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RESEARCH ARTICLE

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The thermal effect analysis in the high power frequency-doubled Nd:YAG Laser

Abstract In the high power frequency-doubled Nd:YAG Laser, the temperature inside a nonlinear crystal will become non-uniform due to the absorbtion of the fundamental and second harmonic waves, leading to phase mismatch, conversion efficiency reduction and output power instability. This phenomenon appears severe in the high-power high-repetition rate laser system. In this paper, temperature distribution inside a KTP crystal was analyzed by solving the thermal conductivity equation. According to the temperature distribution of the KTP, we have theoretically calculated the optimal phase matching angles, tolerance angles and walk-off angles of type II KTP crystal as a function of temperature.

Keywords high power, frequency doubled crystal, thermal effect, phase matched, tolerance angles, walk-off angles

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1 Introduction

High-power all-solid-state green lasers have been widely used in a number of scientific and industrial fields such as pumping source for tunable lasers, flow visualization, ocean exploration, defense and security, pollution detection, high-power material processing, and biomedical equipment. Intracavity frequency doubling with a KTP crystal is one of the promising methods of obtaining the high power green laser. In high-power laser-diode-pumped intracavity frequency doubling lasers, although the high-quality KTP crystal is grown by improved technique and the absorption spectrum coefficient is small, the thermal effect [1,2] also appears under the high-average power infrared laser radiation. This is because the KTP crystal absorbs the fundamental and the second harmonic (SH) waves power, as well as nonlinear absorption such as two-photon absorption etc [3]. Mann and Weber [4] measured the linear and nonlinear absorption coefficients of the KTP crystal in the ns-pulse Q-switched laser based on a laser calorimetric and interferometric technique instead of applying the traditional the z-scan technique. They thought that linear absorption is independent on the laser power density.

Mann *et al.*[5] discussed in detail how the conversion efficiency was affected by the thermally induced mechanical stress in the KTP crystal. Barnes and Williams-Byra [6] analyzed thermally-induced phase mismatch and thermally-induced lensing in a parametric oscillator and amplifier. Using the Gaussian approximation, temperature gradients and thermal lensing had been calculated for both radial and longitudinal heat extraction by defining thermal time constants. In addition, the influence of heating parameters on optical conversion efficiency was numerically simulated, and the effects of thermally induced phase mismatch decreased by a judicious selection of the phase-matching conditions.

To alleviate thermal effect in the KTP crystal, several compensating approaches have been proposed and tested: Hon and Bruesselbach [7] suggested that a KTP crystal with appropriate slab configuration can increase the thermal conductivity dimension and change the distribution of fundamental wave. Eimerl [8] and Wu *et al.* [9] reported that the N-Plate crystal configuration can compensate for the thermally induced phase mismatch, effectively increasing the interaction length of nonlinear optical crystals during harmonic generation under high loading. Yao *et al.* [10] developed an external cooling method, gas cooling at the exit face of the crystal, which initiated a negative temperature profile that compensated for the positive profile

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generated from laser absorption.

All techniques mentioned above are common for longitudinal thermal gradients in KTP crystal, but longitudinal thermal gradients can be ignored in high-power intracavity frequency doubling lasers because of the use of short crystals with side-cooling and the small variation of longitudinal power density in the cavity. And the influence of the temperature on parameters of the frequency doubled crystal was not studied in detail for the high-power intracavity frequency doubling laser. In this paper, we analyzed the temperature distribution of a KTP crystal by solving thermal conduction equation, and calculated the variations of phase matching angles, tolerance angles, walk-off angles with refractive index equations. The results provided some theoretical and practical guidance for high-power frequency-doubling laser.

2 Thermal effect during high-power frequency doubling

In a high-power intracavity frequency doubling laser, the laser power density is very high in the cavity. Since the KTP crystal absorbs the fundamental and the second harmonic (SH) waves power, the temperature distribution in the KTP crystal will change, this can be obtained from the three-dimensional thermal conduction equation:

$$\frac{\partial^2 T(x,y,z)}{\partial x^2}+\frac{\partial^2 T(x,y,z)}{\partial y^2}+\frac{\partial^2 T(x,y,z)}{\partial z^2}=\frac{Q(x,y,z)}{K} \qquad (1)$$

where $Q(x,y,z)$ is the calorific power in the KTP crystal and K is thermal conductivity. The thermal power in the KTP crystal is uniform along the direction of beams because the fundamental wave is comparatively stable and the frequency-double proceeds along the wave direction in the crystal. Moreover, the crystal is water-cooled at the side surface, air cooled at end. Therefore, the variations of temperature along the axial direction will be neglected. The temperature profile inside the KTP crystal can be determined by solution of the two-dimensional conduction equation:

$$\frac{\partial^2 T(x,y,z)}{\partial x^2}+\frac{\partial^2 T(x,y,z)}{\partial y^2}=\frac{Q(x,y,z)}{K} \qquad (2)$$

For a high-power high repetition rate laser system, the absorption thermal is expressed as:

$$Q(x,y)=f_{\mathrm{rep}}\int_{\mathrm{pulse}}(a_\omega I_\omega(x,y,t)+(\alpha_{2\omega}+\beta_{2\omega})I_{2\omega}(x,y,t))\mathrm{d}t \qquad (3)$$

where f_{rep} is the repetition rate; α_ω and $\alpha_{2\omega}$ are linear absorption coefficients for the fundamental and the second harmonic (SH) waves; and $\beta_{2\omega}$ is the nonlinear absorption coefficient for SH wave, which is dependent on the power density of SH wave.

We assume that the incident beam is a Gaussian beam. The electric amplitude is given by

$$E(x,y,t)=E_0\exp\left(-\frac{x^2+y^2}{\omega_0^2}\right)\exp\left(-f_{\mathrm{rep}}t\right) \qquad (4)$$

where w_0 is the beam radius. The optic power and electric-field intensity are given by

$$P=\frac{\pi\omega_0^2}{4}nc\varepsilon_0\left|E^2\right| \qquad (5)$$

$$I=\frac{1}{2}nc\varepsilon_0\left|E^2\right| \qquad (6)$$

By using equations (4) and (5), we obtain for I the following expression

$$I(x,y,t)=\frac{2P}{\pi\omega_0^2}\exp\left(-2\frac{x^2+y^2}{\omega_0^2}\right)\exp\left(-f_{\mathrm{rep}}t\right) \qquad (7)$$

Figure 1 shows the temperature distribution inside a given KTP crystal. In the calculations, we used the following parameters: the size of the KTP crystal is 6×6×9.2 mm^3; the power of the fundamental wave P_{1064} is 180W; the power of the SH wave P_{532} is 110 W; their absorption coefficient are a_{1064} = 0.01 % /cm and a_{532} = 1.0 %/ cm individually; the repetition rate is 10 kHz, and the radius of the fundamental wave beam (ω_0) is 3 mm. From Fig. 1, it can be seen that the temperature difference between the center and the boundary is about 40 K, which is larger than the allowed phase matching temperature of the KTP crystal.

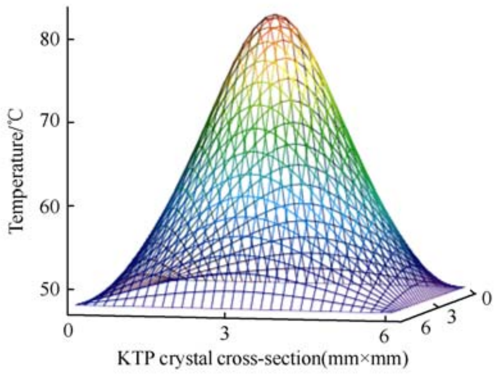


Fig.1 The distribution of temperature inside the KTP crystal

3 The influence of temperature on interaction between three waves during high-power frequency doubling laser [11]

According to the theoretical calculation above, we can see the temperature distribution in frequency-doubled crystal is a Gaussian approximation, and the temperature difference between the center and the boundary is about 40 K. Hence, the parameters of three-wave coupling will change. The refractive index formulas for the KTP crystal at room temperature are also not suitable. Considering the influence of temperature on biaxial crystals, the refractive index formulas of the optical wave in the crystal can be described as:

$$\frac{k_x^2}{n^{-2}(\omega,T)-n_x^{-2}(\omega,T)}+\frac{k_y^2}{n^{-2}(\omega,T)-n_y^{-2}(\omega,T)}$$
$$+\frac{k_z^2}{n^{-2}(\omega,T)-n_z^{-2}(\omega,T)}=0 \qquad (8)$$

where $k_x=\sin\theta\cos\phi, k_y=\sin\theta\sin\phi, k_z=\cos\theta$. Here, θ is the angle between the wave-vector $\boldsymbol{K}$ and z-axis, ϕ is the angle between wave-vector $\boldsymbol{K}$ and the x-axis; $n(\omega,T)$ is the refractive index of the optical wave at frequency ω under the given temperature T; $n_x(\omega,T)$, $n_y(\omega,T)$, and $n_z(\omega,T)$ are the three principal refractive indices of the wave. We define

$$\begin{cases} a=n_x^{-2}(\omega,T)=\left(n_{x,\omega}+\dfrac{\partial n_{x,\omega}}{\partial T}\times\Delta T\right)^{-2} \\ b=n_y^{-2}(\omega,T)=\left(n_{y,\omega}+\dfrac{\partial n_{y,\omega}}{\partial T}\times\Delta T\right)^{-2} \\ c=n_z^{-2}(\omega,T)=\left(n_{z,\omega}+\dfrac{\partial n_{z,\omega}}{\partial T}\times\Delta T\right)^{-2} \\ B=-k_x^2(b+c)-k_y^2(a+c)-k_z^2(a+b) \\ C=k_x^2bc+k_y^2ac+k_z^2ab \end{cases} \qquad (9)$$

where $n_{x,\omega}$, $n_{y,\omega}$ and $n_{z,\omega}$ are the principal refractive indices of the wave at frequency ω under room temperature; $\partial n_{x,\omega}/\partial T$, $\partial n_{y,\omega}/\partial T$ and $\partial n_{z,\omega}/\partial T$ are the temperature derivatives of the refractive indices. By substituting (9) into (8), the refractive indices of the optical wave are given by

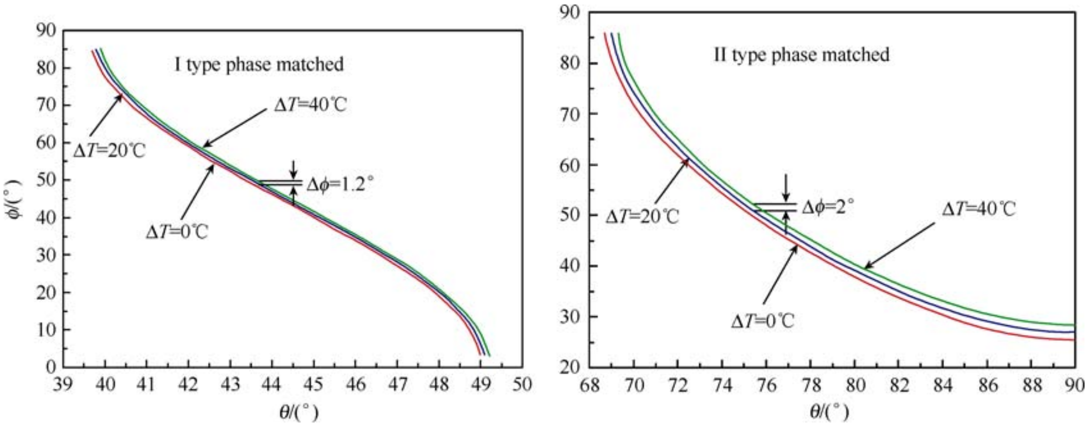
$$\begin{aligned} n^{e_1}(\omega_i,T)&=\sqrt{\frac{2}{-B_i-\sqrt{B_i^2-4C_i}}} \\ n^{e_2}(\omega_i,T)&=\sqrt{\frac{2}{-B_i+\sqrt{B_i^2-4C_i}}} \end{aligned} \qquad (10)$$

where n^{e_1} (ω,T) and n^{e_2} (ω,T) are the refractive indices for the *fast* and *slow* optical wave, corresponding to two probable polarization directions of the electric vector $\boldsymbol{E}$, respectively.

The phase matching conditions must be satisfied in the KTP crystal during high-power second harmonic generation (SHG) of Nd: YAG lasers, which are given as

$$\begin{aligned} &\text{I type} \quad n_{\omega}^{e_1}=n_{2\omega}^{e_2} \\ &\text{II type} \quad n_{\omega}^{e_1}+n_{\omega}^{e_2}=2n_{2\omega}^{e_2} \end{aligned} \qquad (11)$$

Because n^{e1} (ω,T) and n^{e2} (ω,T) are both dependent on temperature, the phase matching angles will change as a function of temperature, as shown in Fig. 2, where phase matching angles increase with the temperature, and θ and ϕ change by about 2° corresponding to the temperature difference of 20°C. Consequently, the influence on phase matching angles due to temperature must be considered for higher second harmonic conversion efficiency.



(a) (b)

Fig.2 The type I phase matching angles ϕ and θ change as a function of temperature of the KTP crystal; the type II phase matching angles ϕ and θ change as a function of temperature of the KTP crystal.

Figures 3 and 4 show that the type-II phase matching angles of the KTP crystal change with the different fundamental wavelength under different temperatures. Figure 3 shows the optimum phase matching direction to be in the plane *xOy* for the wavelength from 1.0 μm to 1.079 4 μm. In the plane *xOy*, the phase matching angles of the KTP crystal are $\theta=90^\circ$, $\phi=70^\circ-0^\circ$. The increase in temperature of the crystal gives rise to an increase in the angle ϕ for a certain wavelength. When the temperature increases 10°C, the phase matching angle ϕ increases 1°. When the range of fundamental wavelength is from 1.079 4 μm to 2 μm, the optimum phase matching direction is in the plane *xOz*. The phase matching angles are $\phi=0^\circ$, $\theta=90^\circ-50^\circ$. From Fig. 4, it can be seen that the phase matching angle θ changes by 2° corresponding to the difference temperature of 100°C. Therefore, the temperature has little effect on phase matching angles in this direction.

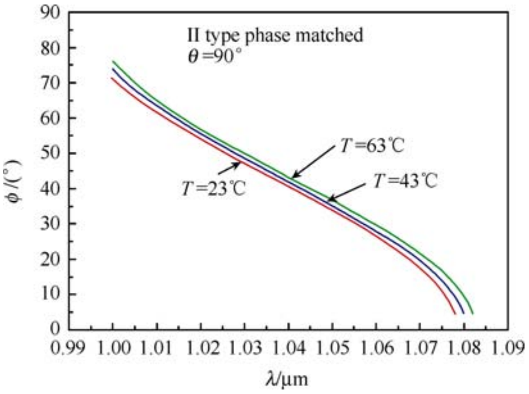


Fig.3 The optimum type II phase matching angles ϕ and λ change as a function of temperature of the KTP crystal.

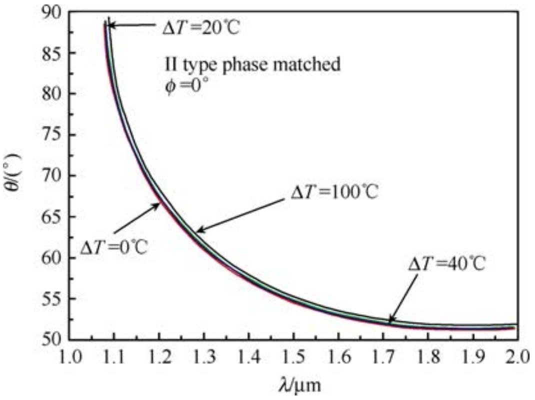


Fig.4 The optimum type II phase matching angles θ and λ change as a function of temperature of the KTP crystal.

It is important to note that changes in optimum phase matching angles (θ_M, ϕ_M) can appreciably change the tolerance parameters of the crystal. Considering the temperature shift, the phase mismatch can be expressed as

$$\begin{aligned}\Delta k(T) &= k(\omega_3,T)-k(\omega_2,T)-k(\omega_1,T)\\ &= \frac{2\pi}{\lambda_3}n(\omega_3,T)-\frac{2\pi}{\lambda_2}n(\omega_2,T)-\frac{2\pi}{\lambda_1}n(\omega_1,T)\end{aligned} \tag{12}$$

Furthermore, the tolerance phase mismatch should satisfy the following condition:

$$|\Delta k| \leqslant \pm\frac{\pi}{l} \tag{13}$$

Hence, the tolerance angles $\Delta\theta$ and $\Delta\phi$ can be determined by solving Eqs. (12) and (13).

Figures 5 and 6 show that tolerance angles $\Delta\phi\cdot l$ and $\Delta\theta\cdot l$ for type II phase matching of the KTP crystal in planes *xOy* and *xOz* changes with different wavelength under difference temperatures, respectively. The variation of $\Delta\phi\cdot l$ is zero under different temperature for the fundamental wavelength from 1.01 μm to 1.05 μm. Hence, the tolerance angle $\Delta\phi\cdot l$ at this wavelength range is not affected by temperature, whereas it is greatly affected by temperature at the wavelength less than 1.01 μm or more than 1.05 μm, especially dramatically near 1.794 μm, as shown in Fig. 5, where the stability of the SH power is dependent on the line width. $\Delta\theta\cdot l$ changes evidently with the temperature near the wavelength of 1.08 μm, which is much smaller than that in the plane *xOy*, whereas it is not influenced by temperature at the wavelength from 1.1 μm to 2.0 μm, as shown in Fig. 6.

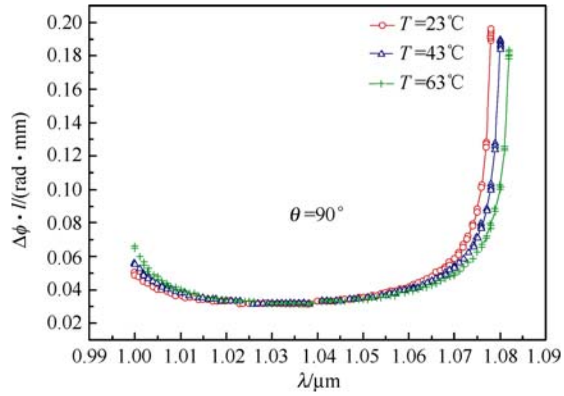


Fig.5 The tolerance angles $\Delta\phi\cdot l$ of the type II KTP crystal phase matching under the difference temperature.

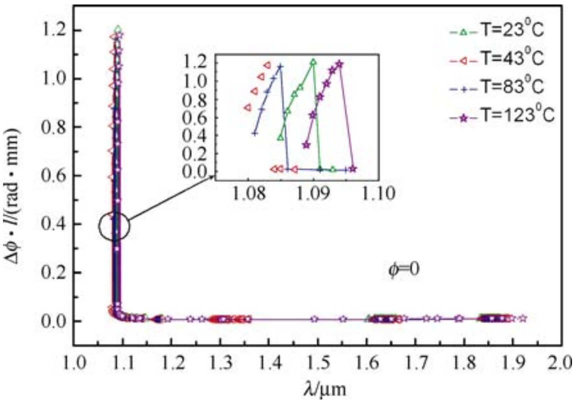


Fig.6 The tolerance angles $\Delta\theta\cdot l$ of the type II KTP crystal phase matching under the difference temperature.

As is well known from crystal optics, the direction of the wave vector and the Poynting vector do not generally coincide inside the crystal because of double refraction. The difference between them is called walk-off angle. There are two polarization directions e_1 and e_2 for a plane wave in a biaxial crystal, so walk-off occurs in two planes with different angles α_1 and α_2. Here, $e_1$and e_2 refer to the polarization directions of the *s* (*slow*) and *f* (*fast*) lights. Figure 7 shows walk-off angles of the fundamental wave and second harmonic wave in plane *xOy*, where ρ_1 and ρ_2 are the angles between the directions of fundamental wave and SH wave with parallel and orthogonal polarization, respectively; ρ_3 is the walk-off angle between the two orthogonally polarized fundamental waves; ϕ_{m} is the phase-matching angle.

The walk-off angles for waves with $e_1$and e_2 polarization are given by the following equation

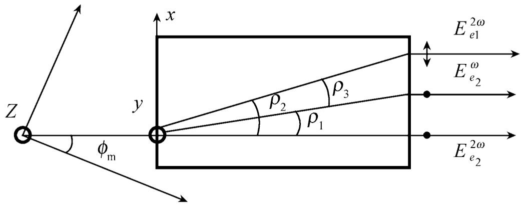


Fig.7 The propagating direction of the fundamental and second harmonic waves in the KTP crystal under the condition of type II phase matching.

$$\begin{aligned}\alpha^{e_1} &= \arccos(a_x^{e_1}(\omega_i,T)b_x^{e_1}(\omega_i,T)+a_y^{e_1}(\omega_i,T)\\ &\quad\cdot b_y^{e_1}(\omega_i,T)+a_z^{e_1}(\omega_i,T)b_z^{e_1}(\omega_i,T))\\ \alpha^{e_2} &= \arccos(a_x^{e_2}(\omega_i,T)b_x^{e_2}(\omega_i,T)+a_y^{e_{21}}(\omega_i,T)\\ &\quad\cdot b_y^{e_2}(\omega_i,T)+a_z^{e_2}(\omega_i,T)b_z^{e_2}(\omega_i,T))\end{aligned} \tag{14}$$

Figure 8 demonstrates the walk-off angles of the three waves for type II phase matching when the temperature of KTP are 23 °C and 73 °C which are denoted by continuous line and dotted line. Plot a and b show the walk-off angles of the fundamental wave and SH wave both with polarization direction e_2. Plot c shows the walk-off angle of the fundamental wave with polarization direction e_1. As is shown in Fig. 8, the direction of polarization e_1 doesn't walk off and the direction of polarization e_2 walks off when the wavelength of the fundamental wave is less than 1.079 4 μm, for which the optimum phase matching direction is in plane *xOy*; the directions of polarization e_1 and e_2 both walk off when the wavelength is more than 1.079 7 um, the optimum phase matching direction for which is in plane *xOz*; the walk-off angles of the waves are zero at the fundamental wavelength of 1.079 4 μm, for which the optimum phase matching direction is along the *x*-axis. In addition, the walk-off angles decrease as the temperature increases regardless of in plane *xOy* and *xOz*. For example, if $\Delta T=$ 50°C, the change of walk-off angle is about 0.06° in plane *xOy*, about 0.1° in plane *xOz* at utmost, which increases as wavelength decreases.

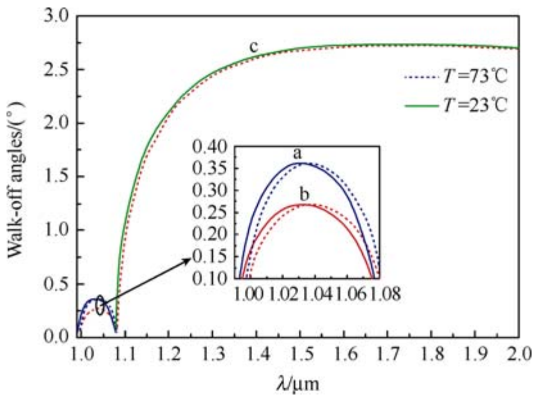


Fig.8 The walk-off angles in the KTP crystal

4 Conclusions

In conclusion, during the process of high-power frequency-doubling, the optical absorption of the fundamental and second harmonic waves in the crystals causes temperature non-uniformity, which can reduce the conversion efficiency and make output power unstable by disturbing phase matching. This is especially severe in the high-power high-repetition rate laser system. In this paper, temperature distribution inside a KTP crystal was analyzed by solving thermal conductivity equation. Depending on the temperature distribution of KTP, we have theoretically calculated the optimal phase matching angles, tolerance angles and walk-off angles of type II KTP crystal as function of temperature, which will help sustain the reliable operation of high power frequency-doubling laser.

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