. Many works have been done and some methods have been developed in the study of the sound propagation in inhomogeneous material [1—9]. Based on the resonance scattering theory, Gaunaurd *et al*. have presented an effecttive medium theory (EMT) which can be recognized as a classic theory. According to the EMT theory, the scattering far field generated by an inhomogeneous media, which distributes air-filled bubbles, droplets or solid inclusions, should be the same with the scattering field generated by the effective media. Thus the effective properties can be obtained through the little variable approximation of a spheric function and the determinant calculation. The use of EMT is limited in some special inhomogeneous media because it attempts to obtain the effective properties of all kinds of composites, and therefore, the specific characteristic of some inhomogeneous media cannot be taken into account. For example, the nonlinear parameter of the slightly compressible porous media, which has the character of porous rubber [7—8], cannot be obtained by EMT. Moreover, the derivation of the EMT is complicated. Therefore, developing a new and effective method to acquire both the linear and nonlinear acoustic properties is necessary.

A new effective medium method (EMM) is presented in this article based on the method of Ostrovsky to obtain the nonlinear coefficient of the slightly compressible porous media [7—8]. Since the stress and strain of the effective media should be the same with the porous media, the EMM for the porous media is developed, which is simpler than EMT, and the nonlinear formula is more standard than Ostrovsky's. It is worth pointing out that the materials permeated with air-filled cavities discussed in this paper differ from the porous media analyzed by Biot's model [10—11], in which the pore fluid can move relative to the solid frame. Scince the bubbles in this paper are isolated from each other, Biot's model cannot be used in this situation.

The viscoelastic medium permeated with air-filled bubbles is studied in detail by Liang Bin *et al*. [12]. For simplicity, the damp of the matrix is not taken into account in this paper.

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2 Theoretical analysis

2.1 The porosity and the slightly compressible elastic media

The implication of the porosity φ and the slightly compressible elastic media should be understood first. The porosity is the total volume of the bubbles in unit volume. According to statistics, the ratio of the air area to the total area for all cross sections should be φ [10]. The variation of the porosity when wave propagate in porous media is neglected since only the small elastic motion is studied for the elastic wave.

The slightly compressible elastic medium includes the water-like or rubber-like media. The assumption of the slight compressibility supposes the condition that the bubble radius R is very small in comparison with the wavelength of the longitudinal wave [13—14], that is

$$R_0 f / c_1 \ll 1 \tag{1}$$

where R_0 is the bubble radius, c_1 is the longitudinal wave speed, f is the frequency. If the resonant frequency of the empty spherical cavity in the slightly compressible medium [15] is selected as the reference frequency, we get

$$f = c_t / \pi R_0 \tag{2}$$

where c_t is the shear wave speed. Substituting Eq. (2) into Eq. (1), one obtains $c_t \ll c_l$ (The effect of the π is neglected). According to the relationship of the wave speed and Lame coefficients, one can obtain $\lambda + 2\mu \gg \mu$, where λ and μ are the Lame coefficients of the material. Therefore, the assumption of slight compressibility corresponds to $\lambda \gg \mu$.

2.2 The effective Lame coefficients for the effective medium

The micro-unit of the porous media, which includes many bubbles, is shown in Fig. 1 (a). The sizes of the micro-unit are dx_1, dx_2 and dx_3, which are much larger than the bubble radius, therefore, many bubbles are included in the micro-unit. The same micro-unit of the effective medium is shown in Fig. 1 (b), and the effective Lame coefficients are expressed as $\bar{\lambda}$, $\bar{\mu}$. The $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ in Fig.1 are the scalar and vector potentials of the incidence wave, respectively, and the plane wave is assumed in this paper. According to the theory of effective medium, the micro-unit of the effective medium should have the same stress and strain with the micro-unit of the porous media for the same disturbance. Using the relationship of stress and strain, the stress of the effective medium can be written as:

$$\begin{cases} \sigma_i = \bar{\lambda}\theta + 2\bar{\mu}\varepsilon_i, & i = 1,2,3 \\ \tau_{ij} = \gamma_{ij}\bar{\mu}, & i,j = 1,2,3 \text{ and } i \neq j \end{cases} \tag{3}$$

whre subscripts 1, 2, 3 express the x, y, z directions, respectively, σ_i, τ_{ij} are the normal and shear stress components, ε_i, γ_{ij} are the normal and shear strain components, respectively, and $\theta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ is the volume perturbation of the effective media.

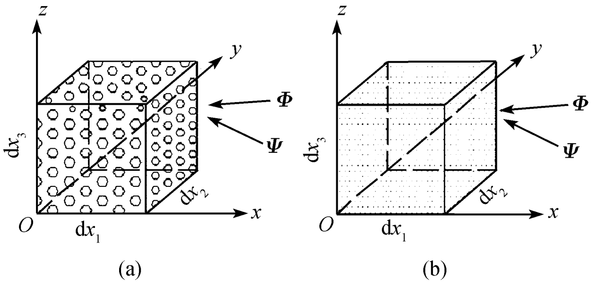

Fig. 1 The micro-unit of porous media **(a)** and the micro-unit of effective media **(b)**.

Because the modulus of the gas in the bubble is much smaller in comparison with the solid media, the stress of the porous media is mainly generated by the solid media [7]. Thus the stress of the porous media can be expressed as the form:

$$\begin{cases} \sigma_i = (\lambda\theta_s + 2\mu\varepsilon_i^s)(1-\varphi), & i = 1,2,3 \\ \tau_{ij} = \gamma_{ij}\mu(1-\varphi), & i,j = 1,2,3 \text{ and } i \neq j \end{cases} \tag{4}$$

where $\theta_s = \varepsilon_1^s + \varepsilon_2^s + \varepsilon_3^s$ is the volume perturbation of the solid phase, ε_i^s is the effective normal strain component of the solid phase, and γ_{ij} are the shear stain components.

Besides the volume variation happening in the solid media, the volume of the bubbles will also be changed with the bubble oscillation when the plane longitudinal wave travels through the porous media. Therefore, the volume perturba tion of the porous media micro-unit has the form:

$$\theta dx_1 dx_2 dx_3 = \theta_s (1-\varphi) dx_1 dx_2 dx_3 + V_c dx_1 dx_2 dx_3$$

where V_c is the volume perturbation generated by the bubble oscillation in unit volume. It can be simplified as:

$$\theta = (1-\varphi)\theta_s + V_c \tag{5}$$

From Eq. (3), the sum of σ_1, σ_2, σ_3 for the effective medium is

$$\sigma_1 + \sigma_2 + \sigma_3 = 3(\bar{\lambda} + 2\bar{\mu}/3)\theta \tag{6}$$

Similarly, the sum of σ_1, σ_2, σ_3 for porous media can be obtained from Eq. (4):

$$\sigma_1 + \sigma_2 + \sigma_3 = 3(\lambda + 2\mu/3)\theta_s(1-\varphi) \tag{7}$$

Since the stress, strain and volume perturbation of the porous media should be the same as the effective media, one can obtain:

$$\bar{\mu} = \mu(1-\varphi) \tag{8}$$

by comparing Eq. (3) with Eq. (4). Comparing Eq. (6) with Eq. (7) and substituting Eq. (5) into Eq. (6) yields,

$$\left(\bar{\lambda} + \frac{2}{3}\bar{\mu}\right)\theta = \left(\lambda + \frac{2}{3}\mu\right)(\theta - V_c) \tag{9}$$

In order to obtain the volume variation V_c generated by the bubble oscillation in unit volume, the volume perturbation of the single bubble must be considered first. The nonlinear oscillation of the single bubble has been described by Ostrovsky in the form of volume perturbation U [7]:

$$\ddot{U}+\omega_0^2 U-\frac{R_0}{c_l}\dddot{U}=GU^2+q\left(\dot{U}^2+2U\ddot{U}\right)-\frac{4\pi R_0}{\rho_s}P^{\text{inc}} \tag{10}$$

where

$$\omega_0=\sqrt{\omega_e^2+\omega_g^2},\ \omega_e=\sqrt{4\mu/\rho_s R_0^2},\ \omega_g=\sqrt{3\gamma P_g/\rho_s R_0^2}$$

$$G=q\left[3(\gamma+1)\omega_g^2+\frac{\omega_e^2(9+2\beta)}{2}\right],\ q=\frac{1}{8\pi R_0^3}$$

Here ρ_s is the density of the elastic media, R_0 is the equilibrium radius of the bubble, P^{inc} is the effective pressure generated by the incidence longitudinal wave, γ is the adiabatic coefficient, P_g is the gas pressure of the bubble, and β is the asymmetry coefficient [16].

For the incidence wave with the time dependence $e^{-i\omega t}$ (ω is the angular frequency of the incident wave), using the perturbation method, the solution of Eq. (10) up to quadratic order has the form [7]:

$$U=-g_1(R_0)P^{\text{inc}}+g_2(R_0)(P^{\text{inc}})^2 \tag{11}$$

where

$$g_1(R_0)=\frac{4\pi R_0}{\rho_s(\omega_0^2-\omega^2-iR_0\omega^3/c_l)} \tag{12a}$$

$$g_2(R_0)=\frac{G-3q\omega^2}{\omega_0^2-4\omega^2-8iR_0\omega^3/c_l}\cdot\left[\frac{4\pi R_0}{\rho_s(\omega_0^2-\omega^2-iR_0\omega^3/c_l)}\right]^2 \tag{12b}$$

For slightly compressible elastic media, $P^{\text{inc}}\approx-\sigma$, where σ is the stress generated by the incident wave [7]. From Eq. (12a), the linear resonance angular frequency is ω_0, which is obtained by setting the real part of the denominator equal to zero. One can find from Eq. (10) that ω_0 is determined by the shear modulus of the elastic media, the gas pressure P_g and the bubble radius R_0.

The strain will be generated when the plane wave travels through the media and only the principle strain in the incident direction will not equal to zero. If ε_I is assumed as the only nonzero principle strain, we have

$$\varepsilon_I=\varepsilon_I+\varepsilon_{II}+\varepsilon_{III}$$

where ε_I, ε_{II}, ε_{III} are the three principle strains. Using Hooke's law, the stress in the incident direction has the form:

$$\sigma=(\bar{\lambda}+2\bar{\mu})\varepsilon_I=(\bar{\lambda}+2\bar{\mu})(\varepsilon_I+\varepsilon_{II}+\varepsilon_{III})$$

According to the invariant relation of strain tensor [17], there is

$$\nabla\cdot\boldsymbol{u}=\varepsilon_x+\varepsilon_y+\varepsilon_x=\varepsilon_I+\varepsilon_{II}+\varepsilon_{III}$$

where $\boldsymbol{u}=\boldsymbol{i}u_x+\boldsymbol{j}u_y+\boldsymbol{k}u_z$ is the displacement vector, and ε_x, ε_y, ε_x are the strain components in the x, y, z directions, respectively. Therefore, the stress in the incident direction can be expressed as:

$$\sigma=(\bar{\lambda}+2\bar{\mu})\nabla\cdot\boldsymbol{u} \tag{13}$$

Substituting Eq. (13) into Eq. (11) yields,

$$U=g_1(R_0)(\bar{\lambda}+2\bar{\mu})\nabla\cdot\boldsymbol{u}+g_2(R_0)\left[(\bar{\lambda}+2\bar{\mu})\nabla\cdot\boldsymbol{u}\right]^2 \tag{14}$$

From Eq. (14), one can find that the bubble oscillation is not affected by the shear wave because $\nabla\cdot\boldsymbol{u}=\varepsilon_x+\varepsilon_y+\varepsilon_x=\theta$ expresses the volume perturbation, which is independent of the shear wave. The volume variation produced by bubble oscillation in unit volume can be obtained from Eq. (14):

$$V_c=V_{c1}(\bar{\lambda}+2\bar{\mu})\nabla\cdot\boldsymbol{u}+V_{c2}\left[(\bar{\lambda}+2\bar{\mu})\nabla\cdot\boldsymbol{u}\right]^2 \tag{15}$$

where

$$V_{c1}=\int g_1(R_0)\,n(R_0)\,dR_0,\quad V_{c2}=\int g_2(R_0)\,n(R_0)\,dR_0$$

$n(R_0)$ is the number of bubbles with equilibrium radii from R_0 to R_0+dR_0 in unit volume. If all the bubbles have the uniform radius R_0, $V_{c1}=Ng_1(R_0)$ and $V_{c2}=Ng_2(R_0)$, where N is the number of bubbles in unit volume. Substituting Eq. (15) into Eq. (9) yields,

$$\bar{\lambda}+2\bar{\mu}/3=(\lambda+2\mu/3)[1-V_{c1}(\bar{\lambda}+2\bar{\mu})-V_{c2}(\bar{\lambda}+2\bar{\mu})^2\nabla\cdot\boldsymbol{u}] \tag{16}$$

Eq. (16) can be rewritten as follows:

$$\bar{\lambda}+2\bar{\mu}=\frac{\lambda+2\mu/3+4\bar{\mu}/3}{1+V_{c1}(\lambda+2\mu/3)}-\frac{\lambda+2\mu/3}{1+V_{c1}(\lambda+2\mu/3)}V_{c2}(\bar{\lambda}+2\bar{\mu})^2\nabla\cdot\boldsymbol{u} \tag{17}$$

Since $\bar{\mu}=(1-\varphi)\mu<\mu$ and $\lambda\gg\mu$, one obtains:

$$\frac{\lambda+2\mu/3+4\bar{\mu}/3}{1+V_{c1}(\lambda+2\mu/3)}\approx\frac{\lambda+2\mu/3}{1+V_{c1}(\lambda+2\mu/3)}$$

Thus, Eq. (17) can be expressed as:

$$\bar{\lambda}+2\bar{\mu}=\bar{C}_{11}-\bar{C}_{11}V_{c2}(\bar{\lambda}+2\bar{\mu})^2\nabla\cdot\boldsymbol{u} \tag{18}$$

where $\bar{C}_{11}=\dfrac{\lambda+2\mu/3}{1+(\lambda+2\mu/3)V_{c1}}$. Using the perturbation method, the solution of the Eq.(18) has the form:

$$\bar{\lambda}+2\bar{\mu}=\bar{C}_{11}(1-\bar{C}_{11}^2V_{c2}\nabla\cdot\boldsymbol{u}) \tag{19}$$

On the another hand, the density of the effective medium $\bar{\rho}$, which should be the same with the porous media, has the form:

$$\bar{\rho}=(1-\varphi)\rho_s+\varphi\,\rho_g \tag{20}$$

where ρ_g is the density of gas in the bubbles.

From the previous discussions, one can find that it is just The nonlinear oscillation of bubbles that results in the nonlinear property for the slightly compressible porous media. Although the propagation of shear wave does not affect the bubble oscillation, the shear modulus affected the bubble resonance frequency and nonlinear oscillation greatly. The less the shear modulus is, the lower the resonance frequency and the stronger the nonlinear oscillation is [7, 13, 18].

2.3 The wave equation for slightly compressible porous media

Since the stress and strain of the effective media are the same with the porous media, the wave equation of the porous media can be obtained from the effective media by substituting $\bar{\lambda}$, $\bar{\mu}$, $\bar{\rho}$ into the classic wave equation:

$$\nabla\left[(\bar{\lambda}+2\bar{\mu})\nabla\cdot\boldsymbol{u}\right]-\bar{\mu}\,\nabla\times\nabla\times\boldsymbol{u}=\bar{\rho}\frac{\partial^2\boldsymbol{u}}{\partial t^2} \tag{21}$$

Because of the relations between displacement vector and the potentials as:

$$\boldsymbol{u}=\nabla\boldsymbol{\Phi}+\nabla\times\boldsymbol{\Psi},\quad \text{and } \nabla\cdot\boldsymbol{\Psi}=0 \tag{22}$$

where $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are the scalar potential and vector potential, respectively, the substitution of Eqs. (19) and (22) into Eq. (21) yields,

$$\bar{C}_{11}(1-\bar{C}_{11}^2V_{c2}\nabla^2\boldsymbol{\Phi})\nabla^2\boldsymbol{\Phi}=\bar{\rho}\frac{\partial^2\boldsymbol{\Phi}}{\partial t^2} \tag{23}$$

$$\bar{\mu}\nabla^2\boldsymbol{\Psi}=\bar{\rho}\frac{\partial^2\boldsymbol{\Psi}}{\partial t^2} \tag{24}$$

Therefore, the effective longitudinal and transverse wave equation is expressed by Eqs. (23) and (24), respectively. Obviously, the effective longitudinal wave equation Eq. (23) is the nonlinear equation and the effective shear wave is a linear equation. From Eqs. (8), (20) and (24), one can also find that the propagation of shear wave was hardly affected by the bubbles.

The linear acoustic property can be analyzed by neglecting the nonlinear part of Eq. (23). The effective wave number can be expressed as:

$$k_1=\omega\sqrt{\bar{\rho}/\bar{C}_{11}}=\mathrm{Re}\,k_1+\mathrm{i}\,\mathrm{Im}\,k_1=\omega/c_{\mathrm{le}}+\mathrm{i}\alpha_{\mathrm{e}} \tag{25}$$

where c_{le} is the effective longitudinal wave velocity, α_{e} is the attenuation. According to the definition of the nonlinear coefficient [7—8] and Eq. (23), the nonlinear coefficient Γ has the form:

$$\Gamma=V_{c2}\bar{C}_{11}^2 \tag{26}$$

In this paper, the modulus of nonlinear coefficient $|\Gamma|$ is used to measure the nonlinearity of the porous media.

3 Numerical results and discussion

According to the theory analysis, besides the linear property, the nonlinear acoustic property, which cannot be obtained by EMT, can be obtained by EMM; moreover, this method is simpler than EMT. However, the validity of the EMM should be confirmed first.

3.1 The parameters in calculation

In order to confirm the validity of the EMM, the results obtained by EMM are compared with the results of EMT. Computations are performed for the slightly compressible media whose parameters are listed in Table 1, in which there are $\lambda \gg \mu$ for both the rubber [2] and the plastic material (plastizole) [8], and the shear modulus of plastizole is much less than the rubber's. In numerical calculations, the air, with an adiabatic coefficient of γ= 1.4, is assumed in the bubble; other parameters include the β= 0, the air pressure is 1.01×10^5 Pa (with density of 1.2 kg/m^3), the concentration of bubbles is 10 %, and the uniform bubble radius R_0 = 20 micron (except special definition).

Table 1 The parameters used in calculation for the rubber and plastizole.

Matrix	Rubber	Plastizole
ρ_s/ (kg · m^{-3})	1130	950
λ/Pa	2.2×10^9	2.01×10^9
μ/Pa	1.0×10^7	1.01×10^4

The frequency range is limited in order to ensure that the Wavelength is much larger than the bubble radius. The wavelength is assumed to be larger than the bubble radius by at least one order of magnitude. If the bubble radius is assumed to be 40 microns (which is used in Fig. 3), the frequency range has the form as below:

$$f \leqslant c_l/(10R_0) \approx 3700\ \text{kHz}$$

3.2 Comparison of the linear property

The effective longitudinal velocity and attenuation of the two kinds of porous media are shown in Figs. 2 (a) and (b), respectively, in which the solid line is acquired by EMM and the dashed line is obtained by EMT. Fig. 2 shows that both the effective velocity and the attenuation obtained by the two methods are in good agreement. It confirms the validity of the EMM. At the same time, the peaks in the effective velocity and attenuation curves move to the lower frequency when the shear modulus becomes less (such as the porous plastizole). The reason is that the bubble oscillation is affected by the shear modulus of the matrix, and the larger the shear modulus is, the larger the resonance frequency is.

The effective velocity and attenuation of porous rubber changing with the bubble radius is shown in Figs. 3 (a) and (b), respectively, in which the solid line is obtained by EMM, and the dashed line is acquired by EMT and the curves number 1, 2, and 3 correspond to the bubble radius 40, 30, and 20 microns, respectively. The comparison confirms the validity of the EMM again. From Fig. 3, one can also find that the peaks of the effective velocity and attenuation curves move to the lower frequency with the increase of the bubble radius. This phenomenon results from the changing of bubble oscillation, and the larger the bubble radius is, the lower the resonance frequency is.

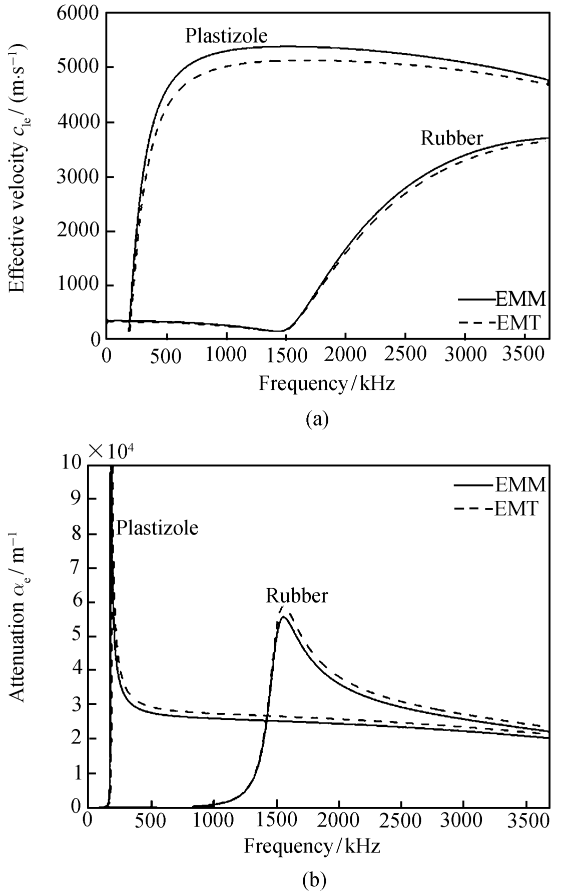

Fig. 2 The effective velocity **(a)** and attenuation **(b)** of the porous rubber and porous plastizole (The uniform bubble radius is 20 micron).

3.3 Nonlinear property

According to Eq. (26), the nonlinear parameter $|\Gamma|$ of the porous rubber and porous plastizole varying with the frequency, which cannot be obtained by EMT, is shown in Fig. 4. From Fig. 4, the nonlinear coefficient in lower frequencies increases and the peak of the nonlinear coefficient curve moves to the lower frequencies if the shear modulus of the matrix becomes small. The difference shows that the nonlinear property of the porous media is affected greatly by the nonlinear oscillation of bubbles in different material.

4 Conclusions

An effective medium method, which can be used to obtain the linear and nonlinear properties of porous media, is developed based on the fact that the micro-unit of the effective medium has the same stress and strain with the micro-unit of the porous media. The linearized properties obtained by this method are confirmed by Gaunaurd's classic theory.

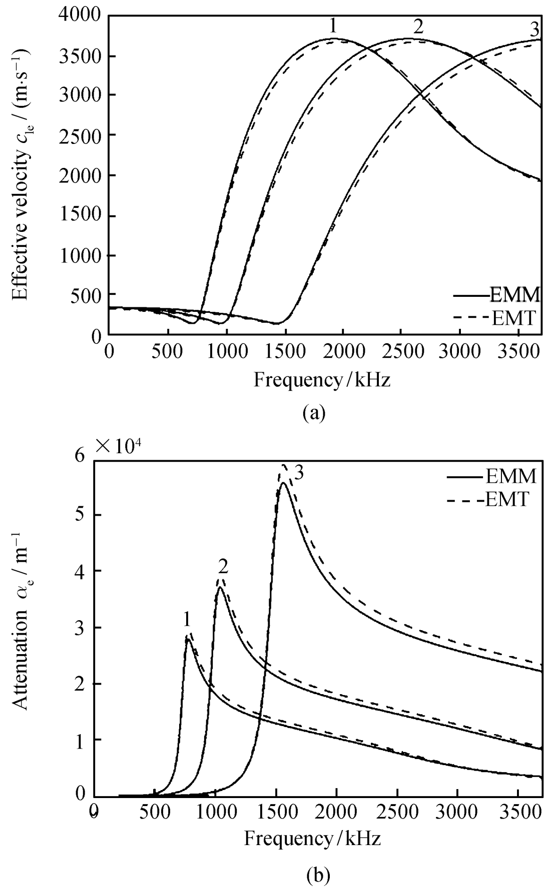

Fig. 3 The effective velocity **(a)** and attenuation **(b)** of the porous rubber varying with the bubble radius (The curves 1, 2, 3 corresponding to the bubble radius 40, 30, 20 micron, respectively).

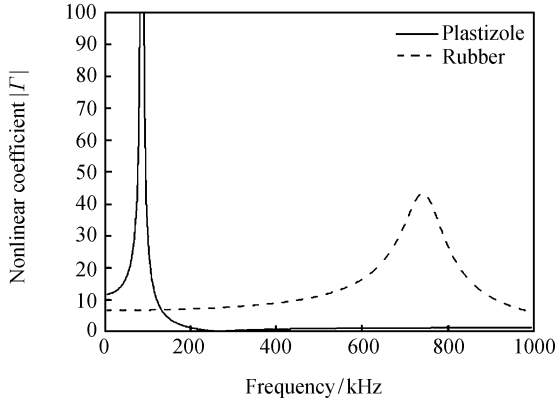

Fig. 4 The moduli $|\Gamma|$ of the nonlinear coefficient of the porous rubber and porous plastizole varying with the frequency (The uniform bubble radius is 20 micron).

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