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RESEARCH ARTICLE

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The first loop for daily bus operation

Abstract We study the basic behaviors of buses using bus route models (BRM) by introducing a special kind of noise induced by impacts of other vehicles. The peak where the maximum velocity of buses exists shrinks in low noise conditions, which is worth the further study. Furthermore, we extend the model to take into consideration more realistic and important parameters, such as the number of passengers, the capacity of buses, and the possibility of overtaking, for performing simulations of the first loop. Suggestions on the choice of the number of buses and the maximum velocity are provided for the practical operation.

Keywords traffic physics, bus route model, first loop, daily bus operation

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1 Introduction

Recently, automobile traffic behaviors have attracted much attention from physicists. Several models are presented to study the traffic flow and jams [1－4]. Thereof, the bus operation is a representative many-body system similar to other automobile traffic systems while its dynamics differ due to the added interaction of buses with passengers at designated bus stops. The main motivation for studying the dynamics of bus routes is that they are so often unstable. Nonetheless, it is scarcely studied compared with traffic flow. The first model [5] was developed by Biham *et al.* early in 1992, but not until 1998, when O'loan *et al.* presented the cellular automaton (CA) BRM [6], the study of the bus transportation had been abeyant. From then on, a few extended versions have been suggested [7, 8], some other approaches have also been developed, for instance, the time-headway model of buses [9, 10] and the car-following model of buses [11－13].

The basic dynamics of bus transportation is explained as follows. Buses are initially designed to be spaced at regular intervals. If a bus is delayed by fluctuations, the gap (time headway) between it and its predecessor becomes larger because the delayed bus has to pick up more passengers. Additionally, more passengers will arrive at subsequent stops waiting for the bus to come. Consequently, the delayed bus becomes further delayed. On the other hand, the bus following the delayed bus finds fewer passengers waiting for it, allowing it to go faster until eventually it meets up with the delayed bus. Clusters of three, four, or more buses have been known to form in this way, resulting in slower service. Studies have shown that the occurrence of the bunching transition between an inhomogeneous jammed phase and a homogeneous phase varies along with the density of buses [6, 10, 13].

In this paper, we first extend the CA BRM by taking into consideration the effects of the other vehicles. We further extend the model by giving up the rule that the passengers on one site are only represented by a binary figure. Instead, we let the number of passengers at a stop vary with time to restitute the real circumstance. By this modification, more useful information, such as the number of passengers on a bus and effects of on-board and off-board of passengers at each stop, can be taken into account for the operation of the whole system. We add in effects of other factors, including the quantity of passengers, the capacity of buses, and the possibility of overtaking, in order to get some more realistic results. Suggestions are given to bus corporations as well as to the public transportation bureau for the practical consideration according to the result of simulations.

2 The bus route model

For the purpose of comparison, we present the primordial CA BRM below. The model is defined on a one-dimension lattice with the periodic boundary condition. Each lattice site denoted by a number i running from 1 to L can be regarded as a bus stop on the bus route. Site i has two binary variables τ_i and ϕ_i associated with it. These variables can be described in the following two terms.

(i) If site i is occupied by a bus, then $\tau_i=1$; otherwise $\tau_i=0$.

(ii) If site i has passengers on it, then $\phi_i=1$; otherwise $\phi_i=0$; A site cannot have both $\tau_i=1$ and $\phi_i=1$ (i.e., if there is a bus moving onto a site i, it is assumed that all the passengers on site i will get on the bus).

There are M buses in the system and the average bus density $\rho=M/L$ is a conserved quantity.

The update rules for the system are as follows.

(i) Pick a site i at random. This rule is replaced by a parallel updating rule in Ref. [8] and we will adopt the latter one when we refer to the primordial BRM.

(ii) If $\tau_i=0$ and $\phi_i=0$, then $\phi_i \to 1$ with probability λ.

(iii) If $\tau_i=1$ and $\tau_{i+1}=0$, define a hopping rate (or roughly say, a velocity) μ by rules of (a) $\mu=\alpha$ if $\phi_{i+1}=0$; (b) $\mu=\beta$ if $\phi_{i+1}=1$, and update $\tau_i \to 0$, $\tau_{i+1} \to 1$, and $\phi_{i+1} \to 0$ with probability μ.

Thus, $\mu=\alpha$ is the hopping rate of a bus moving over a site with no passengers and $\mu=\beta$ is the hopping rate of a bus moving over a site with passengers. It is assumed that β is smaller than α, reflecting the fact that a bus has to slow down to pick up the passengers. The probability that a passenger arrives at an empty site is λ. When a bus hops onto a site with passengers, it removes the passengers. Once a larger gap opens up between two successive buses, the gap is likely to grow further and the steady state in a finite system consists of a single jam of buses.

One feature in this model is worth emphasizing: an ability for buses to overtake each other would have almost no effect. This is because in a jammed situation, the interchange of a fast-moving bus with a slower moving one in front also results in an interchange of their velocities. We addressed it here because in our later discussion (Section 3.2), this feature no longer exists in our realistic model, the velocity of a bus is not solely determined by the situation of passengers on the next site. It is also determined by the number of passengers on the bus for the moment because there is a limit of the capacity.

3 Simulations of the realistic model

The primordial CA BRM simply treats the bus transportation as a self-developed many-body problem and does not embody other (kinds of) vehicles along the bus route, effects of passengers, capacity or overtaking of buses. As a consequence, although it can manifest some features of the bus transportation system, such as the occurrence of the bunching transition from an inhomogeneous jammed phase to a homogeneous phase, it's far from a complete redivivus happened in our everyday life. To overcome this drawback, we take into consideration the influence of other vehicles in Section 3.1 first and then effects of passengers, limited capacity and practical overtaking of buses in Section 3.2.

3.1 The Influence of other vehicles

To deal with features of vehicles other than buses independently, a great quantity of models can be found in Refs. [1—3]. However, it is difficult to directly add the effect of other kinds of vehicles to the bus operation in the CA BRM. In order to make the situation exercisable and compendious while maintaining the primary influence caused by other kinds of vehicles, here we simplify this influence in an effective way by representing it in the form of a noise term adding to the velocity of a bus.

Since the velocity of a bus in the CA BRM is indicated by the hopping rate μ, the noise term added would have impacts on it to a certain extent. Before doing this, we first redefine the hopping rate μ as the average hopping rate, so α represents the average hopping rate of a bus onto a site with no passengers and β represents the one with which a bus moves onto a site with passengers. We still keep the convention of $\alpha=1$ and $\beta<\alpha$ as the primordial BRM without loss of generality. Note that μ can be thought idealistically as the hopping rate in a condition that other vehicles could hardly affect the operation of the buses. In this way, it returns to its former definition in the primordial model.

Now $\theta_i(t)$ stands for the noise term for the ith bus at time t, where the sequence number of a bus is determined by the initial position of the bus along the route. We might as well suppose the number of other vehicles between two successive buses at any time is a constant and keep its initial value for the moment. Nevertheless, it is logical that $\theta_i(t)$ varies as the headway (gap between two successive buses, represented by $h_i(t)$, where i and t have the same meanings as in $\theta_i(t)$) changes from time to time, and we assume that their relation can be described by

$$-\Delta\theta_i(t) \propto \frac{\theta_i(t)\cdot\Delta h_i(t)}{h_i(t)} \tag{1}$$

There is a negative sign because as the time headway increases, there is more free space for each vehicle and influences to buses tend to be smaller. From the above relation we have

$$\theta_i(t) = \frac{c}{h_i(t)} \tag{2}$$

where c is a constant.

However, in the real situation the number of other vehicles between two successive buses also varies from time to time. Therefore, we modify our former supposition and combine this influence and those caused by accidental behaviors and preferences of drivers together as a fluctuation and represent it by a normal distributed term. Combining these two factors into a whole, we have

$$\theta_i(t) = \frac{c}{h_i(t)} + \left[\theta_i(t-1) - \frac{c}{h_i(t-1)}\right](1+\eta) \tag{3}$$

where η is actually similar to the relaxation factor between the consequential times. It is reasonable to suppose it to be a normally distributed random number with zero mean and variance of a small number, say 0.01 in this paper for the test purpose.

Now the hopping rate of the ith bus is represented by $\mu - \theta_i(t)$. In our later discussion, we set c equal to the unit and the maximum absolute value of $\theta_i(t)$ equal to the relative maximum absolute noise θ_m within the range of [0, 1]. Otherwise, if the absolute value exceeds this limit, we set it to θ_m. The initial values of θ_i (i = 1, 2, 3, ⋯) are randomly chosen in the range of $[-\theta_m, \theta_m]$.

To see whether this change for the hopping rate will bring an apparent effect on the bus operation, we perform over 2 000 simulations under a large number of sites, namely L = 700. Results for different values of parameters α, β, λ, and θ_m are shown in Figs.1—5. Note that we always keep $\alpha = 1$, and make some changes for β, λ and θ. These figures represent some typical results in comparison of the primordial BRM.

In Figs. 1 and 2, curves for the velocity μ as a function of the density ρ in large noise conditions ($\theta_m \geqslant 0.5$) are plotted. The curve of the primordial model (with $\theta_m = 0$) is drawn for comparison. We can see that in large noise conditions, the main effect of other vehicles is to reduce the velocity of buses. Slowing down the service completely erases the feature of the bus system itself. One more phenomenon we can find in these two figures is that the change of velocity in the small and middle ranges of the density is more sudden than that in the larger range, which is a natural result of Eq. (1).

Figures 3—5, depicted for the velocity μ as a function of the density ρ correspondingly in small noise conditions ($\theta_m \ll 0.5$), are much more interesting than Figs. 1—2. In both Figs. 3 and 4 we can see clearly that the difference between curves in small noise conditions and that of the primordial model nearly disappeared in high density. In Fig. 3, this phenomenon begins when ρ approaches 0.8 and in Fig. 4 it begins approximately at $\rho \approx 0.7$. It can be explained in this way: when the density is very high, the possibility of the appearance of a large gap is rare (see Fig. 6). This leads to the result that the distribution of buses along the route is quite homogeneous. That is to say, all values of μ are almost the same. Since we take c in Eq. (2) as 1 and the maximum noise θ_m as only 0.1, and $\theta_i(t)$ is mainly determined by the maximum noise. As a result, the probability is large that nearly all the buses are operated in the same noise condition, so the dynamics of our model returns to that of the primordial one. At the same time, such a high density makes it quite possible that the next site of a bus is occupied by another bus, and there is no chance for it to move on, since overtaking is not permitted here. From this fact, we can derive that the average velocity of buses are contributed by those buses which have a free site in front of them. The possibility that the site in front of a bus is not occupied by another bus is solely determined by the density. Although $\theta_i(t)$ has a small effect on α, its contribution to the average velocity is negligible when it comes to a great many of unmoved buses.

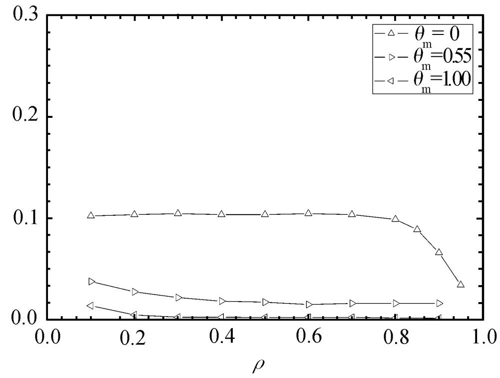

Fig. 1 Velocity μ as a function of density ρ in large noise conditions. The simulation is performed with L = 700, λ = 0.02, α = 1 and β = 0.1. The curve of the primordial model (with the maximum noise $\theta_m = 0$) is drawn for comparison.

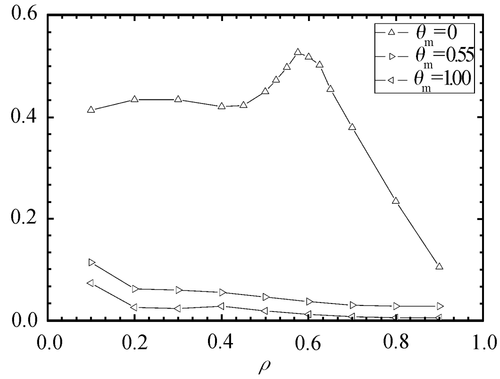

Fig. 2 Velocity μ as a function of density ρ in large noise conditions. The parameter set is the same as in Fig. 1 but $\beta = 0.4$. The curve of the primordial model (with the maximum noise $\theta_m = 0$) is also drawn for comparison.

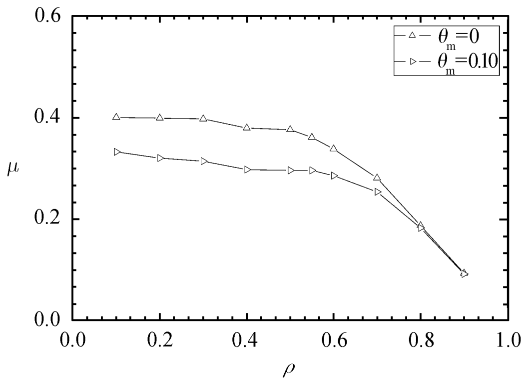

Fig. 3 Velocity μ as a function of density ρ in small noise conditions. The simulation is performed with L = 700, λ = 0.1, α = 1 and β = 0.4.

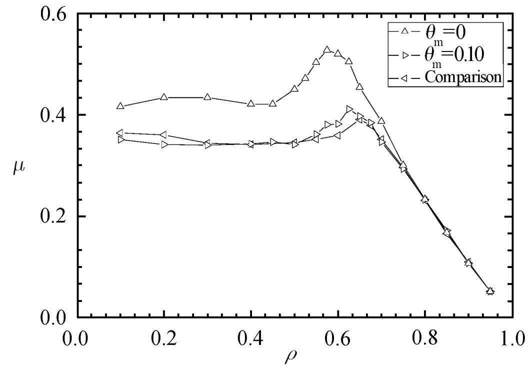

Fig. 4 Velocity μ as a function of density ρ in small noise conditions. The parameter set is the same as in Fig. 3 but λ = 0.02. The 'comparison' curve is drawn with L = 700, λ = 0.02, α =1 and β = 0.32.

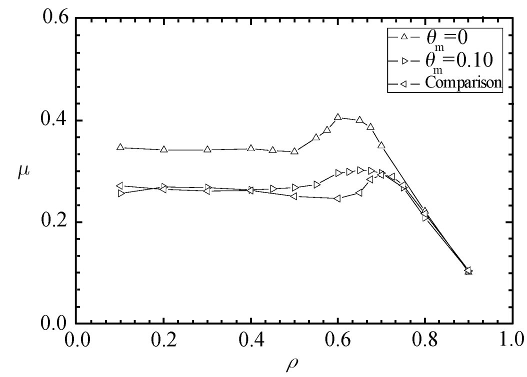

Fig. 5 Velocity μ as a function of density ρ in small noise conditions. The simulation is performed with L = 700, λ = 0.02, α = 1 and β = 0.32. The "comparison" curve is drawn with L = 700, λ = 0.02, β = 1 and β = 0.25.

What is even more interesting is the shape of crests of three curves in Fig. 4. The center of the crest (peak) of the upper curve shifted form $\rho\approx 0.57$ to $\rho\approx 0.65$ when a noise with the maximum absolute value of $\theta_{\rm m}$ = 0.1 is added. This is not all. Note that the width of the crest of the latter one is narrower than the curve denoted by "comparison", which has the same values of L, α, and λ with the other two curves but a smaller value of β. This phenomenon is quite different from our initial thought, since generally after the addition of a noise term, the width of the crest of the curve should become wider, *not* narrower! The phenomenon is also observed in Fig. 5 with an even more smaller value of β.

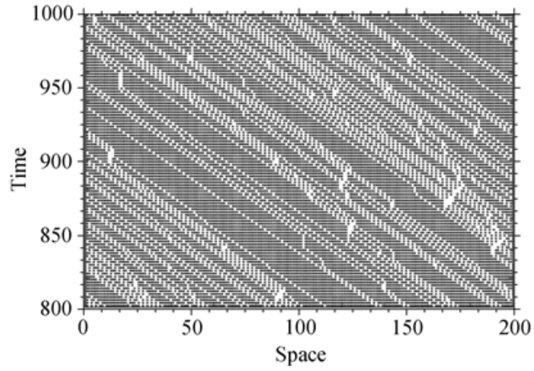

Fig. 6 Space-time plot of the bus position for L = 200, α = 1, β = 0.4, λ = 0.02, ρ = 0.8 and max noise 0.01. The buses and passengers are positioned randomly. There are 4 time steps between each snapshot on the time axis.

It is worth reminding that the real situation of the impacts of other kinds of vehicles to the bus operation is complicated. However, we argue that, in principle, these results of velocity vs. density can be tested experimentally.

3.2 Effects of passengers, capacity of buses and overtaking

As we have pointed out at the beginning of this section, one drawback of the primordial CA BRM is that it neglects the quality of passengers on each site, which may have an important influence on behaviors of the bus system. This has been noticed in an extended time-headway model [9], in which it is suggested that the arrival time of a bus is affected by the stopping time.

In the primordial CA BRM, the time is discrete and the interval between two consecutive times is comparable to the time it takes a bus to travel from one site to the next. Notice that we have adapted a simultaneous update rule. As a result, the stopping time is no longer suitable here. Therefore, we add a term $\Delta\mu$ to the hopping rate μ to delegate on-board and off-board effects, namely

$$\Delta\mu = \max(a_1\varsigma_i(t), a_2Q_i(t)) \tag{4}$$

where $\varsigma_i(t)$ and $Q_i(t)$ stand for number of passengers getting on and off the ith bus at time t, respectively. Here a_1 and a_2 are the related proportional coefficients, which usually take almost the same value. In our discussions, we neglect the difference of a_1 and a_2 and address a as a whole. In Eq. (4), max $(\cdots)$ is the maximum function. The practical meaning here is that the two processes, getting on and off buses, are carried out simultaneously and their influence on the hopping rate is determined by the duration of the longer process. Here with we want to emphasize three points.

(i) Different from the extended time-headway model, where the number of new passengers is proportional to the time headway, the new passengers appear by probability in our model. We define a term W_i represents the designed number of new passengers on site i, and nonetheless let the number of new passengers $\varpi_i(t)$ appearing on site i during two consecutive times from t to $t+1$ be

$$\varpi_i(t) = \text{floor}(RW_i) \quad (5)$$

where R is a uniformly distributed number within the range of [0, 1], floor (X) rounds the elements of X to the nearest integers less than or equal to X.

It is quite true that when W_i is less than 1, $\Delta\varsigma_i(t)$ is zero. But since we can set the time interval comparable to the real situation, it is always true that W_i is bigger than 1. If not, we may wonder why the bus company has chosen such a place to set up a bus stop.

(ii) The passengers getting on a bus at site i at time t is also varied by the remaining capacity of a bus, which is defined as:

$$\Delta M_i(t) \equiv M_0 - M_i(t) + Q_i(t) \quad (6)$$

where M_0 is the maximum capacity of a bus and $M_i(t)$ is the number of passengers on the ith bus at time t. We assume that the relationship between $Q_i(t)$ and $M_i(t)$ is

$$Q_i(t) = bM_i(t) \quad (7)$$

Since in a real situation, the number of passengers getting off is not strictly in positive proposition with $M_i(t)$, we let b vibrate between zero and $b_{\max}$. We will call $b_{\max}$ as b for convenience in our later discussion.

If only one bus arrives at site i (the case where more than one bus come at the same time will be discussed later) and the number of passengers on this site is less than $\Delta M_i(t)$, all the passengers will get on the bus. If the number of passengers is more than $\Delta M_i(t)$, only $\Delta M_i(t)$ of the passengers will be able to get on the bus. The other passengers will have to wait for the next one.

(iii) An additional natural thought is that the number of passengers on a bus may have influence on its velocity. We set $\mu_i(t)$ as follows:

$$\mu_i(t) = \mu_0 \cos(M_i(t)/M_0) - \max(a\varsigma_i(t), bQ_i(t)) \quad (8)$$

where μ_0 is the maximum velocity of a bus. It is reached when there is no passenger on the bus as well as on the site in front of it. Since the velocity is delegated by a hopping rate, it is possible that within a short period of time, the bus will remain on the same site. This is not a flaw of the model and it corresponds to the phenomenon that a bus is moving quite slowly somewhere between one site and the next in the real situation. What we need to do is just carefully set the update rules to guarantee that passengers get off and on the bus only at the time the bus arrives at the site.

We emphasize here that now the velocity of the bus is not only determined by the number of passengers on a site, but also by the number of passengers on the bus. As we have pointed out, the overtaking effect *cannot* be omitted any longer (This point has been overlooked by other models). A problem now comes with this change: what is likely to happen if two or more buses arrive at a site at the same time? According to our observation, if this happens, three possibilities may exist.

(i) When the number of passengers on this site is less than the sum of the remaining capacities of the two buses that has the fewest and the second fewest passengers $\Delta M_{\max}(t) + \Delta M_{\max*}(t)$, then both of the buses will be fully occupied.

(ii) If the number of the waiting passengers is smaller than $\Delta M_{\max}(t) - \Delta M_{\max*}(t)$, only the bus with the larger free space will pick up the passengers.

(iii) In other cases, the result of loading passengers will make $\Delta M_{\max}(t)$ and $\Delta M_{\max*}(t)$ to be the same.

In the second case, the passengers getting on must be more than the passengers getting off, hence $\mu_i(t)$ has the form

$$\mu_i(t) = \mu_0 \cos(M_i(t)/M_0) - a\varsigma_i(t) \quad (9)$$

An important difference in our model is the choice of the initial condition. We replace the random distribution of buses and passengers by the following rule: The first bus leaves the first site at $t = 0$ and the following buses leave with the time interval t_{int}. This will be explained in detail in Section 3.3.

3.3 First loop of the bus operation

3.3.1 The short-range bus system

In our following discussions, the short-range bus system (i.e., city bus system) is considered and the first loop of the bus operation is focused on. The long-range bus system (i.e., interstate bus system) and whole day's operation will be discussed elsewhere.

Suppose that in the real world the bus transportation starts at 6:00 in the morning and the last bus leaves the starting site at 10:00 in the evening. We call the duration of this process as a whole day's operation time T. If we make the period between two consecutive time t and $t+1$ in our model, a unit of t_{int}, stand for 5 minutes, then T corresponds to 193 times in our model. Since the simulation of a real bus system is not only a purely theoretical analysis, we can take the quantity of sites along the route, which is denoted by L in our following discussion, far smaller than what we used in Sections 3.1 and 3.2 (however, even larger quantity of sites is used in Ref. [6]).

For a short-range bus system, a practical value of the period between two consecutive times, represented by t_{int}, from 5 to 15 minutes is a good approximate for the simulation. Note that the case where buses travel around a short route by several loops is not equivalent to the case where buses travel around a much longer route just once. On one hand, in the latter case, the number of passengers on sites in front of the leading bus keeps on accumulating until a bus arrives. On the other hand, there is an obvious front and back of the bus queue and chances for buses in the front and in the back to interact with each other are quite small, while in the former case, one cannot tell that buses apart only by their initial positions and every bus has a similar chance to interact with the others.

3.3.2 The first loop

The first loop is defined as the process starting from $t = 0$ till the time a bus returns to the first stop. Below we give two parameters that are needed to describe this process:

(i) T_0: the duration of the first loop.

(ii) N_0: the number of buses in operation along the route at time T_0.

These two parameters can help us when choosing proper values for the time interval t_{int} and the maximum velocity μ_0. The value of T_0 gives us an approximation for the average time a bus travels a loop, and the other parameter, N_0, can be referred when deciding the number of buses needed to operate along the route. The number of needed buses reflects the fixed cost of the operation, not only for the amount of money to purchase the buses but also the cost to employ a necessary number of drivers, conductors and mechanics, etc.

In short, effects of passengers and capacity on the global characteristics (T_0, N_0, and μ_0) of the first loop are linked by parameters (L, W_i, a, b, and t_{int}) of our extension model. We need to know their relations in some detail for further consideration for possible practical applications. Actually these parameters are not of the same importance in different conditions.

In Figs. 7 and 8, the five curves are drawn with $t_{\text{int}} = 3$, $a = 0.007$; $t_{\text{int}} = 3$, $a = 0.003$; $t_{\text{int}} = 3$, $a = 0.001$; $t_{\text{int}} = 2$, $a = 0.001$; and $t_{\text{int}} = 1$, $a = 0.001$; respectively. We carry out the simulation over 200 times and show the average results. In Fig. 7 we can see that the general trend of the T_0-μ_0 curves decreases as μ_0 gets larger. Besides the drop as a whole, there are some small fluctuations, which reflect the random arrival of new passengers and probabilistic updating rules of the BRM. Also in Fig. 7, the differences between the three curves indicated by $a = 0.001$ are very small, and the difference of T_0 caused by the variance of t_{int} never exceeds 20 percent in our model. From this we can see that there is no finite relationship between T_0 and t_{int}. The relationship is so weak and it is covered by the innate random mechanics of the model.

What is more, in Fig. 8 we cannot find any notable difference of N_0 caused by the variance of parameter a. The result indicates that when the number of new passengers, W_i, is fixed, the influence on the velocity by passengers getting on and off a bus is quite small compared with other influences.

The fact that t_{int} has a strong influence on N_0, shown in Fig. 9, is easy to understand, since for a fixed period of time, N_0 is in negative proportion with t_{int}. Although here the period of time (T_0) is not exactly the same (see Fig. 8), the difference has not reached such an extent that it can change the relation between N_0 and t_{int} here.

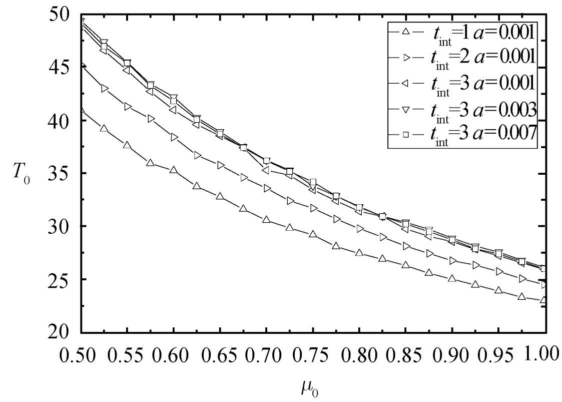

Fig. 7 T_0 as a function to μ_0 with $L = 20$, $W_i = 3$ and $b = 0.1$. The five curves are drawn with $t_{\text{int}} = 3$, $a = 0.007$; $t_{\text{int}} = 3$, $a = 0.003$; $t_{\text{int}} = 3$, $a = 0.001$; $t_{\text{int}} = 2$, $a = 0.001$; and $t_{\text{int}} = 1$, $a = 0.001$, respectively.

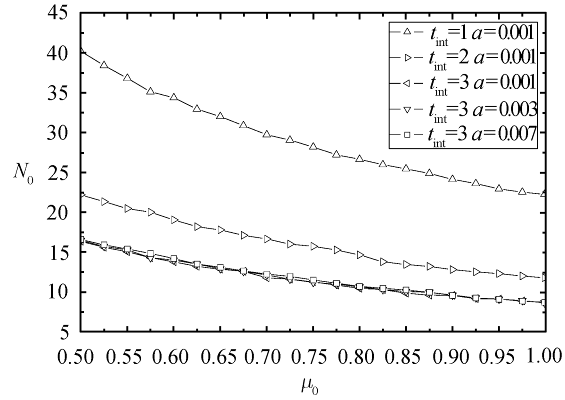

Fig. 8 N_0 as a function to μ_0 with the same data set of Fig. 7.

In Figs. 9 and 10 we show the influence of W_i on T_0 and N_0, respectively. We can find that the gratitude of the effect of W_i is comparable to the effect of t_{int}. This gives us a strong support for the alteration of the rule of passengers in the primordial CA BRM for our analysis of daily bus operation.

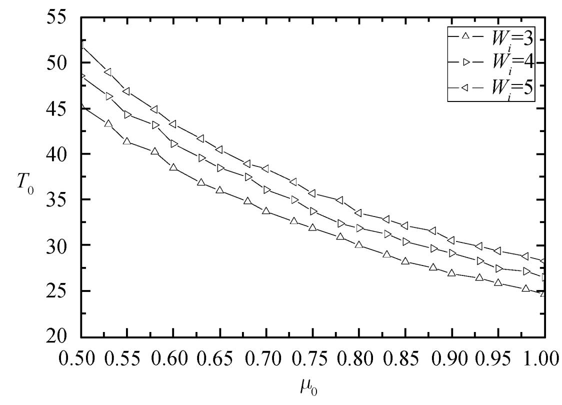

Fig. 9 T_0 as a function to μ_0 with $L = 20$, $t_{\text{int}} = 2$, $a = 0.001$, and $b = 0.1$. The three curves are drawn with $W_i = 3$, $W_i = 4$, $W_i = 5$, respectively.

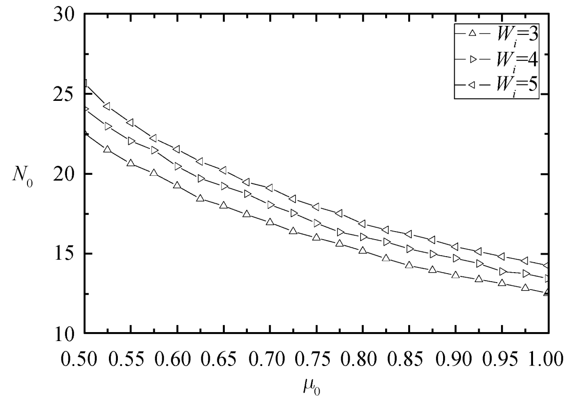

Fig. 10 N_0 as a function to μ_0 with L = 20, t_{int} = 2, a = 0.001 and b = 0.1. The three curves are drawn with W_i = 3, W_i = 4, W_i = 5, respectively.

One more feature we want to stress is that margins of the curves here decrease less rapidly as those in Figs. 7 and 9. It means that when μ_0 is fixed, the effect of W_i on T_0 and N_0 less than the effect of t_{int}.

Now let's see whether our numerical results correspond to the real situation. We take the data of t_{int} to be equal to 2 and μ_0 = 0.75. In this situation, buses set out from the starting point every 10 minutes and the minimum traveling time between two consecutive sites is a little bit less than 7 minutes. In Fig. 10 (W_i = 3), we can see that about 17 buses are needed to carry out the transportation and the time for the first loop is 2 hours and 40 minutes. It agrees with the real situations quite well.

An interesting problem that draws our attention is which bus returns to the starting site first. The logical thought would be that as t_{int} becomes larger, it becomes harder for the later buses to catch up with those in front, so the sequence of the buses is hard to disturb. This is corroborated by our simulations.

In Fig. 11 we show the result of the number of occurrences for the first returning bus among 2000 simulations performed with W_i = 3, μ_0 = 0.5, and a = 0.001. The plot is drawn for t_{int} being 2 and 3. From here we can find that when t_{int} = 2, it is the first bus that comes back first. But when t_{int} = 3, the first three buses nearly have the same chances to return first and the chance for other buses increases slightly. A different possible factor of μ_0 may sway the process. The result caused by the decrease of μ_0 is that more buses have the probability to return first. These phenomena have been actually observed in our daily bus operation.

The evidence in Fig. 12 is that the "tail" of the plot with μ_0 = 0.5 extends much wider than the plot with μ_0 = 0.75. The influence of μ_0 on the first few buses is more complicated. For example, in the upper plot of Fig. 12 where μ_0 = 1, the probability for the third bus to return first is much greater than other buses and the chance for the other buses declines gradually as their sequence number becomes far from 3, while in the middle plot of Fig. 12, where μ_0 = 0.5, the probability for the first bus to return first is quite small. Since we know that once an overloaded bus is slowed down, another bus with less passengers on board may overtake it. However, the overtaking bus has to pick up more passengers, so it also becomes overloaded and slows down and is overtaken by other buses. This is quite true for the first loop, because the sites before the leading bus have not been visited before and there are a great number of waiting passengers for the overtaking bus to pick up. In this situation, the velocity of the overtaking bus slows down quickly. Together with the chosen values of L and W_i, it provides chances of the velocity for some buses to increase and for others to decrease. But if the overtaking happens at those sites that have been visited, for example, after the first loop, it takes a much longer time for the overtaking bus to slow down to such an extent that it is overtaken by another bus. Notice that the number of sites along the route is small, one can understand this easily. This distinct character of the first loop is another reason why we pick it out for this special description.

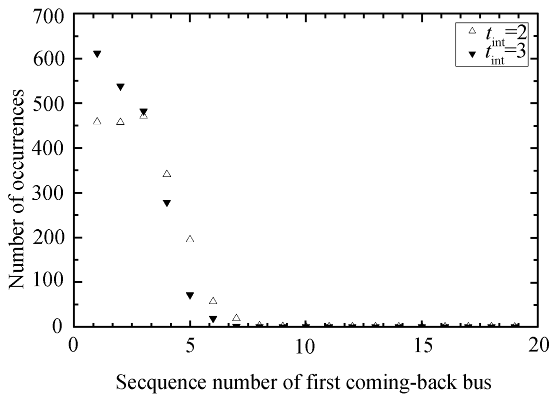

Fig . 11 The plot of 2000 times simulations of the first coming-back bus with W_i = 3, a = 0.001, μ_0 = 0.75 and t_{int} = 2 and 3, respectively.

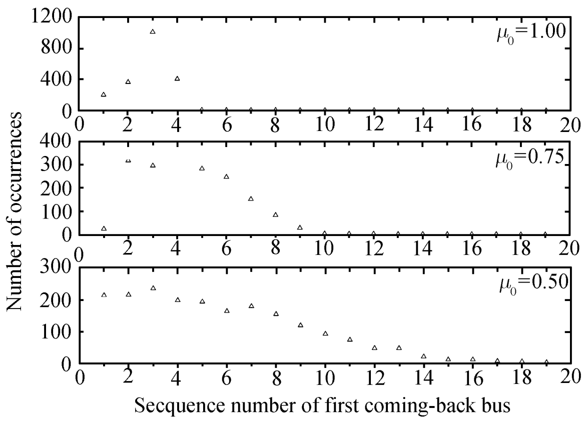

Fig. 12 The plot of 2000 times simulations of the first coming-back bus with W_i = 3, a = 0.001, t_{int} = 1, μ_0 = 1, 0.75, and 0.5, respectively.

4 Summary and outlook

We studied the behaviors of buses in the extension of the approach of bus route models by introducing noise from other vehicles. The daily bus operation in both high and low noise conditions is simulated mainly in random functions so as to tally with realistic conditions. In large noise conditions, the main effect of the other vehicles is to heavily slow down the velocity of buses in all range of the density. Notwithstanding, it is interesting that in low noise conditions the peak of velocity shrinks in width in the middle range, which is worth further studying. Furthermore, we extend the model to take into consideration more parameters, for simulations of impacts of on- and off-board passengers, the capacity, and overtaking of buses, for the first loop of a real bus operation.

With a combination of the above aspects, which can be regarded as an extension of the primordial BRM, it is possible to provide the proper choice of the required number of buses and the maximum bus velocity in a real situation.

The number of the needed buses related to N_0 to carry out the transportation task reflects the fixed cost of the operation. The total effective mileage, also related to T_0 on the other hand, is a good parameter to denote the variable cost of the operation. Through the analysis of the total mileage, one can make an approximation of the cost of gasoline, maintenance of buses, and so on, which can help balance profit and cost. It is also an important factor, the average flux, roughly represented by the value of $N_0 T/T_0$, is also an important factor to show the extent of the burden the bus transportation adds to the traffic. Finally, we know that complaints of passengers usually come from the long waiting time. To solve this problem, the result of simulations of the average waiting time can give us some merits. By setting μ_0 and $t_{\rm int}$, one can adjust the average waiting time to achieve the proper outcome. All these parameters, closely related to the current work and for further study, are very useful for bus corporations as well as the public transportation bureau for practical consideration.

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