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## Feedback control of quantum system

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**Abstract** Feedback is a significant strategy for the control of quantum system. Information acquisition is the greatest difficulty in quantum feedback applications. After discussing several basic methods for information acquisition, we review three kinds of quantum feedback control strategies: quantum feedback control with measurement, coherent quantum feedback, and quantum feedback control based on cloning and recognition. The first feedback strategy can effectively acquire information, but it destroys the coherence in feedback loop. On the contrary, coherent quantum feedback does not destroy the coherence, but the capability of information acquisition is limited. However, the third feedback scheme gives a compromise between information acquisition and measurement disturbance.

**Keywords** quantum feedback control, information acquisition, state recognition, coherence

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### 1 Introduction

Control of quantum system is a most important task in atomic physics, molecular chemistry and quantum information [1–5]. Especially in the rapidly developing quantum information

science [6], since information is represented with quantum bit (qubit) and quantum information processing is essentially the manipulation of quantum state, the ability to control quantum system at will is necessary. Hence quantum control theory rapidly develops in recent years, where it expects to determine how to drive quantum mechanical systems from an initial given state to a pre-determined target state with some given time  $T$  [7]. The current main fields of quantum control research include controllability [8–11], quantum optimal control [12–14], coherent control [15–17] and quantum feedback control [18–24]. The controllability answers whether a controller can drive a quantum system to a desired state. Some necessary and sufficient conditions on controllability of some finite-dimensional Hamiltonian quantum systems have been obtained. Quantum optimal control theory is applied to find the best control fields to achieve the desired target. Coherent control is a particularly powerful quantum control method and it has been widely applied to chemical reaction, especially selectively breaking and making chemical bonds in polyatomic molecules.

Moreover, quantum feedback control has interested many scientists. As we all know, feedback is a most effective strategy in classical control theory and the aim of feedback is to compensate the effects of unpredictable disturbances on a system under control, or to make automatic control possible when the initial state of the system is unknown. In quantum feedback, scientists expect to obtain information about the system, process the information and feed it back to the system to complete active control of quantum system in a desired way. To control a system, one must obtain the information about the state of system through measurement. However, a quantum system is essentially different from a classical one. The measurement on a classical system doesn't change its state, but commonly a measurement of a quantum system will disturb its quantum state. Hence quantum feedback control may have many characteristics different from classical feedback. The first theoretical work on quantum feedback was presented by Yamamoto *et al.* [19, 25], where they treated the fluctuations of the photocurrent in negative feed-

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back way to generate amplitude squeezed state. Since then, scientists have proposed many quantum feedback schemes such as Markovian quantum feedback [20], Bayesian quantum feedback [21], non-Markovian quantum feedback with time delay [22], coherent quantum feedback [23] and quantum feedback based on cloning and recognition [24]. In this paper, we review different quantum feedback control strategies and divide them into three classes according to the different methods of information acquisition: quantum feedback control with measurement, coherent quantum feedback control and quantum feedback control based on cloning and recognition.

This paper is organized as follows. In Section 2, we briefly introduce several information acquisition methods based on measurement including projective measurement, POVM measurement and QND measurement. Section 3 reviews some quantum feedback strategies with quantum measurement including Markovian quantum feedback, Bayesian quantum feedback and non-Markovian quantum feedback with time delay. In Section 4 and Section 5, we introduce coherent quantum feedback and quantum feedback based on cloning and recognition. Concluding remarks are given in Section 6.

## 2 Information acquisition in quantum feedback control

In quantum control, the systems to be controlled are mainly some quantum systems, whose states are called quantum states. According to quantum theory, the measurement on a quantum system will make its state collapse, which is the greatest difficulty for information acquisition in quantum feedback control. However, measurement is a main method for acquiring feedback information. Hence we will briefly introduce several quantum measurement including projective measurement, POVM measurement and QND measurement [26].

### 2.1 Projective Measurement

A projective measurement is described by an observable,  $M$ , a Hermitian operator on the state space of the system being observed [27]. The observable has a spectral decomposition,

$$M = \sum_m m P_m \quad (1)$$

where  $P_m$  is the projector onto the eigenspace of  $M$  with eigenvalue  $m$ . The possible outcomes of the measurement correspond to the eigenvalues,  $m$ , of the observable. Upon measuring the state  $|\psi\rangle$ , the probability of getting result  $m$  is given by

$$p(m) = \langle \psi | P_m | \psi \rangle \quad (2)$$

If the outcome  $m$  occurred, the state of the quantum system immediately after the measurement is  $\frac{P_m |\psi\rangle}{\sqrt{p(m)}}$ .

### 2.2 POVM Measurement

POVM stands for ‘‘Positive Operator-Valued Measure’’. Let's define

$$E_m \equiv M_m^\dagger M_m \quad (3)$$

where  $\{M_m\}$  are the general measurement operators and satisfy the completeness equation:

$$\sum_m M_m^\dagger M_m = I \quad (4)$$

Thus  $E_m$  is a positive operator satisfying

$$\sum_m E_m = I \quad (5)$$

and

$$p(m) = \langle \psi | E_m | \psi \rangle \quad (6)$$

The operator  $E_m$  is a POVM element and the complete set  $\{E_m\}$  is a POVM [27]. Here different operators  $E_m$  are not necessarily orthogonal, however, the operators  $P_m$  must be orthogonal in projective measurement.

In fact, POVM measurement is a kind of generalized quantum measurement. Under some certain circumstances, it is possible to get more information through POVM measurement than through projective measurement. However, projective measurement and POVM measurement inevitably destroy the quantum state to be measured. Hence some scientists show interest in quantum nondemolition (QND) measurement [28].

### 2.3 QND Measurement

Consider a system that has some observable  $\hat{A}$ . We define a QND measurement of  $\hat{A}$  as a sequence of precise measurement of  $\hat{A}$  such that the result of each measurement is completely predictable from the result of the first measurement-plus, perhaps, other information about the initial state of the system [29]. Quantum nondemolition measurements are ideal tools for use in the detection of weak external forces that act on the system. However, most observables cannot, even in principle, be monitored in a QND way. A general necessary and sufficient condition that  $\hat{A}$  can be monitored in a QND way is that the QND meter must satisfy [28]:

$$[\hat{A}, \hat{U}] |\psi\rangle = 0 \quad (7)$$

where  $|\psi\rangle$  is the initial state of the quantum meter and  $\hat{U}$  is the operator of the joint evolution of the quantum meter and the object under study.

QND measurement is a kind of precise measurement and can be used to quantum feedback control [30]. Unfortunately, only some specific examples can use QND to acquire feedback information since only some specific observables are QND observables.

### 3 Quantum feedback control with measurement

Based on the discussion in Section 2, we know that quantum measurement generally destroys the state of system to be monitored. Even though measurement, as in classical feedback control, is still a main way for information acquisition in quantum feedback control. Hence quantum feedback control with measurement is first proposed. Since the first theoretical work on quantum feedback was presented [19, 25], some other quantum feedback strategies including Markovian quantum feedback [20], Bayesian quantum feedback [21] and non-Markovian quantum feedback with time delay [22] also have been proposed.

#### 3.1 Markovian quantum feedback

Markovian quantum feedback scheme is first proposed by Wiseman and Milburn. In 1993, they presented a quantum theory of optical feedback via homodyne detection and solved the problem of quantum-limited optical cavity feedback for the case of instantaneous feedback of the homodyne photocurrent. In their quantum feedback theory, the homodyne photocurrent is immediately fed back onto optical cavity to alter the dynamics of the source cavity and may then be forgotten, so the master equation describing the resulting evolution is Markovian and the feedback scheme is called Markovian quantum feedback [20]. The feedback control system is an open system and its state can be described by density operator  $\rho$ . They obtained the general master equation for homodyne-mediated feedback [20]:

$$\dot{\rho} = -i[H_0, \rho] + \mathbf{D}[a]\rho + \mathcal{K}(a\rho + \rho a^\dagger) + \frac{1}{2\eta}\mathcal{K}^2\rho \quad (8)$$

where  $H_0$  is internal Hamiltonian,  $a$  and  $a^\dagger$  are annihilation operator and creation operator respectively, and  $\mathbf{D}[a]$  is a superoperator satisfying

$$\mathbf{D}[a]\rho = a\rho a^\dagger - \frac{1}{2}(a^\dagger a\rho + \rho a^\dagger a) \quad (9)$$

$\mathcal{K}$  is a Liouville superoperator satisfying  $\mathcal{K}\rho\mathcal{K} = -i[A, \rho]$  for arbitrary Hermite operator  $A$ , and  $\eta$  is the efficiency of the detection system.

This feedback scheme has been applied to lock the phase of a regularly pumped laser and the result shows that it can produce perfect squeezing on resonance in the output light [20, 31]. It is also used to complete some other tasks such as stabilizing the internal state of atoms [32] and cavity-QED test [33].

#### 3.2 Bayesian quantum feedback

Markovian quantum feedback uses each instant measurement signal to control the Hamiltonian of a quantum system

and doesn't use the previous knowledge about the system to be controlled. In fact, there are many other ways in which the measurement signal may be fed back to affect the system. Therefor Doherty and Jacobs presented a quantum feedback scheme using continuous state estimation in 1999, where they used certain integrals of the measurement records to provide specific information [21]. They made the best of the detailed information from measurement, and divided quantum feedback control process into two steps: a state estimation step and a feedback control step. Since the best state estimation will use all previous measurement results, not just the latest one, which is analogous to classical Bayesian estimation, the feedback is called Bayesian quantum feedback [21, 34, 35].

Recently, Doherty *et al.* also tried to consider separately optimizing state estimation step and feedback control step [36]. They considered optimizing desirable properties of each under the minimal constraint that the available strength of both is limited and identified an information tradeoff between information and disturbance in quantum feedback control. Wiseman *et al.* have done comparative research on Bayesian quantum feedback and Markovian quantum feedback, and the result has shown that Bayesian quantum feedback is never inferior, and is usually superior, to Markovian quantum feedback in stabilizing the quantum state of the simplest nonlinear quantum system [34].

Bayesian quantum feedback has been used to cool and confine a single quantum degree of freedom [21], switch the state of a particle in a double-well potential [35] and stabilize the internal state of atoms [34]. Besides, Korotkov *et al.* used it to control the decoherence of solid-state qubit [37, 38]. The scheme is shown as Fig. 1. The state of qubit is described by  $\rho$  and it interacts with environment. The current  $I(t)$  from the detector is used to calculate  $\rho(t)$  through evolution equation, then compare it with desired  $\rho$  and obtain the feedback signal. Hence one can produce suitable control to suppress qubit decoherence. The result showed that Bayesian quantum feedback can be used to control decoherence [37].

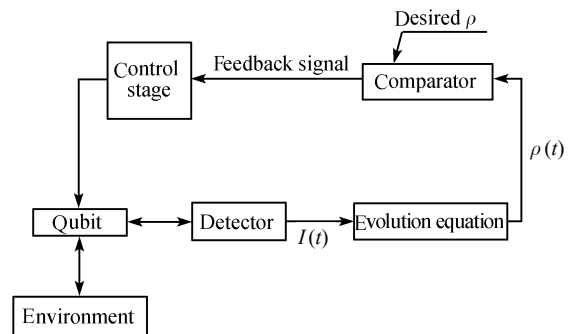


Fig. 1 Control of decoherence of solid-qubit using quantum feedback [37].

#### 3.3 Non-Markovian quantum feedback with time delay

Both Markovian quantum feedback and Bayesian quantum

feedback ignore the effect of feedback time delay, that is to say, we always assume the feedback delay  $\tau \rightarrow 0$ . As a result, the effect of feedback can be expressed in terms of an effective master equation and the problem is much easier to handle. The treatment is justified whenever the feedback delay time is much smaller than the system time scale. When we consider some stationary state phenomenon, the feedback delay time  $\tau$  has to be compared with the cavity relaxation time  $\gamma^{-1}$  and for some sufficiently good systems the condition  $\gamma\tau \ll 1$  is usually satisfied [22]. However, the delay can't be ignored in some situations. For example, when we use the quantum feedback for decoherence control, the delay  $\tau$  and the decoherence time  $t_{\text{dec}}$  must be considered simultaneously. In many cases, the unavoidable nonzero feedback delay time may have important effects and it would be important to deal with the exact non-Markovian problem with  $\tau \neq 0$  [22].

Giovannetti and co-workers first studied the non-Markovian quantum feedback with time delay in 1999 [22]. They completely solved the non-Markovian dynamics in the presence of a nonzero feedback delay by considering the time evolution of the probability distribution of the measured field quadrature and of the characteristic function. They also considered in particular the possibility of inhibiting the decoherence of a Schrödinger-cat state initially generated in the cavity and their results showed that feedback can also improve the dynamics of quantum systems for the delay time not too large [22]. In 2001, Wang *et al.* also gave a rigorous analysis of an anti-decoherence feedback scheme in a two-level atom with nonzero feedback time delay [39]. Using numerical simulations they found that the effect of nonzero time delay is qualitatively similar to the results obtained for no time delay but with inefficient detection and the time delay  $\tau$  which is not too large is not fatal to decoherence control.

#### 4 Coherent quantum feedback

Markovian quantum feedback, Bayesian quantum feedback and non-Markovian quantum feedback with time delay all use the feedback information from measurement results. According to the discussion of Section 2, we know that measurement destroys the quantum characteristics of feedback information, so feedback information becomes classical information. As a result, although the system under control is quantum system, feedback controller processes classical information and effective quantum channel in feedback loop isn't constructed, so the strategies can be called quantum control with classical feedback.

To remain the coherence in feedback loop, Lloyd proposed a coherent quantum feedback scheme in which the sensors, controller, and actuators all are quantum systems that interact coherently with the system to be controlled [23, 40]. In this

feedback strategy, the controller gets, processes and feeds back quantum information, so the quantum characteristics in feedback loop have not been destroyed and the strategy can accomplish some tasks such as entanglement transfer that are not possible using classical feedback.

Consider the entanglement transfer in the case of spin system. The goal of the control process is to put the system spin in an entangled state [23]

$$\frac{1}{\sqrt{2}} |\uparrow\rangle|\uparrow\rangle_a + \frac{1}{\sqrt{2}} |\downarrow\rangle|\downarrow\rangle_a$$

where  $|\uparrow\rangle_a$  and  $|\downarrow\rangle_a$  are states of the "ancilla". If the direct interaction of the two spins is not allowed, they can not become entangled through the exchange of classical information alone. Hence quantum control with classical feedback can not accomplish the task. However, coherent quantum feedback can achieve the goal. First, prepare the ancilla spin in the state  $(1/\sqrt{2})(|\uparrow\rangle_a + |\downarrow\rangle_a)$ , then entangle the ancilla and controller spins by performing a CNOT on the controller spin with the ancilla spin as control. If the initial state of system spin is  $(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$ , the state of the three spins is [23]:

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)(1/\sqrt{2})(|\uparrow\rangle|\uparrow\rangle_a + |\downarrow\rangle|\downarrow\rangle_a) \quad (10)$$

Now perform the quantum feedback control and apply a third  $\pi$  pulse to flip the controller spin if and only if the system spin is in the state  $|\uparrow\rangle$ . As a result, the final state becomes [23]

$$(1/\sqrt{2})(|\uparrow\rangle|\uparrow\rangle'' + |\downarrow\rangle|\downarrow\rangle'')(\alpha|\uparrow\rangle' + \beta|\downarrow\rangle') \quad (11)$$

Thus the coherent quantum feedback completes the goal of producing the desired entanglement between the system spin and the ancilla despite no direct interaction.

Moreover, since quantum control with classical feedback involves measurement, it is typically stochastic and destructive. However, coherent quantum feedback is deterministic and nondestructive [23].

#### 5 Quantum feedback control based on cloning and recognition

Quantum feedback control with measurement has strong ability in information acquisition, but it destroys the coherence in feedback loop. On the contrary, coherent quantum feedback preserves the coherence of feedback loop, but its capability of information acquisition is limited. Therefore we consider a compromise between information acquisition and coherence maintenance [24, 41]. We separate quantum feedback control design into a state recognition step involving measurement and a feedback control step without measurement. This is obviously different from traditional quantum feedback since there the information acquisition necessarily results in destroying the state of quantum system. In the feedback

scheme, quantum cloning and recognition are two important aspects, so we call it quantum feedback control based on cloning and recognition.

The general picture of quantum feedback control based on cloning and recognition is as follows (Fig. 2) [24]. The quantum system to be controlled is called controlled object. The actuator generates input signal to drive the object and its output is sent into an optimal cloning machine (quantum cloner). The quantum cloner clones the state, generates many copies. Recognizer receives  $N$  unknown copies, makes some measurements on them and obtains some information about their states. Then, one can recognize the  $N$  copies through appropriate recognition algorithm. If the  $N$  copies are “good” enough, the recognizer gives an “on” signal to the actuator, and the actuator receives another  $M$  copies from the quantum cloner as feedback and generates new quantum signal to drive the controlled object until the given control target is reached. If the  $N$  copies are not “good” enough, the recognizer will send an “off” signal to the actuator, and the actuator will not receive feedback information determined by feedback controller.

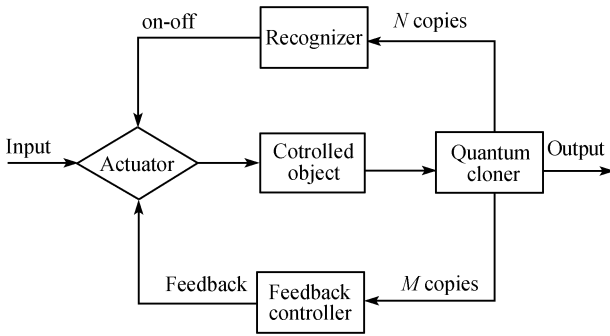


Fig. 2 Quantum feedback control based on cloning and recognition [24].

In the feedback control scheme, our objective is both to acquire information and not to destroy quantum information channel. Hence quantum cloning and feedback can not involve general quantum measurement. On the other hand, according to no-cloning theorem [42], arbitrary unknown quantum state cannot be copied exactly, and this is also an important limit on quantum control theory. However, this doesn't exclude the possibility of approximately or probabilistically cloning quantum state [43]. In fact, there are two kinds of quantum cloning machines: probabilistic cloning machine [44] and optimal universal quantum cloning machine [45, 46]. Probabilistic cloning machine is only suitable for linearly independent quantum states, where the machine yields faithful copies of the input state with a postselection of the measurement result [44]. However, optimal universal quantum cloning machine can approximately copy arbitrary unknown quantum state with unity probability [45]. It can be decomposed into rotations and CNOT gates and doesn't involve measurement. Hence we use approximate cloning to amplify the output state of the object, that is to say, the output state is approximately copied, some copies are used

to obtain information through quantum measurement and other copies are used to determine feedback.

The  $1 \rightarrow 2$  cloning process can be described by the transformation:

$$|s\rangle_a |M\rangle_x \rightarrow |s\rangle_a |s\rangle_b |\tilde{M}\rangle_x \quad (12)$$

where  $|s\rangle_a$  is the state of the original mode,  $|s\rangle_b$  is the copied state,  $|M\rangle_x$  is the original state of the cloning machine and  $|\tilde{M}\rangle_x$  is the final state of the cloning machine. The whole process of quantum cloning is to produce at the output of the cloning machine two identical states  $|s\rangle_a$  and  $|s\rangle_b$ . Considering  $1 \rightarrow M+N+1$  cloning process, it can be described as follows:

$$|s\rangle_a |M\rangle_x \rightarrow |s\rangle_a |s\rangle_b \cdots |s\rangle_{M+N+1} |\tilde{M}\rangle_x \quad (13)$$

The quantum cloner generates  $(M+N+1)$  copies and sends  $N$  copies into the state recognizer. Since these copies are approximate, we must recognize them using suitable recognition method [47, 48]. For generalization, represent these copies with density matrices  $\{\rho_1, \rho_2, \dots, \rho_N, \rho_{N+1}, \dots, \rho_{N+M+1}\}$ . Assume the unknown copies that the recognizer receive are  $U_R = \{\rho_1, \rho_2, \dots, \rho_N\}$ . We can acquire part information and determine parameters of  $\rho_i (1 \leq i \leq N)$  through suitable measurement. To recognize the copies, we should create a “objective state”  $\rho_o$ . For example, the simplest method is equal weight average:

$$\rho_o = \frac{1}{N} \sum_{i=1}^N \rho_i \quad (14)$$

Look upon  $\rho_o$  as “objective state”, one can calculate the trace distance  $D(\rho_i, \rho_o)$  between  $\rho_i$  and  $\rho_o$  [6, 49]:

$$D(\rho_i, \rho_o) \equiv \frac{1}{2} \text{Tr} |\rho_i - \rho_o| \quad (15)$$

Compare them with the beforehand given trace distance threshold  $D_0$ . If all trace distances satisfy  $D(\rho_i, \rho_o) \leq D_0$ , recognize them as a class, the state recognizer generates an “on” signal, and the actuator receives another  $M$  copies from the cloner as feedback. According to the feedback information, the actuator generates new quantum signal to drive the controlled object. Otherwise, the state recognizer will send an “off” signal to the actuator, and the actuator will not receive feedback copies from the quantum cloner [24].

In this feedback scheme, the design of quantum feedback algorithms is separated into two steps: a state recognition step and a feedback control step. The aim of state recognition step is to obtain information and the process involves quantum measurement, so it is destructive. However, the feedback process doesn't necessarily acquire information and doesn't involve measurement, so it can preserve quantum coherence [24]. In fact, we give a compromise between information acquisition and measurement disturbance in view of the characteristics of quantum measurement. The compromise

is realized through approximate cloning and renouncing precise feedback. The relations between different quantum feedback control schemes can be shown as Fig. 3.

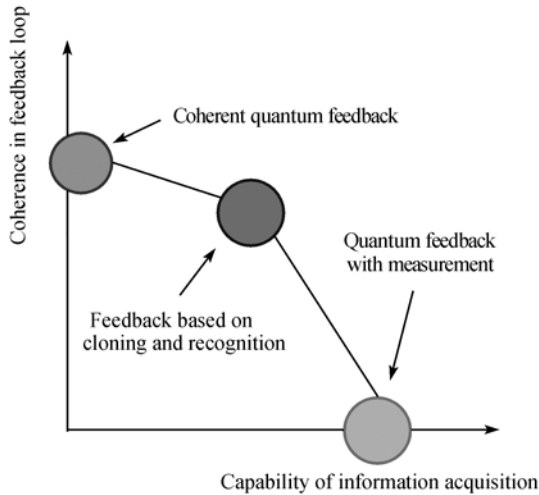


Fig. 3 Relations between different quantum feedback control schemes.

## 6 Concluding remark

Feedback control of quantum system is a rapidly developing research field [50–56]. In this paper, we have discussed several kinds of quantum feedback control schemes including Markovian quantum feedback, Bayesian quantum feedback, non-Markovian quantum feedback with time delay, coherent quantum feedback, and quantum feedback control based on cloning and recognition. These feedback schemes have different advantages. For example, the experiment realization is easiest for Markovian quantum feedback and the ability of coherence maintenance of coherent quantum feedback is the best.

In practical applications, the most important aspects for quantum feedback are information acquisition and coherence maintenance. Quantum feedback control based on cloning and recognition achieves a compromise between them. With the rapid development of quantum information technology, it is possible to use some other information technology approaches to design quantum feedback scheme. Simultaneously, all kinds of quantum feedback strategies can also be applied to large-scale quantum computation, practical quantum communication, quantum robot [57] and quantum intelligence [58–60].

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