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RESEARCH ARTICLE

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Relativistic temperature and Higgs-like coupling of thermodynamic interactions

Abstract The thermodynamic interaction at thermodynamic equilibrium in the free fermion gas is described in an alternative way by the coupling of particles with a scalar thermodynamic field that features self-interaction. This alternative coupling is similar to the Higgs coupling and is helpful in understanding the temperature transformation at thermodynamic equilibrium under the Lorentz boost. As this coupling is applied in the abelian interaction fermion gas, nothing nontrivial is obtained. However, an interesting thing happens in the nonabelian interaction fermion gas where the difference appears for the diagonal and off-diagonal intermediate bosons as the Higgs-like coupling is added.

Keywords thermodynamic interactions, temperature, gauge fields, Lorentz scalar

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1 Introduction

In thermodynamics, it was once a big controversy how the apparent temperature transformed under the Lorentz boost (e.g., see Refs. [1－7]). The discussion on this controversy still seems to be going on (e.g., see Refs. [8－11]). This controversy may be necessarily related to how to define the proper temperature and how to treat the thermodynamic interaction in a covariant framework. The non-linearity as well as non-inertia due to bremsstrahlung processes and multiple collisions in the thermal system is quite beyond the kinetics described by equations of motion for free fermions. The usual way is to use the statistical approach in the thermodynamic interaction. Alternatively, one may try other ways. Considering that the interaction is mediated by intermediate bosons, we may mathematically introduce the appropriate intermediate boson (or denoted as the thermodynamic field) to describe thermodynamic interactions for the fermion gas at thermodynamic equilibrium. The purpose of introducing a thermodynamic field is that we believe problems involving the apparent temperature transformation are much easier carried out in a covariant framework. It is well known that it is easy to involve a boson-mediated interaction in the relativistic covariant framework. To fulfill this attempt, it is technically necessary to begin with two senarios for the proper mass at thermal environment and thermodynamic equilibrium. The first senario is that the mass modified by the heat carried is still observed as the proper mass at the rest frame, which abides by the idea of defining the proper mass. The second senario is that the thermodynamic equilibrium is unchanged under the Lorentz boost, which was once adopted in Ref. [9].

The mass increase (or acquisition) is usually related to the spontaneous-symmetry-breaking property of the field. It is necessary to investigate the corresponding property of the thermodynamic field. Besides the perfect fermion gas, we can also go into the interacting fermion gas. It is convenient to investigate the interactions in the fermion gas by introducing the covariant derivative, since the interactions of fermions are fixed by the gauge invariance [12]. The gauge bosons acquire the mass through coupling with the field which displays the spontaneous breakdown of symmetry under the internal Lie group [13－15]. This leads to an interesting exploration: On what will take place for gauge fields as the thermodynamic field increases the mass of fermions.

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We will restrict ourselves to the point under the classical limit, and that allows us apply the equipartition of thermodynamic energy conveniently. The arrangement of the paper is as follows. In the next section, thermodynamic interactions in the free fermion gas are described in the relativistic field theory and the property of the thermodynamic field is investigated. The transformation of the apparent temperature under the Lorentz boost is given as an evidence of the applicability of our treatment. In Section 3, the thermal influence on the abelian and nonabelian gauge interactions will be then investigated in turn. A brief summary is given in the last section.

2 Thermodynamic interactions and temperature transformation

The most favorable bosons are the scalar and vector ones. In the relativistic frame, the vector meson has four components. The spatial part of the vector boson will couple to the non-zero thermodynamic current, which violates thermodynamic equilibrium. One may choose a special frame where the current is zero, but the current may appear by performing a Lorentz boost. Since we consider that the second scenario is appropriate, the existence of the vector meson is excluded. Consequently, only the scalar boson mediates the thermodynamic interaction at thermodynamic equilibrium. The coupling of the scalar boson with the fermion may be considered as the heating, which increases the fermion mass. The usual scalar coupling decreases the fermion mass. In order to realize the mass increment, the self-interaction of the scalar boson has to be introduced. The idea for doing so is similar to that for the pseudoscalar field [16] and for the Higgs scalar field [13, 14].

The scalar boson field, noted as the thermodynamic field below, is described by the following Lagrangian:

$$\mathcal{L}_\tau = \frac{1}{2}(\partial_\mu \phi_\tau \partial^\mu \phi_\tau - \mu_\tau^2 \phi_\tau^2) - \frac{\lambda}{4}\phi_\tau^4 \quad (1)$$

where μ_τ and λ are the constants with $\mu_\tau^2 < 0$ and $\lambda > 0$. where, we need to obtain the real mass of ϕ_τ. The vacuum solution of field ϕ_τ is obtained as $\phi_\tau^{0^2} = -\mu_\tau^2 / \lambda$, by minimizing the potential. The constant vacuum field is symmetry-breaking under the reflection of internal field space. Substituting

$$\phi_\tau = \phi_\tau^0 + \phi = \sqrt{-\mu_\tau^2 / \lambda} + \phi \quad (2)$$

into Eq. (1), it becomes

$$\mathcal{L}_\tau = \frac{1}{2}(\partial_\mu \phi\, \partial^\mu \phi - m_\tau^2 \phi^2) - \lambda \phi_\tau^0 \phi^3 - \frac{\lambda}{4}\phi^4 \quad (3)$$

where the real mass of ϕ is obtained to be $m_\tau = \sqrt{-2\mu_\tau^2}$. Now, the vacuum of the thermodynamic field is actually redefined and is symmetric under field reflection, while the reflection symmetry of $\mathcal{L}_\tau$ is broken due to the term $-\lambda \phi_\tau^0 \phi^3$. The scalar thermodynamic field defined here is in a form similar to the Higgs field, and its spontaneoussymmetry-breaking property under internal Lie groups can be found in Refs. [13－17].

The total Lagrangian which contains the fermion, thermodynamic field, and the Yukawa coupling between them is

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\gamma_\mu \partial^\mu - m - g_\tau \phi_\tau)\psi + \mathcal{L}_\tau \quad (4)$$

where ψ is the Dirac spinor, and m is the proper mass of the fermion. In homogeneous thermal matter, the scalar field ϕ is given as:

$$\phi = \frac{1}{m_\tau^2}(-g_\tau \bar{\psi}\psi - 3\lambda \phi_\tau^0 \phi^2 - \lambda \phi^3) \quad (5)$$

In an alternative description for the thermodynamic interaction, the real-mass boson does not always suggest the definite existence of the boson. Actually, a scalar boson that serves the role like Higgs bosons has not been found yet. Alternatively, one may take the boson mass m_τ as large enough. In this way, the field ϕ will be as small enough. Now, the thermal heat of the system may just be determined by the term $g_\tau \bar{\psi}\phi_\tau^0 \psi$ in Eq. (4). The homogeneous thermodynamic equilibrium of the fermion system can be elaborated by the coupling with the isotropic field vacuum ϕ_τ^0. Thus, it is actually the broken vacuum of the scalar field that plays the very role, in analogy to that of the Higgs field in gauge theories [13－15].

The potential $g_\tau \phi_\tau^0$ is a homogeneous quantity, which indicates the application of the equipartition theorem that holds for the classical limit is consistent with our treatment. The fermion mass measured at the rest frame is thus

$$m^* = m + g_\tau \phi_\tau^0 = m + \kappa T_0 \quad (6)$$

with κ=3 and 3/2 for the relativistic and nonrelativistic limits, respectively. The derivation of Eq. (6) implies that it is reasonable to take m_τ as large enough. Otherwise, it will lead to the inequality $m^* \neq m + \kappa T_0$ for finite m_τ not large enough. The proper temperature T_0 in the free fermion gas, which is proportional to the modification of the proper mass of fermions as given in Eq. (6), is a Lorentz scalar, consistent with the first senario. This definition of the proper temperature is also consistent with the idea of introducing a proper temperature demonstrated in Ref. [11].

It may be straightforward to work out various Lorentz ransformations for the apparent temperature based on the Lorentz structure of Lagrangian. For a moving body, one needs to measure the apparent temperature T. It is able to obtain various apparent temperatures without violating the relativity principle and the structure of the Dirac equation. There is the following relation for the total energy E of one particle in a co-moving system:

$$E = \sqrt{p^2 + m^{*2}} = \gamma m^* = \gamma m + \kappa T_0 \gamma \quad (7)$$

where the apparent temperature is given as $T = T_0\gamma$, which is the result of Refs. [2—5]. On the other hand, we note that the density of thermal heat $\rho_\tau = g_\tau \bar{\psi}\phi_\tau^0\psi$, is a Lorentz scalar, while the total thermal heat $Q^\circ = \int d^3x\rho_\tau$ is not a Lorentz scalar due to the volume element d^3x. According to the definition $T = \partial Q/\partial S$ with S the entropy, it has the relation $T = T_0/\gamma$ which is the result of Refs. [6, 7]. The distinct difference arises from the different measurement. The former is by means of the mass measurement, while the latter is through measuring the volume. The different measurement actually corresponds to different definitions or deduction of the apparent temperature.

3 Higgs-like coupling in gauge fields

The thermodynamic interaction has been alternatively described through the scalar boson coupling in the free fermion gas above. Now, let's investigate a more realistic system the interacting fermion gas. The interaction forms of the interacting fermions are fixed by the gauge invariance [12]. The applications of gauge theories can be found, for instance, in Refs. [15, 18]. In the following we will in turn investigate gauge interactions in U (1) and SU (3) symmetries as two examples.

The abelian interaction in U (1) symmetry (e.g., the electromagnetic interaction) is simple. In this case, the coupling of fermions with the thermal bath is through the exchange of the charge neutral thermodynamic field. Hence, there is no coupling between the gauge boson and thermodynamic field in U(1) symmetry, that is, the covariant derivative D_μ equals to the partial derivative ∂_μ. It is obvious that the Higgs-like thermodynamic coupling has no influence on the gauge field in U (1) symmetry. Considering the different structure of the nonabelian gauge field from the abelian one, we turn to investigate whether there is the thermodynamic effect on nonabelian gauge fields.

We take the thermodynamic interaction in the color $SU_c(3)$ symmetry as an example. In the Lagrangian the partial derivative ∂_μ with the covariant derivative $D_\mu = \partial_\mu - \mathrm{i}gA_\mu$ with g the coupling constant. The gauge field is expressed with respect to the generators λ in matrix form as $A_\mu = \lambda^\alpha A_\mu^\alpha/2$, with $\alpha = 1, \cdots, 8$ in SU_c (3) symmetry. The classical Lagrangian involving the gauge field can be given as

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\gamma_\mu D^\mu - m - g_\tau\phi_\tau)\psi + \mathrm{Tr}\left\{\frac{1}{2}[(D_\mu\phi_\tau)^\dagger D^\mu\phi_\tau - \mu_\tau^2(\phi_\tau^\dagger\phi_\tau)]\right\} - \frac{\lambda}{4}(\mathrm{Tr}[\phi_\tau^\dagger\phi_\tau])^2 - \frac{1}{4}F_{\mu\nu}^\alpha F^{\alpha\mu\nu} \quad (8)$$

where the trace is over the internal color space. Mathematically, there can be terms linear in $\mathrm{Tr}\phi_\tau$, $\mathrm{Tr}\phi_\tau^3$, and $\mathrm{Tr}(\phi_\tau^\dagger\phi_\tau)^2$. Assuming the reflection symmetry for the thermodynamic field $\phi_\tau \to -\phi_\tau$, the terms having the odd powers are ruled out. Without involving the term linear in $\mathrm{Tr}(\phi_\tau^\dagger\phi_\tau)^2$, one may obtain the field vacuum

$$\phi_\tau^0 = \left\langle\sqrt{\mathrm{Tr}(\phi_\tau^\dagger\phi_\tau)}\right\rangle_0 = \sqrt{-\mu_\tau^2/\lambda} \quad (9)$$

which is a natural extension of the vacuum of O (1) symmetry given by Eq. (4). In this way, it is simple to perform the rotation in the internal space without adding more assumption on the vacuum value of thermodynamic fields. In order to keep the hermitian constraint for the Lagrangian, the field ϕ_τ should be hermitian, i.e., $\phi_\tau^\dagger = \phi_\tau$. The gauge field strength tensor is

$$F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + gf^{\alpha\beta\gamma}A_\mu^\beta A_\nu^\gamma \quad (10)$$

with $f^{\alpha\beta\gamma}$ the antisymmetric structure constants. The covariant derivatives for the fermion and thermodynamic field are defined, respectively,

$$D_\mu\psi = (\partial_\mu - \mathrm{i}gA_\mu^\alpha\lambda^\alpha/2)\psi$$
$$D_\mu\phi_\tau = \partial_\mu\phi_\tau - \mathrm{i}\frac{g}{2}[A_\mu^\alpha\lambda^\alpha, \phi_\tau] \quad (11)$$

Under the gauge transformation

$$\psi' = S^{-1}\psi,\ \phi_\tau' = S^{-1}\phi_\tau S$$
$$A_\mu' = S^{-1}A_\mu S + \frac{\mathrm{i}}{g}S^{-1}\partial_\mu S \quad (12)$$

where $S = \exp[\mathrm{i}\theta^\alpha(x)\lambda^\alpha/2]$ with $\theta^\alpha(x)$ the gauge functions, the Lagrangian (8) is invariant.

Because thermodynamic interactions exist for fermions with various charges (colors), the hermitian matrix of thermodynamic fields is not required to be diagonalized. Similar to the non-interacting case, the apparent consequence resulting from the thermodynamic interaction is the modification of the fermion mass. We therefore need to perform the diagonalization for ϕ_τ. The hermitian matrix can be diagonalized through the unitary transformation as follows:

$$\phi_\tau = \exp\left(\mathrm{i}\frac{\xi^\alpha\lambda^\alpha}{2}\right)\begin{pmatrix}\phi_{\tau1} & & \\ & \phi_{\tau2} & \\ & & \phi_{\tau3}\end{pmatrix}\exp\left(-\mathrm{i}\frac{\xi^\alpha\lambda^\alpha}{2}\right) \quad (13)$$

In diagonalized form, the ϕ_τ vacuum quantity is $\phi_\tau^0 = \sqrt{\sum_i \phi_{\tau i}^{0^2}}$, and one may choose any of the following forms to express the vacuum in the tensor form

$$v = \begin{pmatrix}\phi_\tau^0 & & \\ & 0 & \\ & & 0\end{pmatrix}, \begin{pmatrix}0 & & \\ & \phi_\tau^0 & \\ & & 0\end{pmatrix}, \begin{pmatrix}0 & & \\ & 0 & \\ & & \phi_\tau^0\end{pmatrix} \quad (14)$$

Any one of tensor vacua can be rotated to other forms by the symmetry-breaking generator λ^β such as performing $\lambda^\beta v_{\beta\beta}\lambda^\beta$. The thermodynamic field can be parametrized based on the given vacuum. The parametrization is simple as knowing that the diagonal elements in Eq. (13) are the real quantities. We just need to replace the medium matrix in Eq. (13), and it is

$$\phi_\tau = \exp\left[\mathrm{i}\frac{\xi^\alpha \lambda^\alpha}{2} \right](u+v)\exp\left[-\mathrm{i}\frac{\xi^\alpha \lambda^\alpha}{2} \right] \quad (15)$$

where u is also in the matrix form as v but with the zero vacuum quantity is substituted by the field ϕ. In order to give the modification to the fermion mass of all colors by thermodynamic interactions, three forms of vacuum are needed. Thus the Lagrangian should involve thermodynamic fields with all of three vacuum choices, and for simplicity it is not rewritten here.

In a standard parametrization of Higgs mechanism [15], the fermion mass can be modified and massless Goldstone bosons [16] will be exorcised. In the parametrized expression of ϕ_τ given in Eq. (15), Goldstone bosons are described by tensors $[\lambda^\beta, v]\xi^\beta$ which satisfy the relations $\lambda^\beta v \neq 0$ and $v\lambda^\beta = (\lambda^\beta v)^\dagger \neq 0$ with M (< 8) choices for β. To exorcise massless Goldstone bosons we just need to perform the gauge transformation given in Eq. (12) under the unitary gauge $\theta^\alpha = \xi^\alpha$.

A straightforward consequence of the coupling between the gauge and thermodynamic fields is the generating of gauge boson masses, abiding by the Higgs mechanism. The mass term of the gauge field is obtained from the term $(D_\mu \phi)^\dagger D^\mu \phi$ in Eq. (8). To acquire the effective gluon mass, we just need sum up the terms linear in v^2 contained in $(D_\mu \phi)^\dagger D^\mu \phi$ over three vacua as

$$m_\alpha^2 A_\mu^\alpha A^{\alpha\mu} = -\frac{g^2}{4}\sum_v \mathrm{Tr}\{[\lambda^\alpha A_\mu^\alpha, v][\lambda^\sigma A_\mu^\sigma, v]\} \quad (16)$$

with $\alpha, \sigma = 1, \cdots, 8$. Thus, the effective gluon masses are

$$m_\alpha = g\phi_\tau^0, \quad \text{for } \alpha \neq 3, 8 \quad (17)$$

For A_μ^3 and A_μ^8, they remain massless.

An important procedure adopted above is the diagonalization, which is actually the decolorization of thermodynamic fields. Through the decolorization of thermodynamic fields, six off-diagonal gluons acquire effective masses. It is difficult to say that the acquired mass is related to the Debye screening since in U (1) symmetry where the same procedure is applied nothing is acquired for the gauge boson. However, it is interesting to note that the difference of diagonal and off-diagonal gauge bosons is observed in our treatment.

4 Summary

This work is an exploration based on some considerations on the thermodynamic equilibrium under the classical limit and temperature transformation under the Lorentz boost. Different from the conventional statistical treatment, the thermodynamic interaction in free fermion gases is alternatively described by the exchange of a scalar boson, based on notions of the relativistic field theory and the senarios for thermodynamic equilibrium. The scalar boson field (thermodynamic field) features self-interaction like the Higgs scalar field. The modification of the fermion proper mass in the thermal bath is equivalently described by the coupling with the symmetry-breaking vacuum of the scalar thermodynamic field. It is helpful for the introduction of the Higgs-like coupling to understand the apparent temperature transformation under the Lorentz boost. In our description, different transformations of the apparent temperature can be obtained from the various measurement.

We further make an exploration of the thermodynamic influence on the gauge fields based on the same ideas. No thermodynamic influence is obtained for the abelian gauge field. The interesting thing happens in the nonabelian interaction fermion gas [in SU_c (3) symmetry]: The different mass acquisition appears for the diagonal and off-diagonal intermediate bosons as the Higgs-like thermodynamic interaction is added. No evidence is shown that the acquired mass of off-diagonal gauge bosons is related to the Debye screening.

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