Front. Phys. China (2006) 4: 412—418
DOI 10.1007/s11467-006-0044-6

REVIEW ARTICLE

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Solution to causality paradox upon total reflection

Abstract A dispute about the existence of an additional time (named as the Goos-Hänchen time) associated with the Goos-Hänchen shift in total reflection has recently arisen. At the same time, an inconsistency between the optical ray model and the electromagnetic theory also appears in the optical planar waveguide. By analyzing light propagation in an optical planar waveguide with both the zigzag-ray model and the electromagnetic theory, this paper shows that the Goos-Hänchen time really exists, and the total time delay upon total reflection upon an ideal nonabsorbing plasma mirror is the sum of the group-delay time and the Goos-Hänchen time. The causality paradox of total reflection of a TM wave upon an ideal nonabsorbing plasma mirror is also solved taking into consideration the negative Goos-Hänchen shift. Finally, the expression of the group velocity of the guided mode in optical planar waveguide was obtained, which clearly shows that the time delay upon total reflection is the sum of the group-delay time and the Goos-Hänchen time at given any time.

Keywords total reflection, optical planar waveguide, Goos-Hänchen shift, ideal nonabsorbing plasma mirror

PACS numbers 42.25.Gy, 01.55.+b, 42.79.Gn

1 Introduction

When a light beam is totally reflected at the interface of two different media, the reflected beam undergoes a lateral shift from the path usually expected from geometrical optics. This is well known as the Goos-Hänchen effect [1, 2]. Since its discovery in 1947, the Goos-Hänchen effect has been a subject of great interest [3—14]. But till date, most investigations are focused on the Goos-Hänchen shift. Few studies are directed to analyze the time delay of the light caused during this shift.

Kogelnik and Weber [15] have investigated the time delay in the optical planar waveguide and indicated that the total time delay during total internal reflection can be expressed as $\tau = -\partial\phi/\partial\omega$ where ϕ is the phase shift and ω is the angular frequency. But this expression has two difficulties. One is that this expression cannot distinguish between the time delay caused by the dispersion through media and the time of origin from the Goos-Hänchen shift. The other is that, in derivation of this expression (in Appendix A of Ref. [15]), the authors treat the propagation constant of optical waveguide as an independent variable and think that the partial derivative of the propagation constant with respect to the angular frequency is zero. Therefore, those derivations are correct only for guided modes in a waveguide, and cannot be applied to the general case of total reflection upon a semi-infinite medium. Recently, the time delay upon total reflection has attracted considerable attention. A calculation [16] has predicted that the disturbed Gires-Tournois interferometer exhibits a negative delay time for total reflection, which seems to contradict causality by considering its 100 % reflectivity. To solve this causality paradox, Kevin J. Resch *et al.* [17] showed that the Goos-Hänchen shift contributes an extra positive time, the Goos-Hänchen time, which is always large enough to make the total time delay of the frustrated Gires-Tournois interferometer positive. But recently, another example [18] has been presented, that is, the total reflection of a plane p (or TM) wave from vacuum upon an ideal nonabsorbing plasma mirror, in which, if the Goos-Hänchen time is included, the total time delay can become negative. Based on the example mentioned above, it is indicated [18] that the existence of the Goos-Hänchen time is doubtful and the causality paradox upon total reflec-

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Received September 12, 2006

tion remains open. And now, the situation being faced is that the total reflection upon the disturbed Gires-Tournois interferometer contradicts causality if the Goos-Hänchen time is not included, and the total reflection upon an ideal nonabsorbing plasma mirror violates causality if the Goos-Hänchen time is included.

In addition to the paradox mentioned above, the inconsistency between the optical ray model and the electromagnetic theory also appears in a simple optical planar waveguide. Based on an optical planar waveguide, the problems mentioned above have been solved in our previous works [19—21]. In this paper, those works are reviewed and a clear explanation about the Goos-Hänchen time and the causality paradox is given. A demonstration that the Goos-Hänchen time upon total reflection really exists is made [19]. For the case of total reflection of a plane TM wave upon an ideal nonabsorbing plasma mirror, the fact that the location where total reflection occurs is not at the interface between two relevant media, but in front of it is pointed out. By considering this special effect, the time delay is always positive. As a result, it is suggested that there is no problem with the relativistic causality [20]. Finally, based on the expression of the group velocity of the guided mode in optical planar waveguide, it is clearly shown that at any time the time delay upon total reflection is the sum of the group delay time and the Goos-Hänchen time [21].

2 Inconsistency between the ray model and the electromagnetic theory in optical planar waveguide

The causality paradox in Gires-Tournois interferometer and ideal nonabsorbing plasma mirror can be found in Refs. [16—18]. In this section, the inconsistency between the ray model and the electromagnetic theory in optical planar waveguide is pointed out.

Figure 1 shows an optical planar waveguide where a guiding layer of high refractive index is sandwiched between two semi-infinite cladding layers of low refractive indices. Guiding is achieved by total reflection of the optical rays upon two cladding layers. The dispersion relation of a guided mode for both polarizations is given by:

$$2\kappa h + \varphi_{12} + \varphi_{13} = 2m\pi \tag{1}$$

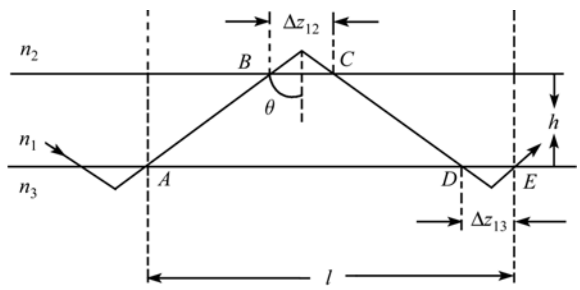

Fig. 1 Rays propagation in optical waveguide.

with

$$\kappa = (k_0^2 n_1^2 - \beta^2)^{1/2} = k_0 n_1 \cos\theta \tag{2}$$

$$\tan\frac{-\varphi_{1j}}{2} = \begin{cases} \left(\dfrac{N_m^2 - n_j^2}{n_1^2 - N_m^2}\right)^{1/2} & \text{(TE modes)} \\ \dfrac{n_1^2}{n_j^2}\left(\dfrac{N_m^2 - n_j^2}{n_1^2 - N_m^2}\right)^{1/2} & \text{(TM modes)} \end{cases} \tag{3}$$

$$\beta = k_0 N_m = k_0 n_1 \sin\theta \tag{4}$$

where m is the mode order, N_m is the effective index of the guided mode, h is the thickness of the guiding layer, β is propagation constant, κ is the component of wavevector normal to the propagation direction in the guiding layer, θ is the angle of incidence, and j = 2, 3 represents two cladding layers respectively.

The propagation of a guided mode with a propagation constant of β is taken into consideration. According to the electromagnetic theory of planar waveguide, the field distribution of the guided mode can be expressed as:

$$E(x, y, t) = E(y)\exp[\mathrm{i}(\beta x - \omega t)] \tag{5}$$

where E (y) is the amplitude of the guided mode. Figure 2 illustrates the propagation of the guided mode by employing the zigzag-ray model. The distance between A and B, which corresponds to one period of propagation of the zigzag-rays in guiding layer, is written as:

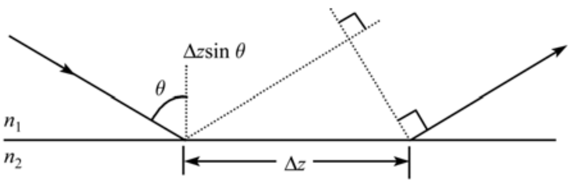

Fig. 2 A beam undergoing reflection from an interface at an angle of incidence beyond critical.

$$l = 2h\tan\theta + \Delta x_{12} + \Delta x_{13} \tag{6}$$

where Δx_{12} and Δx_{13} are the Goos-Hänchen shifts at two boundaries. By considering Eq. (5), the phase shift of the guided mode in this distance is

$$\phi_{\mathrm{mode}} = \beta l \tag{7}$$

In zigzag-ray model, the total phase shift of the rays in this range is

$$\phi_{\mathrm{ray}} = 2k_0 n_1 \frac{h}{\cos\theta} + \varphi_{12} + \varphi_{13} \tag{8}$$

According to Eq. (1), Eq. (8) can be expressed as:

$$\phi_{\mathrm{ray}} = 2\beta h\tan\theta + 2m\pi \tag{9}$$

and then,

$$\phi_{\mathrm{ray}} - \phi_{\mathrm{mode}} = 2m\pi - \beta(\Delta z_{12} + \Delta z_{13}) \tag{10}$$

is obtained.

If the two theories are consistent, the two phases should differ by $2m\pi$. From this example, it is clearly shown that the phase shifts of total reflection are not φ_{12} and φ_{13}.

3 Phase shift of total reflection and Goos-Hänchen time

In this section, the group delay time and the Goos-Hänchen time defined in Refs. [17—18] is introduced. Figure 2 shows a beam undergoing reflection from an interface at an angle of incidence beyond critical. In this diagram, Δx is the Goos-Hänchen shift and $k_x\Delta x$ is the phase accumulated during that shift, where k_x is the x-component of the wavevector. The reflection phase shift φ_R [17] can be calculated by employing the Fresnel formula. This paper states that this phase shift is not the total phase shift, because the Fresnel formula only deals with the components of wavevectors normal to the interface. The total phase shift φ_{TOT} [17] upon total reflection should be written as:

$$\begin{aligned}\varphi_{\mathrm{ToT}} &= \varphi_R + k_x\Delta x\\ &= \varphi_R + \frac{n_1\omega}{c}\sin\theta\Delta x\end{aligned} \tag{11}$$

where ω is angular frequency of the light, θ is angle of incidence, n_1 is index of refraction of the first medium, and c is the speed of light in vacuum. By using the stationary phase theory, the total time delay [17] is

$$\tau = \frac{\partial\varphi_R}{\partial\omega} + \frac{n_1}{c}\sin\theta\Delta x \tag{12}$$

The time delay in Eq. (2) consists of two components. The first term, which is caused by dispersion of the reflection phase shift φ_R, is defined as the group delay time τ_g [18]. The second term is an additional contribution caused by the phase $k_x\Delta x$, and is defined as the Goos-Hänchen time. The Goos-Hänchen shift and the Goos-Hänchen time [17] can be written as:

$$\Delta x = -\frac{c}{n_1\omega\cos\theta}\frac{\partial\varphi_R}{\partial\theta} \tag{13}$$

$$\tau_{\mathrm{GH}} = \frac{n_1}{c}\sin\theta\Delta x = -\frac{\tan\theta}{\omega}\frac{\partial\varphi_R}{\partial\theta} \tag{14}$$

4 Confirmation of the phase shift upon total reflection

If the total phase shift upon total reflection is expressed as Eq. (11), the total phase shift of the rays in length l shown in Fig. 1 in zigzag-ray model is

$$\phi_{\mathrm{ray}} = 2k_0n_1\frac{h}{\cos\theta} + \varphi_{12} + \beta\Delta x_{12} + \varphi_{13} + \beta\Delta x_{13} \tag{15}$$

which can be expanded as:

$$\begin{aligned}\phi_{\mathrm{ray}} &= 2k_0n_1h\cos\theta + 2k_0n_1h\sin\theta\tan\theta\\ &\quad + \varphi_{12} + \beta\Delta x_{12} + \varphi_{13} + \beta\Delta x_{13}\end{aligned} \tag{16}$$

Substituting Eqs. (2), (4), and (6) into Eq. (16),

$$\phi_{\mathrm{ray}} = (2\kappa h + \varphi_{12} + \varphi_{13}) + \beta l \tag{17}$$

is obtained.

The first term in the right hand of Eq. (17) equals $2m\pi$ for a guided mode, which is exactly the dispersion relation of Eq. (1). And the second term is the same as the phase shift in Eq. (7) obtained from the electromagnetic theory. Therefore, the zigzag-ray model of the waveguide coincides with the electromagnetic theory only with the condition that the phases arising from the Goos-Hänchen shifts are included. Only then, the expression of total phase shift in Eq. (11) is correct.

5 Confirmation of the Goos-Hänchen time

By employing the dispersion equation of an optical planar waveguide, it can be demonstrated that, in total internal reflection, the total time delay is exactly the sum of the group delay time and the Goos-Hänchen time. And the general form of the Goos-Hänchen time is also obtained in this section.

An optical planar waveguide as shown in Fig. 3 was taken into consideration, where a guiding layer of high refractive index n_1 is sandwiched between two semi-infinite claddings with low refractive index n_2. Dispersion equation of the waveguide for both TE and TM modes is

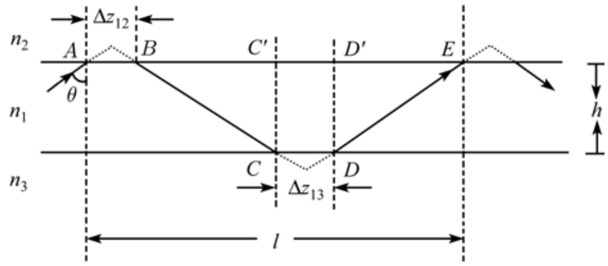

Fig. 3 Optical planar waveguide and ray propagation.

$$2\frac{\omega}{c}n_1h\cos\theta + \phi_{12} + \phi_{13} = 2m\pi,\ \ m = 0,1,2\cdots \tag{18}$$

with

$$\tan\frac{-\phi_{1j}}{2} = \gamma\frac{(n_1^2\sin\theta - n_j^2)^{1/2}}{n_1\cos\theta},\ \ \gamma = \begin{cases}1 & \text{(TE modes)}\\ \dfrac{n_1^2}{n_j^2} & \text{(TM modes)}\end{cases} \tag{19}$$

where ω is the angular frequency of the light, c is the velocity of the light in vacuum, h is the thickness of the guiding layer, θ is the angle of incidence of light, ϕ_{1j} is the phase shift of light in total internal reflection with j=2,0, and m is the mode order.

By performing the derivative of Eq. (18) with respect to angular frequency,

$$2\frac{m_1 h}{c}\cos\theta-2\frac{\omega}{c}n_1 h\sin\theta\cdot\frac{\mathrm{d}\theta}{\mathrm{d}\omega}+\frac{\partial\varphi_{12}}{\partial n_1}\frac{\mathrm{d}n_1}{\mathrm{d}\omega}+\frac{\partial\varphi_{12}}{\partial n_2}\frac{\mathrm{d}n_2}{\mathrm{d}\omega}$$
$$+\frac{\partial\varphi_{12}}{\partial\theta}\frac{\mathrm{d}\theta}{\mathrm{d}\omega}+\frac{\partial\varphi_{13}}{\partial n_1}\frac{\mathrm{d}n_1}{\mathrm{d}\omega}+\frac{\partial\varphi_{13}}{\partial n_2}\frac{\mathrm{d}n_2}{\mathrm{d}\omega}+\frac{\partial\varphi_{13}}{\partial\theta}\frac{\mathrm{d}\theta}{\mathrm{d}\omega}=0 \quad (20)$$

is obtained. Where $m_1 = n_1 + \omega\,\mathrm{d}n_1/\mathrm{d}\omega$ is the group index of the guiding layer. According to Ref. [17, 18], the group-delay times during total reflections upon two boundaries of the guiding layer are

$$\begin{aligned}\tau_{\mathrm{g}}^{(1,2)}&=\frac{\partial\varphi_{12}}{\partial\omega}=\frac{\partial\varphi_{12}}{\partial n_1}\frac{\mathrm{d}n_1}{\mathrm{d}\omega}+\frac{\partial\varphi_{12}}{\partial n_2}\frac{\mathrm{d}n_2}{\mathrm{d}\omega}\\ \tau_{\mathrm{g}}^{(1,3)}&=\frac{\partial\varphi_{13}}{\partial\omega}=\frac{\partial\varphi_{13}}{\partial n_1}\frac{\mathrm{d}n_1}{\mathrm{d}\omega}+\frac{\partial\varphi_{13}}{\partial n_2}\frac{\mathrm{d}n_2}{\mathrm{d}\omega}\end{aligned} \quad (21)$$

And then, the formula derived is:

$$\frac{\mathrm{d}\theta}{\mathrm{d}\omega}=\frac{2\frac{m_1 h}{c}\cos\theta+\tau_{\mathrm{g}}^{(1,2)}+\tau_{\mathrm{g}}^{(1,3)}}{2\frac{\omega}{c}n_1 h\sin\theta-\frac{\partial\varphi_{12}}{\partial\theta}-\frac{\partial\varphi_{13}}{\partial\theta}} \quad (22)$$

The propagation constant of the guided mode is

$$\beta=\frac{\omega}{c}n_1\sin\theta \quad (23)$$

Based on Eq. (23),

$$\frac{\mathrm{d}\beta}{\mathrm{d}\omega}=\frac{m_1\sin\theta}{c}+\frac{\omega}{c}n_1\cos\theta\cdot\frac{\mathrm{d}\theta}{\mathrm{d}\omega} \quad (24)$$

is obtained.

From Eqs. (22) and (24), the group velocity of the guided mode is

$$\begin{aligned}v_{\mathrm{g}}&=\frac{1}{\mathrm{d}\beta/\mathrm{d}\omega}\\&=\frac{2h\tan\theta-\frac{c}{\omega n_1\cos\theta}\frac{\partial\varphi_{12}}{\partial\theta}-\frac{c}{\omega n_1\cos\theta}\frac{\partial\varphi_{13}}{\partial\theta}}{\frac{2m_1 h}{c\cos\theta}-\frac{m_1\tan\theta}{\omega n_1}\frac{\partial\varphi_{12}}{\partial\theta}-\frac{m_1\tan\theta}{\omega n_1}\frac{\partial\varphi_{13}}{\partial\theta}+\tau_{\mathrm{g}}^{(1,2)}+\tau_{\mathrm{g}}^{(1,3)}}\end{aligned} \quad (25)$$

Next, the physical meaning of the group velocity in optical planar waveguide from the point of view of ray model is analyzed. The term $2h\tan\theta$ in Eq. (9) is the sum of BC' and $D'E$ in Fig. 1. According to Ref. [17], the following equation is obtained:

$$\begin{aligned}-\frac{c}{\omega n_1\cos\theta}\cdot\frac{\partial\varphi_{12}}{\partial\theta}&=\Delta z_{12}\\ -\frac{c}{\omega n_1\cos\theta}\cdot\frac{\partial\varphi_{13}}{\partial\theta}&=\Delta z_{13}\end{aligned} \quad (26)$$

which is the sum of two Goos-Hänchen shifts AB and CD. So, the numerator in the right hand of Eq. (25) is the length AE in which the rays in guiding layer complete one period of zigzag propagation. In the denominator of Eq. (25), because $2h/\cos\theta$ is the sum of two paths BC and DE, the term $2m_1h/(c\cos\theta)$ is the sum of light propagation times in BC and DE. The term $\tau_{\mathrm{g}}^{(1,2)}+\tau_{\mathrm{g}}^{(1,3)}$ is the sum of two group-delay times during total internal reflection on AB and CD. By using Eq. (10), the second and third terms in the denominator of Eq. (9) can be expressed as:

$$\begin{aligned}-\frac{m_1\tan\theta}{\omega n_1}\cdot\frac{\partial\varphi_{12}}{\partial\theta}&=\frac{m_1\sin\theta}{c}\Delta z_{12}\\ -\frac{m_1\tan\theta}{\omega n_1}\cdot\frac{\partial\varphi_{13}}{\partial\theta}&=\frac{m_1\sin\theta}{c}\Delta z_{13}\end{aligned} \quad (27)$$

Eq. (27) should be investigated in detail to find the physical meaning. If the n_1 medium has dispersion in Eq. (11), Eq. (12) should be expressed as:

$$\tau=\frac{\partial\varphi_R}{\partial\omega}+\frac{m_1}{c}\sin\theta\Delta x \quad (28)$$

From Eq. (28), it is seen that the physical meaning of Eq. (27) is the sum of two Goos-Hänchen times on AB and CD. Because the dispersion of the incident medium n_1 is also considered, Eqs. (11) and (13) are slightly different from Resch's expression for Goos-Hänchen time. Therefore, in general, the Goos-Hänchen time during total reflection is

$$\tau_{\mathrm{GH}}=\frac{m_1\sin\theta}{c}\Delta z=-\frac{m_1\tan\theta}{\omega n_1}\frac{\partial\phi_R}{\partial\theta} \quad (29)$$

Based on the analyses mentioned above, the group velocity of the guided mode in optical planar waveguide can be expressed as:

$$\frac{\mathrm{d}\omega}{\mathrm{d}\beta}=\frac{2h\tan\theta+\Delta z_{12}+\Delta z_{13}}{\frac{2m_1 h}{c\cos\theta}+\tau_{\mathrm{GH}}^{(1,2)}+\tau_{\mathrm{g}}^{(1,2)}+\tau_{\mathrm{GH}}^{(1,3)}+\tau_{\mathrm{g}}^{(1,3)}} \quad (30)$$

which shows that the time during total internal reflection upon AB is $\tau_{\mathrm{GH}}^{(1,2)}+\tau_{\mathrm{g}}^{(1,2)}$, and the time during total internal reflection upon CD is $\tau_{\mathrm{GH}}^{(1,3)}+\tau_{\mathrm{g}}^{(1,3)}$. Therefore, according to Fig. 3, Eq. (30) clearly demonstrates that the total time delay during total internal reflection includes two parts: the group-delay time and the Goos-Hänchen time.

6 Total reflection of TM plane wave upon ideal nonabsorbing plasma mirror

Aiming at Tournois's example of total reflection of TM plane wave upon ideal nonabsorbing plasma mirror [18], in this section, a study of a symmetric planar waveguide where the guiding layer is a vacuum and two cladding layers the ideal nonabsorbing plasma mirrors is made. Let ω_{p} be the plasma frequency and $u=\omega/\omega_{\mathrm{p}}$ where ω is the angular frequency of the light. The refractive index of the plasma is $n_{\mathrm{p}} = \mathrm{i}\,(1-u^2)^{1/2}$ with $0 < u < 1$. As the total reflection of TM plane wave have causality paradox, TM guided mode of the waveguide is discussed. Figure 4 shows the slab waveguide and zigzag-propagation of the rays for a guided mode with TM polarizations.

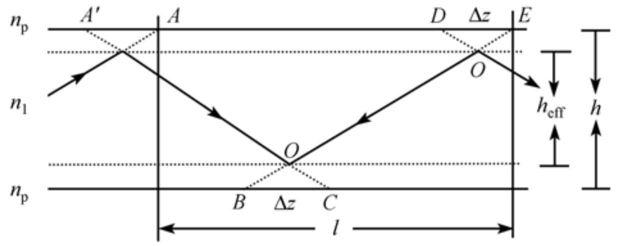

Fig. 4 Symmetric planar waveguide with both cladding layers constituted by ideal nonabsorbing plasma mirrors and zigzag-propagation of the rays for a TM guided mode.

In the ray model of the slab waveguide, the phase shift φ associated with total reflection of TM optical rays from vacuum upon two plasma mirrors with an angle of incidence θ is given by:

$$\tan\frac{\varphi}{2} = \frac{u\left(1-u^2\cos^2\theta\right)^{1/2}}{\left(1-u^2\right)\cos\theta} \tag{31}$$

According to Eq. (31), the group delay time [3] defined by $t_g = \partial\varphi/\partial\omega$ is

$$t_g = \frac{2\cos\theta}{\omega_p\left(1-u^2\cos^2\theta\right)^{1/2}}\frac{\left(1-u^2\cos 2\theta\right)}{\left(\cos^2\theta - u^2\cos 2\theta\right)} \tag{32}$$

And the Goos-Hänchen time is

$$t_{GH} = \frac{2\tan\theta\sin\theta}{\omega_p\left(1-u^2\cos^2\theta\right)^{1/2}}\frac{\left(u^2-1\right)}{\left(\cos^2\theta - u^2\cos 2\theta\right)} \tag{33}$$

For this case of total reflection, it is seen from Eq. (33) that the Goos-Hänchen time is always negative for TM wave. The negative Goos-Hänchen time is caused by the negative Goos-Hänchen shift that is

$$\Delta z = \frac{2\tan\theta}{k_0\left(\sin^2\theta - n_p^2\right)^{1/2}}\frac{n_p^2\left(1-n_p^2\right)}{n_p^4\cos^2\theta + \left(\sin^2\theta - n_p^2\right)} \tag{34}$$

where k_0 is the wave number of the light in vacuum.

A TM guided mode in the planar waveguide with an effective index of $N \equiv \beta/k_0 = \sin\theta$ is studied. The length l between A and E illustrated in Fig. 4, which corresponds to one period propagation of the zigzag-rays in the guiding layer, is the sum of 2htanθ and two Goos-Hänchen shifts at two boundaries of the guiding layer, that is

$$l = 2h\tan\theta + 2\Delta z = 2h_{eff}\tan\theta \tag{35}$$

where h is the thickness of the guiding layer and h_{eff} is the effective thickness [15] of the guided mode with the form of

$$h_{eff} = h + \frac{2}{k_0\left(N^2 - n_p^2\right)^{1/2}}\frac{n_p^2\left(1-n_p^2\right)}{n_p^4\left(1-N^2\right) + \left(N^2 - n_p^2\right)} \tag{36}$$

From Eq. (35) and Fig. 4, it is seen that the rays behave [15] as if they were propagating in a guide of effective thickness. Because the Goos-Hänchen shifts are negative for TM polarization, $h_{eff} < h$ holds true for TM mode, whereas $h_{eff} > h$ holds true for TE mode because of the positive Goos-Hänchen shifts.

Dispersion relation of the TM guided mode is

$$2\kappa h + 2\varphi = 2m\pi, \quad m = 0,1,2\cdots \tag{37}$$

where $\kappa = k_0(1-N^2)^{1/2}$ is the component of wave vector normal to the interface in the guiding layer, m is the mode order, and φ defined by Eq. (31) can be rewritten as:

$$\tan\frac{\varphi}{2} = -\frac{1}{n_p^2}\left(\frac{N^2 - n_p^2}{1-N^2}\right)^{1/2} \tag{38}$$

The group velocity is defined by:

$$\frac{1}{v_g} = \frac{\partial\beta}{\partial\omega} = \frac{N}{c} + k_0\frac{\partial N}{\partial\omega} \tag{39}$$

where c is the velocity of light in vacuum and $\partial N/\partial\omega$ can be calculated by performing partial derivative of Eq. (37) with respect to the angle frequency. The partial derivative of κ with respect to the angular frequency is

$$\frac{\partial\kappa}{\partial\omega} = \frac{\left(1-N^2\right)^{1/2}}{c} - \frac{k_0 N}{\left(1-N^2\right)^{1/2}}\frac{\partial N}{\partial\omega} \tag{40}$$

The partial derivative of φ with respect to ω is

$$\frac{\partial\varphi}{\partial\omega} = -\frac{n_p^2\left(1-n_p^2\right)}{n_p^4\left(1-N^2\right)+\left(N^2-n_p^2\right)}\frac{2N}{\left(1-N^2\right)^{1/2}\left(N^2-n_p^2\right)^{1/2}}\frac{\partial N}{\partial\omega} + \frac{\left(1-N^2\right)^{1/2}}{n_p^4\left(1-N^2\right)+\left(N^2-n_p^2\right)}\frac{2N^2-n_p^2}{\left(N^2-n_p^2\right)^{1/2}}\frac{\partial n_p^2}{\partial\omega} \tag{41}$$

where $\partial n_p^2/\partial\omega = 2\left(1-n_p^2\right)/\omega$. And then,

$$\frac{\partial N}{\partial\omega} = \frac{1-N^2}{k_0 c N h_{eff}}\left[h + 2\frac{1-n_p^2}{n_p^4\left(1-N^2\right)+\left(N^2-n_p^2\right)}\frac{2N^2-n_p^2}{k_0\left(N^2-n_p^2\right)^{1/2}}\right] \tag{42}$$

is obtained.

From Eq. (39), the group velocity of TM guided mode is

$$\frac{1}{v_g} = \frac{N}{c} + \frac{1-N^2}{cNh_{eff}}\left[h + 2\frac{1-n_p^2}{n_p^4\left(1-N^2\right)+\left(N^2-n_p^2\right)} \cdot \frac{2N^2-n_p^2}{k_0\left(N^2-n_p^2\right)^{1/2}}\right] \tag{43}$$

And then, the total propagation time of the TM guided mode in length l illustrated in Fig. 4 is

$$\tau_{\text{total}} = 2h_{\text{eff}} \tan\theta / v_{\text{g}}$$

$$= \frac{2\tan\theta}{cN}\left[h + 2\frac{n_{\text{p}}^2\left(2N^2-1\right)+2N^2\left(1-N^2\right)}{n_{\text{p}}^4\left(1-N^2\right)+\left(N^2-n_{\text{p}}^2\right)} \cdot \frac{1-n_{\text{p}}^2}{k_0\left(N^2-n_{\text{p}}^2\right)^{1/2}}\right] \tag{44}$$

With the help of Eqs. (32), (33), and (36), Eq. (44) can be rewritten as:

$$\tau_{\text{total}} = \frac{2h}{c\cos\theta} + \frac{4\tan\theta\sin\theta}{\omega_{\text{p}}\left(1-u^2\cos^2\theta\right)^{1/2}}\frac{\left(u^2-1\right)}{\left(\cos^2\theta-u^2\cos2\theta\right)}$$

$$+\frac{4\cos\theta}{\omega_{\text{p}}\left(1-u^2\cos^2\theta\right)^{1/2}}\frac{\left(1-u^2\cos2\theta\right)}{\left(\cos^2\theta-u^2\cos2\theta\right)}$$

$$= \frac{2h}{c\cos\theta} + 2t_{\text{GH}} + 2t_{\text{g}} \tag{45}$$

It is obvious that τ_{total} includes three parts: the propagation time of the rays in guiding layer 2h/(c cosθ), two Goos-Hänchen times, and two group-delay times occurring at two boundaries of the guiding layer. Therefore, according to the ray model of the guided mode demonstrated in Fig. 4, the total time delay associated with total reflection is exactly the sum of the group-delay time and the Goos-Hänchen time for TM polarizations. Although the Goos-Hänchen time is negative, the existence of this time coincides well with the rigorous frequency-domain electromagnetic theory of the waveguide. The same conclusion as Eq. (45) can also be obtained with the same method for TE guided mode where the Goos-Hänchen shift and time are both positive.

7 Solution to causality paradox

It has been shown that the total time delay of total reflection includes the Goos-Hänchen time. Therefore, the causality is preserved in Gires-Tournois interferometer [17]. But in the case of total reflection of a plane TM wave from a vacuum upon an ideal nonabsorbing plasma mirror, the negative Goos-Hänchen time can result in negative total time delay [18], which seems to violate relativistic causality. In this section, this problem is being solved.

In fact, the negative Goos-Hänchen time, which must be associated with the negative Goos-Hänchen shift, has its profound physical meaning. From the point of view of electromagnetic theory, because of $n_{\text{p}}^2 < 0$, the direction of the time-averaged Poynting vectors and its flux lines in the two cladding layers of the slab waveguide shown in Fig. 1 (b) is opposite to the propagation direction of the guided mode, resulting in the power flow of the guided mode concentrating in the range of effective thickness that is less than the thickness of the guiding layer. In the ray model, this effect can be explained as total reflections that occur at two boundaries of the effective thickness (see Fig. 4). This consideration is also supported by a calculation [10] that shows time-averaged Poynting vectors and its flux lines in the two media for a Gaussian beam in the case of a negative Goos-Hänchen shift. It is indicated that the incoming flux lines do not pass through the interface between two media at all, and the closed-loop flux lines exist around the interface. It is also shown [10] that the intersection of the incident beam axis and the reflected beam axis lies in front of the interface, and is located exactly at O as shown in Fig. 4. Therefore, the negative Goos-Hänchen shift and time are reasonable by considering that the points labeled as A, C, and E in Fig. 4 are selected as the reference points, whereas total reflections occur at the points labeled as O. From this point of view, Eq. (45) can be rewritten as:

$$\tau_{\text{total}} = \frac{2h_{\text{eff}}}{c\cos\theta} + \frac{-4\cos\theta}{\omega_{\text{p}}\left(1-u^2\cos^2\theta\right)^{1/2}}\frac{\left(u^2-1\right)}{\left(\cos^2\theta-u^2\cos2\theta\right)} + \frac{4\cos\theta}{\omega_{\text{p}}\left(1-u^2\cos^2\theta\right)^{1/2}}\frac{\left(1-u^2\cos2\theta\right)}{\left(\cos^2\theta-u^2\cos2\theta\right)} \tag{46}$$

The first term at the right hand of Eq. (46) is the propagation time of the rays in a guide of effective thickness in length l. The third term is the sum of two group-delay times. The second term, which is the sum of the two times associated with two negative Goos-Hänchen shifts, is obtained by considering that total reflections occur at the points O. Therefore, for total reflection of a plane TM wave from a vacuum upon an ideal nonabsorbing plasma mirror, the time associated with the negative Goos-Hänchen shift should be

$$t'_{\text{GH}} = \frac{-2\cos\theta}{\omega_{\text{p}}\left(1-u^2\cos^2\theta\right)^{1/2}}\frac{\left(u^2-1\right)}{\left(\cos^2\theta-u^2\cos2\theta\right)} \tag{47}$$

which is always positive. By employing Eq. (14), Eq. (47) can also be expressed as:

$$t'_{\text{GH}} = \frac{n_1 OC}{c} + \frac{n_1 BO}{c} + t_{\text{GH}}$$
$$= n_1\Delta x\left(\sin\theta - 1/\sin\theta\right)/c \tag{48}$$

which is exactly the sum of the three times: two propagation times of rays in OC and in BO, and the Goos-Hänchen time t_{GH} at the interface. Expression (48) is then the general form of the time caused by a negative Goos-Hänchen shift if calculated from the location where total reflection occurs, and is always positive.

8 Conclusions

It has been demonstrated that the total time delay upon total reflection is the sum of the group-delay time and the

Goos-Hänchen time. IIn the case of total reflection with a negative Goos-Hänchen shift, it has been proved that the time is still positive if taking the location where total reflection exactly occurs into account. Therefore, the causality paradox does not exist both in the disturbed Gires-Tournois interferometer case and in the case of total reflection of a plane TM wave upon an ideal nonabsorbing plasma mirror.

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