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RESEARCH ARTICLE

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Dynamical behaviors of an exciton in an asymmetric double coupled quantum dot

Abstract Dynamical behaviors of an exciton in an asymmetric double coupled quantum dot and an alternatingcurrent (ac) electric field have been analyzed based on the two-level approximation theory, and the conditions under which dynamical localization occurs are obtained. It shows that when the amplitude of the ac electric field is small, the Coulomb interaction plays an important role. The dynamical behaviors of the exciton are mainly confined in the low-level subspace. When the ratio of the field intensity to frequency is the root of Bessel function, electron and hole are localized in one dot, and they can be divided with the increasing amplitude of the ac electric field.

Keywords exciton, quantum dots, localization,quasienergy, Floquet states

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1 Introduction

Much attention has been focused on the coherent control of quantum systems both in theory and experiment. Using the Floquet theory [1], Grossmann and Dittrich [2] studied the dynamical properties of the electron in the double coupled quantum dot and high-frequency alternating-current (ac) electric field, and then first brought forward the conception of "dynamical localization." They reported that, at the quasienergy and avoidance intersections, the electron was confined in a single quantum dot. Their calculation showed that when the ratio of the field strength to the frequency was a root of the zeroth Bessel function, this localization would occur.

Adding another electron to the system complicates the problem due to the Coulomb interaction. This system, however, presents an excellent model to study the effect of Coulomb. Zhang and Zhao [3–5] have investigated the dynamical behaviors of two interacting electrons in the symmetric double coupled quantum dot with a weak ac electric field. They concluded that localization also could happen at the quasienergy intersection. Recently, Liu et al. [6,7] have studied the interacting electron-hole pair (exciton) in a symmetric double coupled quantum dot. They demonstrated that the above conclusion still holds true for excitons.

In this paper, we theoretically analyze the phenomenon of localization of the exciton in an asymmetric double coupled quantum dot. The kinetic properties of this system are investigated using Hubbard model and two-level approximation theory. With the Floquet theory and numerical method, the time evolution factor of one periodic is diagonalized to obtain the quasienergy. We calculate the evolution of external field, and the result indicates that there would be some avoidance intersection in the quasienergy. Finally, we study the kinetic behaviors of the system at the intersection and the influences of the external field.

2 Theoretical model

For the sake of analytic simplicity, we take the assumption

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that there is only one electron (hole) level in each quantum dot. The Hamiltonian for the system can be written as [8]:

$$H(t)=\sum_{\sigma=e,h;i=L,R}\varepsilon_{i\sigma}(t)d_{i\sigma}^{+}d_{i\sigma}+\sum_{\sigma=e,h}w_{\sigma}(d_{L\sigma}^{+}d_{R\sigma}+d_{R\sigma}^{+}d_{L\sigma})$$
$$+U_1(n_{eL}n_{hL}+n_{eR}n_{hR})\,U_2(n_{eL}n_{hR}+n_{hL}n_{eR}) \qquad (1)$$

here, $d_{i\sigma}^{+}(d_{i\sigma})$ creates an electron (hole) in left (right) dot. If the external electric field is applied only to the dot, it will cause a proportionate shift on the energy level. $\varepsilon_{L(R)\sigma}(t)=-(+)\frac{1}{2}(E+V_0\cos(\omega t))$; where E is the detuning between two dots, which characterize the asymmetry, and V_0 is the amplitude of the ac electric field (Here, we regard E as an applied constant field); w_σ is the tunneling coefficient of the electron (hole) between two dots; U_1 and U_2 are the intradot and interdot Coulomb interactions, respectively. There are four possible states for the system: $|LL\rangle$, $|RR\rangle$, $|LR\rangle$ and $|RL\rangle$. In Fig. 1, the possible states of electron and hole are demonstrated in two quantum dots.We choose the two-body basis as $|mn\rangle=d_{m,e}^{+}d_{n,h}^{+}|0\rangle$ (m,n=L,R), where $|m,n\rangle$. represents the state of the electron in the quantum dot m and the hole in the dot n, and $|0\rangle$. denotes the vacuum state. Hence, the wave function of the system can be expanded as:

$$\psi(t)=a_1(t)|LL\rangle+a_2(t)|RR\rangle+a_3(t)|LR\rangle+a_4(t)|RL\rangle \qquad (2)$$

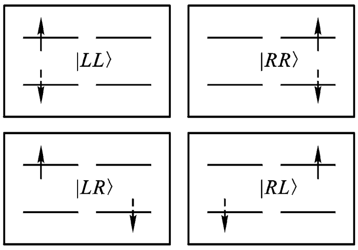


Fig. 1 Scheme of the electron and hole configuration in two quantum dots. In each panel, the *left side* corresponds to dot *L*, the *right side* to dot *R*. The *solid arrows* and the *dashed arrows* stand for the electron and the hole, respectively.

localized states, $|LL\rangle$ and $|RR\rangle$, construct a low-energy subspace, and two delocalized states $|LR\rangle$ and $|RL\rangle$. construct a high-energy subspace. Here, $a_1(t)$, $a_2(t)$, $a_3(t)$, and $a_4(t)$ are determined by:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}a_1\\a_2\\a_3\\a_4\end{pmatrix}=\begin{pmatrix}K-(E+V_0\cos(\omega t)) & 0 & w_h & w_e\\ 0 & K+(E+V_0\cos(\omega t)) & w_e & w_h\\ w_h & w_e & 0 & 0\\ w_e & w_h & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}a_1\\a_2\\a_3\\a_4\end{pmatrix} \qquad (3)$$

where $K=U_1-U_2$, with the unit of $\hbar=1$.

When the amplitude of the ac electric field is small, the tunneling coefficient is also small and the Coulomb interaction plays an important role. It satisfies the condition that $|K|$ is much larger than w_h and w_e; thus, $a_3(t)$ and $a_4(t)$ can be obtained approximately:

$$a_3(t)\approx\frac{w_h}{K}a_1(t)+\frac{w_e}{K}a_2(t),\ a_4(t)\approx\frac{w_e}{K}a_1(t)+\frac{w_h}{K}a_2(t) \qquad (4)$$

then,the system will be reduced into an effective two-level system with $a_3(t)$ and $a_4(t)$ eliminated.The Schrödinger Eq.(2)is simplified:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}b_1\\b_2\end{pmatrix}=\begin{pmatrix}-(E+V_0\cos(\omega t)) & \frac{2w_hw_e}{K}\\ \frac{2w_hw_e}{K} & +(E+V_0\cos(\omega t))\end{pmatrix}\begin{pmatrix}b_1\\b_2\end{pmatrix} \qquad (5)$$

where $|a_i(t)|^2=|b_i(t)|^2$(i=1,2);for this two-level system,the Coulomb interaction K is absorbed into the tunneling matrix elements. The larger the Coulomb interaction is, the stronger the localization is. The Coulomb interaction enhances the localization effect. The exciton moves between the two dots in a way similar to an electron in such aquantum system with an ac field.

In what follows, we make a further transformation. For large frequency $\omega>>\left|\frac{2w_hw_e}{K}\right|$, we average Eq.(5)over a complete cycle to obtain the high-frequency approximation:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}0 & \frac{2w_hw_e}{K}J_n(x)\\ \frac{2w_hw_e}{K}J_n(x) & 0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} \qquad (6)$$

where $|b_i(t)|^2=|c_i(t)|^2$ (i=1,2), $J_n(x)=\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\mathrm{d}\tau$ $\exp((\mathrm{i}x\sin(\tau)-n\tau)$ $x=\frac{V_0}{\omega}, n=\frac{2E}{\omega}$, $\tau=\omega t$, $J_n(x)$ is

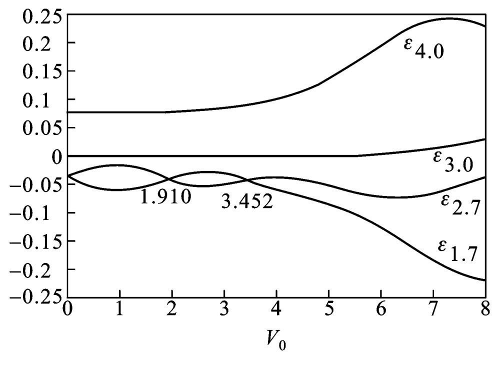


Fig. 2 The Floquet spectrum as a function of the field strength of the ac field V_0 with ω =1, K=−6.5, E =0.5, w_e= 0.4, w_h= 0.3.

the nth-order Bessel function. It is obvious that the dynamical localization occurs in the case of high-fre-quency approximation $\left(\omega\gg\left|\frac{2w_kw_e}{k}\right|\right)$ when $J_n(X_n)=0$ (n=1,2,3, ⋯, $n=\frac{2E}{\omega}$ and $X_n=\frac{V_0}{\omega}$);that is ,Eq.(6) implies that the exciton will be localized in the initially populated dot, when X_n is a root of the n th-order Bessel function. The electron and hole move between the two coupled dots together, and they can't be divided forever.

3 Results and conclusion

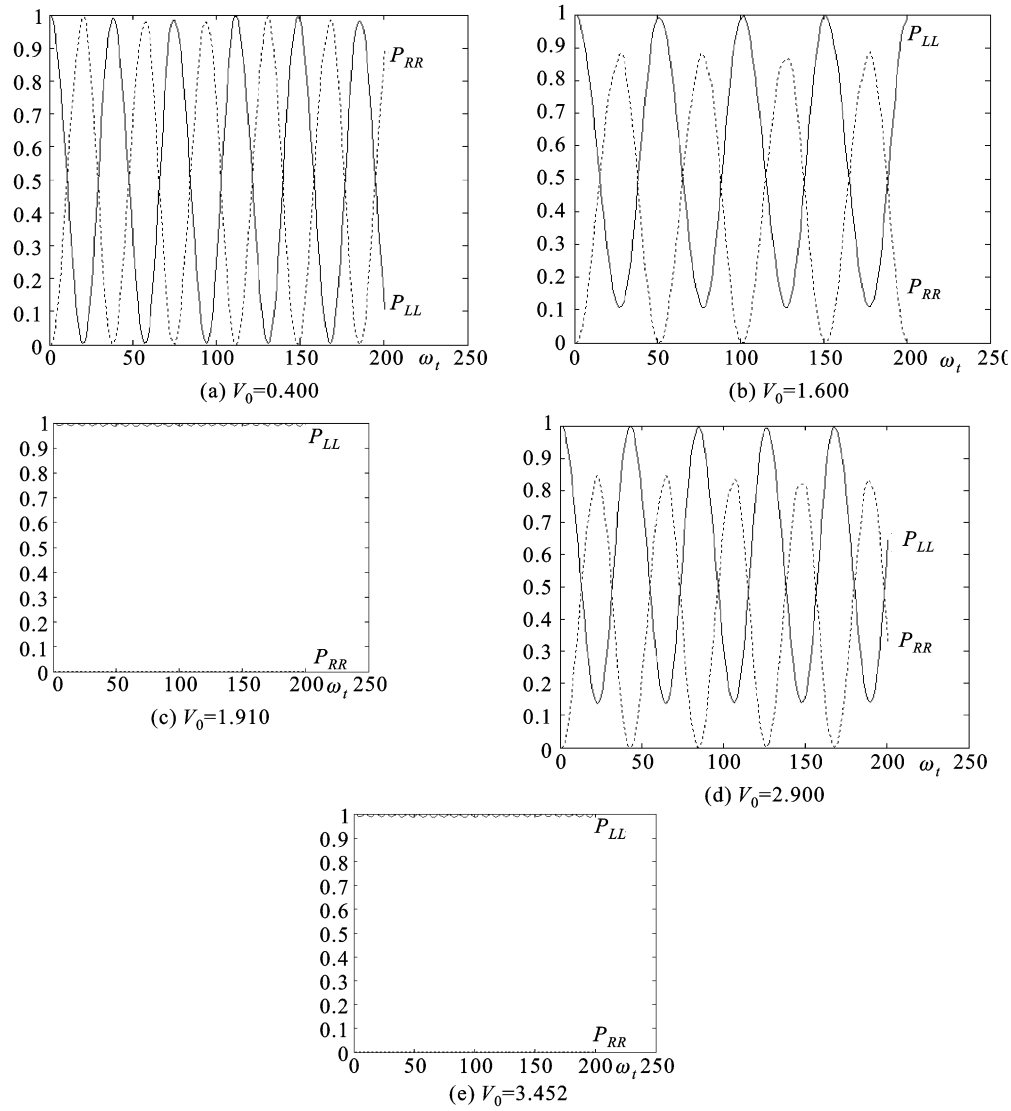
For the T-periodic Hamiltonian, the Floquet theory implies that a state vector of the quantum system can be written as $\left|\psi_\alpha(t)\right\rangle = e^{-i\varepsilon_\alpha t}\left|\phi_\alpha(t)\right\rangle$, with $\left|\phi_\alpha(t)\right\rangle = \left|\phi_\alpha(t+T)\right\rangle$, where $\left|\phi_\alpha(t)\right\rangle$ where $\left|\phi_\alpha(t)\right\rangle$ is a Floquet state and ε_α is quasienergy.To get the quasienergies and Floquet states, we can diagonalize one-period evolution factor $U(T,0)$, but this problem cannot be treated exactly because $[H(t_1),H(t_2)]\neq 0$. Hence, we use the numerical integration method to solve the equation:

$$\mathrm{i}\frac{\partial}{\partial t}U(t,0) = H(t)U(t,0) \tag{7}$$

With the initial condition $U(0,0)=I_{4\times 4}$, we obtain the quasienergies $\{\varepsilon_{\alpha,l}\}$ and Floquet states $\{\left|\phi_{\alpha,l}(0)\right\rangle\}$. $\left|\phi_{\alpha,l}(t)\right\rangle$ can be derived from the intrinsic equation:

$$\left(H(t) - \mathrm{i}\frac{\partial}{\partial t}\right)\left|\phi_{\alpha,l}(t)\right\rangle = \varepsilon_{\alpha,l}\left|\phi_{\alpha,l}(t)\right\rangle \tag{8}$$

Fig. 3 The time evolution of the probabilities P_{LL} (*solid lines*) and P_{RR} (*dashed lines*) for the following values of the strength of ac field: ω =1, K=−6.5, E=0.5,w_e= 0.4, w_h =0.3.
(**a**) V_0= 0.400,
(**b**) V_0 = 1.600,
(**c**) V_0 = 1.910,
(**d**) V_0 = 2.90,
(**e**) V_0 = 3.452.



Quasienergies $\{\varepsilon_{\alpha,l}\}$ are confined in the first Brillouin zone [−0.5 ω ,0.5 ω]. When the ac field is slowly diminished to zero, the relationship between the $\{E_\alpha\}$ and $\{\varepsilon_{\alpha,l}\}$ is described by $\varepsilon_{\alpha,l} \to \varepsilon^0_{\alpha,l} = E_\alpha - l\omega$ and the relationship between $\left|\phi_{\alpha l}\right\rangle$ and the stationary eigenfunction φ_α is $\left|\phi_{\alpha,l}\right\rangle \to \left|\phi^0_{\alpha,l}\right\rangle = \varphi_\alpha e^{il\omega t}$. The index l indicates how many photons must be subtracted from the unperturbed energy level E_α to arrive at the first Brillouin zone. Given an initial state $|\psi(0)\rangle$ of the system, the time evolution of the system in terms of Floquet states is expressed as [9]:

$$|\psi(t)\rangle = \sum_\alpha \exp(-i\varepsilon_\alpha t)|\phi_\alpha(t)\rangle\langle\phi_\alpha(0)|\psi(0)\rangle \quad (9)$$

Here, the evolution of the system begins with the localized $|LL\rangle$. In Fig. 2, we present the Floquet quasienergies as the function of the external field. It can be seen that $\varepsilon_{1,7}$ and $\varepsilon_{2,7}$ develop into an avidance intersection when $J_0(X_n)=0$. As for symmetric quantum dots, $\varepsilon_{3,0}$ and $\varepsilon_{4,0}$, however, almost remain unchanged on the whole, and the intersection satisfies zeroth-order Bessel function $J_0(X_0)=0$ .It indicates that the dynamical behaviors of the exciton are mainly determined by the two Floquet states in lower energy level.

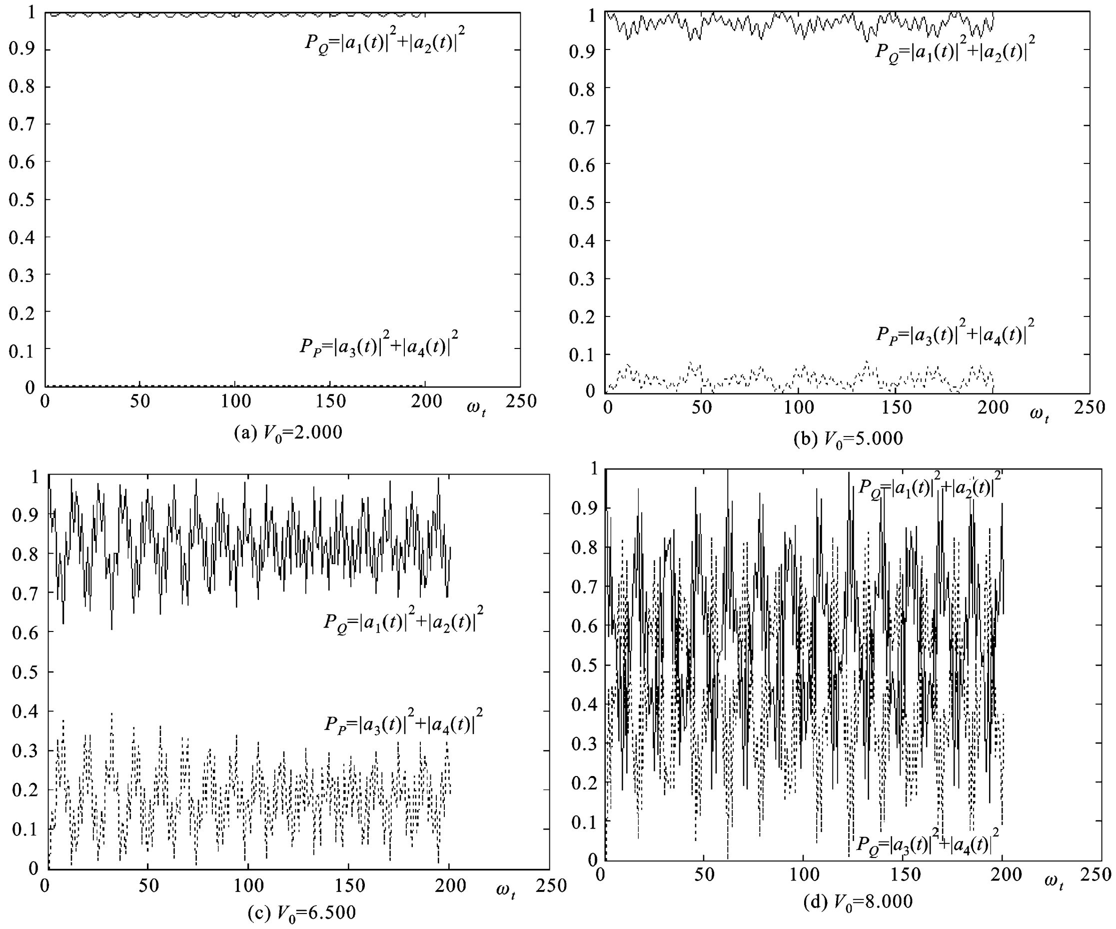


Fig. 4 The time evolution of probabilities P_Q (*solid lines*) and P_P (*dashed line*) of the localized and delocalized states for the following values of the ac field: $\omega=1, K=-6.5, E=0.5, w_e=0.4, w_h=0.3$, (**a**) $V_0=2.000$, (**b**) $V_0=5.000$, (**c**) $V_0=6.500$, (**d**) $V_0=8$.

To elucidate the localization of the exciton, we calculate the probability of two local states P_{LL} and P_{RR}, where $P_{LL}=|a_1(t)|^2$ ($P_{RR}=|a_2(t)|^2$) is the probability of finding the exciton in the left (right) quantum dot.Figures 3(a)–(e) presents the time evolution of the probabilities P_{LL} and P_{RR} for the different values of the strength of an ac electric field.We find that the dynamical localization happens when the strength of the field arrives at an avoidance intersection of $\varepsilon_{1,7}$ and $\varepsilon_{2,7}$ ($V_0=1.910, 3.452$).In other words, an initially localized exciton in the left dot would remain in the dot, as if the electron and hole were frozen by the ac electric field. Quantum tunneling is restrained. However, it emerges apart from the intersections. Equation (6) makes a perfect explanation for the phenomenon of dynamical localization:in the case of $J_n(2V_0/\omega)=0$, $\frac{dc_1(t)}{dt}=\frac{dc_2(t)}{dt}=0$,with the initial conditions $|c_1(0)|^2=|a_1(0)|^2=1$, $|c_2(0)|^2=|a_2(0)|^2=0$, $|c_2(t)|^2=|a_2(t)|^2$, the invariable probability $P_{LL}=|a_1(t)|^2=1$, $P_{RR}=|a_2(t)|^2=0$; that is, the electron and hole will be localized in the left dot all through. It is just the result of numerical calculation.

Next, we analyze the influence of the external driving ac field on the quantum system. Figure 4 shows the time evolution of the probabilities P_Q and P_p ,where $P_Q = |a_1(t)|^2 + |a_2(t)|^2$ ($P_P = |a_3(t)|^2 + |a_4(t)|^2$) is the probability of exciton appearing in the localized (delocalized)states. It is clear from Fig. 4 that when the amplitude of the ac electric field is small ($V_0 \leqslant |K|, |K| = 6.5$), P_Q approaches 1, which means that under the above condition, the Coulomb interaction plays an important role and the particles (electron and hole) tunnel between the two dots as if they were bound together. However, when the amplitude increases until $V_0 \geqslant |K|$, the approximation performed on Eq. (4) will not be feasible, and the initial system cannot be transformed into the two-level system, which results in the separation of particle movement.

In summary, with the Floquet theory, we investigate the dynamical properties of the exciton in an asymmetric double coupled quantum dot and an ac electric field. Despite the energy detuning between the two quantum dots, there will be avoidance intersections in the quasienergy, and the dynamical localization of the exciton appears at the intersection. This conclusion sheds light on the comprehension of the dynamical behavior of the exciton in low-dimensional system and may be applied to quantum calculation and information transaction.

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