REVIEW ARTICLE

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Theory of Atom Optics: Feynman's Path Integral Approach

Abstract The present theory of atom optics is established mainly on the Schrödinger equations or the matrix mechanics equation. The authors present a new theoretical formulation of atom optics: Feynman's path integral theory. Its advantage is that one can describe the diffraction and interference of atoms passing through slits (or grating), apertures, and standing wave laser field in Earth's gravitational field by using a type of wave function and calculation is simple. For this reason, we derive the wave functions of particles in the following configurations: single slit (and slit with the van der Waals interaction), double slit, N slit, rectangular aperture, circular aperture, the Mach–Zehnder-type interferometer, the interferometer with the Raman beams, the Sagnac effect, the Aharonov–Casher effect, the Kapitza–Dirac diffraction effect, and the Aharonov–Bohm effect. The authors give a wave function of the state of particles on the screen in abovementioned configurations. Our formulas show good agreement with present experimental measurements.

1 Introduction

We know that there are three equivalent theories in quantum mechanics: Schrödinger's wave mechanics, Heisenberg's matrix mechanics, and Feynman's path integral theory. The Schrödinger equations and the matrix mechanics equation are differential equations and are local theories describing the wave property of particles. Feynman's path integral is a functional integral equation, and its theory is a global theory describing the wave property of particles. Its position corresponds to Huygens's wave optics theory in optics theory. Hence, Feynman's path integral theory is better than the Schrödinger equation for the diffraction and interference of atoms passing through slits (or grating), apertures, and standing wave laser field.

The basic problem of atom optics theory is determining the positions of interference fringe, fringe spacing, and the probability distribution of atoms on the screen. The present theory of atom optics is not established as a systematic theory, like optics theory [1–3]. At present, it is not a unified calculation formula of fringe spacing in Young's two-slit interference of atoms [4,5]. For example, Catnal and Mlynek [4] use a similar formula for optics: $\Delta z = \lambda_{dB} f/d$, but Shimizu et al. [5] use a different formula: $\Delta x = (h/mv_s)(l/d)[2(\sqrt{1+\alpha}-1)/\alpha]$, $\alpha = 2gl/v_s^2$, where g is the acceleration of gravity. What is the influence of acceleration due to gravity on interference fringes? In addition, Young's interference experiment for atoms discussed in [1,2], but the probability density for atoms do not include diffraction factor. For the Kapitza-Dirac diffration effect of atoms in the gravitational field, it gives too explicit formulas of the output momentum probability distribution in a finite region of standing wave laser field in [2,3]. In addition, the probability distribution of atoms on the screen does not have a unified calculation formula yet. Not having the aid of analogy of optics, we must derive all wave functions and their probability distribution of atoms in the abovementioned configurations from the first principles (Feynman's path integral theory). Taking into account the acceleration of gravity, we will study separately the diffraction and interference of atoms in the abovementioned configurations.

2 Single slit diffraction

Feynman assumed two types of slit functions to calculate the single-slit diffraction of a particle [6]. Its integral form corresponds to the Fresnel integral. Here, we do not assume forms of the slit function. From the first principles (Feynman's

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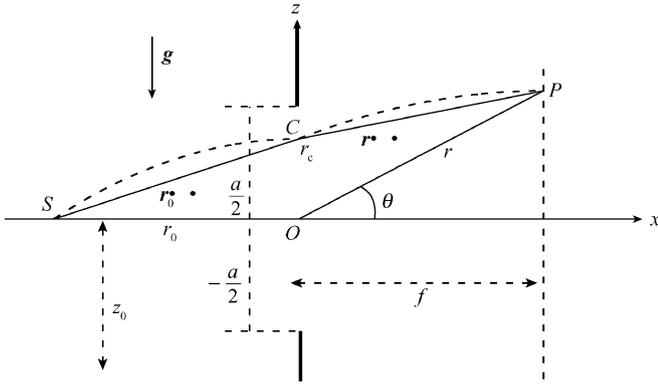Nanjing 210096, China
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Fig. 1 Single-slit diffraction of an atom in the gravitational field

path integral theory), we directly calculate a wave function of diffraction state of an atom passing through a single slit.

Let an atom move from a source point S passing a single slit of width a to a point P on a screen (see Fig. 1). The time that it took the particle to run from S to C is denoted by t, while that from S to P is denoted by T. The wave function of the diffraction state of the atom passing through a single slit is [6]

$$\Psi(\boldsymbol{r}, T) = \int^c \mathrm{d}\boldsymbol{r}_c \int_0^T \mathrm{d}t U(\boldsymbol{r'}, T - t; \boldsymbol{r}_c, t) U(\boldsymbol{r}_c, t; \boldsymbol{r}_0, 0) \tag{1}$$

where the propagators of the free atom moving from S to C and from C to P (parabola) in the gravitational field are, respectively [6],

$$U(\boldsymbol{r}_c, t; \boldsymbol{r}_0, 0) = (\frac{m}{\mathrm{i}ht})^{3/2} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mr_c^2}{2t} - \frac{1}{2}mgt(z_c + |z_0|) - \frac{mg^2t^3}{24}]\} \tag{2}$$

and

$$U(\boldsymbol{r}, T - t; \boldsymbol{r}_c, t) = (\frac{m}{\mathrm{i}h(T - t)})^{3/2} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mr'^2}{2(T - t)} - \frac{1}{2}mg \times (T - t)(z + z_c) - \frac{mg^2(T - t)^3}{24}]\} \tag{3}$$

where m is the mass of the atom, g is the acceleration of gravity, and z_0 is the height from the earth's surface.

In the following approximation conditions:

$$r'_0 \approx r_0 + \hat{\boldsymbol{r}}_0 \cdot \boldsymbol{r}_c, \quad r' \approx zr - \hat{\boldsymbol{r}} \cdot \boldsymbol{r}_c = r - z_c \sin\theta \tag{4}$$
$$\hat{\boldsymbol{r}}_0 = \boldsymbol{r}_0/r_0, \quad \hat{\boldsymbol{r}} = \boldsymbol{r}/r, \quad \sin\theta = z/r$$
where $\hat{\boldsymbol{r}} = \dfrac{\boldsymbol{r}}{|\boldsymbol{r}|}$.

the integration limits for C are $(-a/2, a/2)$. Then, integrating Eq. (1), we obtain

$$\Psi(\boldsymbol{r}, T) = (\frac{m}{\mathrm{i}\hbar})^{5/2} \frac{av}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mv^2}{2}T - \frac{mgT(z + |z_0|)}{2} - \frac{mg^2T^3}{12}]\} \frac{\sin[\dfrac{mva\sin\theta}{2\hbar} + \dfrac{mgTa}{2\hbar}]}{\dfrac{mva\sin\theta}{2\hbar} + \dfrac{mgTa}{2\hbar}} \tag{5}$$

where $v = (r_0 + r)/T$ (average velocity of the particle).

3 Single-slit diffraction with the van der Waals interaction

When the atoms go through a nanoslit, we have to take into account the van der Waals $-C_3/l^3$ interaction of atoms with solid surface. Grisenti et al. [7] used at first the Kirchoff integral formula to study the diffraction of atoms passing through the slits with the van der Waals interaction. They introduced artificially the slit function, which depends on the van der Waals attractive potential, into the Kirchoff integral. Here, we introduce naturally the van der Waals attractive potential into the action of the propagator. Hence, we study this topic by using Feynman's path integral theory. Taking the van der Waals interaction into account, and after adding an additional action $S = V\tau$, Eq. (1) becomes

$$\Psi(\boldsymbol{r}, T) = \int^c \mathrm{d}\boldsymbol{r}_c \int_0^T \mathrm{d}t U(\boldsymbol{r'}, T - t; \boldsymbol{r}_c, t) \mathrm{e}^{-\mathrm{i}V\tau/\hbar} U(\boldsymbol{r}_c, t; \boldsymbol{r}_0, 0) \tag{6}$$

where the van der Waals potential is $V = -C_3/l^3$; l is the distance from the surface and C_3 is the potential constant. τ Is the time that elapsed when an atom passes through the slit. Integrating Eq. (6), we obtain

$$\Psi(\boldsymbol{r}, T) = (\frac{m}{\mathrm{i}\hbar})^{5/2} \frac{av}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mv^2}{2}T - \frac{mgT(z + |z_0|)}{2} - \frac{mg^2T^3}{12}]\} \times \{\frac{\sin K}{K} + \sum_{n=1}^{\infty} \frac{2\pi(-1)^{3n}\mathrm{i}^{n+1}C^n}{n!(3n - 1)!} \cos[K + (3n - 1)\frac{\pi}{2}]\} \tag{7}$$

where $K = (\sin\theta + \dfrac{mgt}{\hbar k})\pi\dfrac{a}{\lambda}$; $C = \dfrac{8C_3\tau}{\hbar a^3}$.

From the above equation, we know that the effect of the van der Waals Interaction is equal to the decreasing slit width. It changes not only the fringe spacing but also the intensity distribution of the interference fringes.

4 Double-slit interference

The double-slit interference of an atom in the gravitational field is shown in Fig. 2. Through the abovementioned method, we obtain a wave function of the state of an atom passing through a double slit on the screen in the gravitational field. Catnal and Mlynek [4] and Shimizu et al. [5] observed experimentally the double-slit interference of atoms. However, they calculated the spacing of the interference fringes using different formulas. The intensity distribution of the interference fringes was calculated using the superposition of quantum states of the Schrödinger equation in [2], but there was no diffraction factor. Whereas it was obtained using the Fourier transform of double-slit function in [8]. Young's double-slit experiment of atoms is plotted in Fig. 2, where we used Feynman's path integral method:

$$
\begin{aligned}
\Psi(\boldsymbol{r}, T) =& (\frac{m}{\mathrm{i}h})^{5/2} \frac{2av}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mv^2}{2}T - \frac{mgT(z+|z_0|)}{2} \\
&- \frac{mg^2 T^3}{12}]\} \times \frac{\sin[\frac{mva\sin\theta}{2\hbar} + \frac{mgTa}{2\hbar}]}{\frac{mva\sin\theta}{2\hbar} + \frac{mgTa}{2\hbar}} \\
&\times \cos[\frac{mvd\sin\theta}{2\hbar} + \frac{mgTd}{2\hbar}]
\end{aligned}
\tag{8}
$$

where a is the single-slit width and d is the distance between two slits.

We can obtain the following in terms of the maximum value of the fringe intensity in Eq. (8):

The position of the maximum value of the fringe:

$$
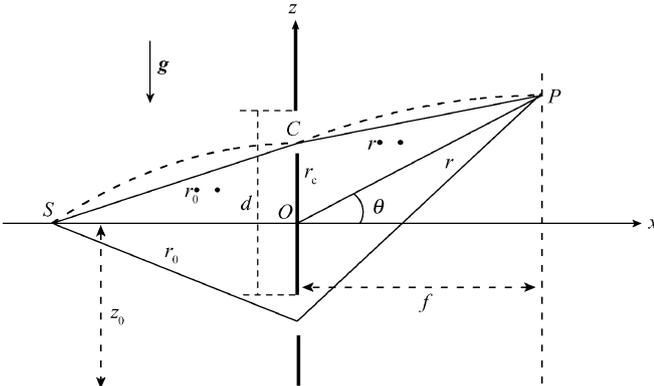z_n = \frac{(2n\hbar k - mgT)f}{mv}, \quad n = 0, \pm 1, \pm 2, \cdots
\tag{9}
$$

Fig. 2 Double-slit interference of an atom in the gravitational field

The fringe spacing:

$$
\Delta z = \frac{\lambda f}{d}, \quad \lambda \approx \frac{h}{mv}
\tag{10}
$$

The shift of the fringe in gravity:

$$
z_0(n=0) = -\frac{gTf}{v}
\tag{11}
$$

The above equations show that the spacing between two consecutive interference fringes is rarely dependent on the acceleration of gravity, but the fringe shift is dependent on it.

5 N-slit interference

With the abovementioned method, we obtain a wave function of the state of an atom passing through N slit on the screen in the gravitational field

$$
\begin{aligned}
\Psi(\boldsymbol{r}, T) =& (\frac{m}{\mathrm{i}h})^{5/2} \frac{2av}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mv^2}{2}T - \frac{mgT(z+|z_0|)}{2} \\
&- \frac{mg^2 T^3}{12}]\} \times \frac{\sin[\frac{mva\sin\theta}{2\hbar} + \frac{mgTa}{2\hbar}]}{\frac{mva\sin\theta}{2\hbar} + \frac{mgTa}{2\hbar}} \\
&\times \frac{\sin[\frac{Nmvd\sin\theta}{2\hbar} + \frac{NmgTd}{2\hbar}]}{\frac{mvd\sin\theta}{2\hbar} + \frac{mgTd}{2\hbar}}
\end{aligned}
\tag{12}
$$

where N is the number of slits.

6 Grating with the van der Waals interaction

Combining N-slit interference and single-slit diffraction with the van der Waals interaction, we obtain a wave function of the state of an atom passing through grating with the van der Waals interaction on the screen in the gravitational field:

$$
\begin{aligned}
\Psi(\boldsymbol{r}, T) =& (\frac{m}{\mathrm{i}h})^{5/2} \frac{2av}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mv^2}{2}T \\
&- \frac{mgT(z+|z_0|)}{2} - \frac{mg^2 T^3}{12}]\} \times \{\frac{\sin\theta}{K} \\
&+ \sum_{n=1}^{\infty} \frac{2\pi(-1)^{3n}\mathrm{i}^{n+1}C^n}{n!(3n-1)!} \cos[K + (3n-1)\frac{\pi}{2}]\} \\
&\times \frac{\sin(\frac{Nmvd\sin\theta}{2\hbar} + \frac{NmgTd}{2\hbar})}{\frac{mvd\sin\theta}{2\hbar} + \frac{mgTd}{2\hbar}}
\end{aligned}
\tag{13}
$$

7 Rectangular aperture

With the abovementioned method, we obtain a wave function of the state of atoms passing through a rectangular aperture of width a and length b on the screen in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = (\frac{m}{\mathrm{i}h})^{5/2} \frac{ab\upsilon}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{m\upsilon^2}{2}T$$
$$- \frac{mgT(z + |z_0|)}{2} - \frac{mg^2 T^3}{12}]\}$$
$$\times \frac{\sin[\frac{m\upsilon a \sin \boldsymbol{\theta_1}}{2\hbar} + \frac{mgTa}{2\hbar}]}{\frac{m\upsilon a \sin \theta_1}{2\hbar} + \frac{mgTa}{2\hbar}} \qquad (14)$$
$$\times \frac{\sin[\frac{m\upsilon b \sin \boldsymbol{\theta_2}}{2\hbar} + \frac{mgTb}{2\hbar}]}{\frac{m\upsilon b \sin \theta_2}{2\hbar} + \frac{mgTb}{2\hbar}}$$

where θ_1 and θ_2 represent the angles of diffraction.

8 Circular aperture

With the abovementioned method, we obtain a wave function of the state of atoms passing through a circular aperture of radius a on the screen in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = (\frac{m}{\mathrm{i}h})^{5/2} \frac{\pi a^2 \upsilon}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{m\upsilon^2}{2}T$$
$$- \frac{mgT(z + |z_0|)}{2} - \frac{mg^2 T^3}{12}]\} \qquad (15)$$
$$\times \frac{2J_1(\frac{m\upsilon a \sin \theta}{\hbar} + \frac{mgTa}{\hbar})}{\frac{m\upsilon a \sin \theta}{\hbar} + \frac{mgTa}{\hbar}}$$

where J_1 is the Bessel function of the first order.

9 Mach–Zehnder-type interferometer

The Mach–Zehnder-type interferometer of an atom in the gravitational field is shown in Fig. 3.

With the abovementioned method, we obtain a wave function of the state of atoms on the screen in the Mach–Zehnder-type interferometer in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = (\frac{m}{\mathrm{i}h})^{5/2} \frac{2\upsilon}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{m\upsilon^2}{2}T - \frac{mgT(z + |z_0|)}{2}$$
$$- \frac{mg^2 T^3}{12}]\} \times \cos[\frac{m\upsilon d \sin \boldsymbol{\theta}}{2\hbar} + \frac{mgTd}{2\hbar}] \qquad (16)$$
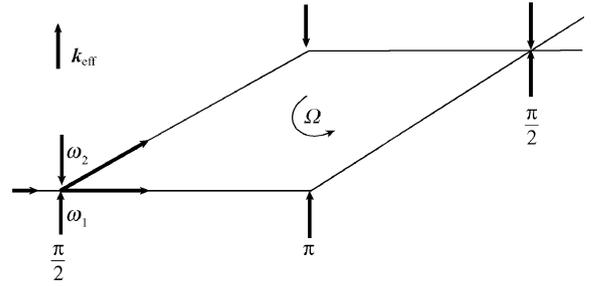

Fig. 3 Mach–Zehnder-type interferometer of an atom in the gravitational field

where d is the distance between symmetric top points in the middle of the two paths.

10 Mach–Zehnder-type interferometer with the Raman beams

The Mach–Zehnder-type atom interferometer with the Raman Beams in the gravity field is shown in Fig. 3. With the abovementioned method, we obtain a wave function of the state of atoms with the Raman beams on the screen in the Mach–Zehnder-type interferometer in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = (\frac{m}{\mathrm{i}h})^{5/2} \frac{2\upsilon}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{m\upsilon^2}{2}T - \frac{mgT(z + |z_0|)}{2}$$
$$- \frac{mg^2 T^3}{12}]\} \times \cos[\frac{m\upsilon d \sin \theta}{\hbar} + \frac{mgTd}{\hbar}$$
$$+ \frac{\hbar}{2}k_{\mathrm{eff}}gT^2 - \frac{\hbar}{2}2\pi T \Delta f] \qquad (17)$$

where the effective wavenumber is $k_{\mathrm{eff}}=|k_1|+|k_2|$ for counterpropagating laser beams. The two additional terms in the last factor of the above equation are due to the atom interaction with the Raman beams in the gravitational field and to the change in the Raman frequency, respectively. Adjusting Δf to make the sum of these two terms disappear, we obtain

$$g = \frac{2\pi\Delta f}{k_{\mathrm{eff}}T} \qquad (18)$$

which can determine the acceleration of gravity. Presently, this method has the highest accuracy (3×10^{-9}) for determining g [9].

11 Sagnac effect for atom interferometer

In the Mach–Zehnder-type atom interferometer, if the interferometer is rotating with angular velocity Ω, about an axis through the center and perpendicular to the plane of the interferometer, a shift of the interference fringes is observed (see Fig. 3). This is the well-known Sagnac effect

for interferometers [10]. We introduce two additional actions into the propagators along two paths of the interferometer, respectively. Using the calculation in Section 8, we obtain a wave function of the state describing the Sagnac effect of atoms in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = \left(\frac{m}{\mathrm{i}h}\right)^{5/2} \frac{2\upsilon}{r_0 r \sqrt{T}} \exp\left\{\frac{\mathrm{i}}{\hbar}\left[\frac{m\upsilon^2}{2}T - \frac{mgT(z + |z_0|)}{2}\right.\right.$$
$$\left.\left. - \frac{mg^2 T^3}{12}\right]\right\} \times \cos\left[\frac{m\upsilon d \sin\theta}{2\hbar}\right.$$
$$\left. + \frac{mgTd\sin\phi}{2\hbar} + \frac{m\boldsymbol{\Omega}\cdot\boldsymbol{A}}{\hbar}\right]$$

(19)

where A is the area enclosed by two paths in the Mach–Zehnder-type atom interferometer and ϕ is the angle included between the interferometer plane and the gravitational direction.

12 Aharonov–Casher effect of atoms

Aharonov and Casher pointed out that a geometrical phase is produced when matter waves associated with neutral particles having magnetic moment μ_{m} encircle a closed path around a charged wire [11]. We introduce two additional actions into the propagators along two paths of the interferometer, respectively. We obtain a wave function of the state describing the Aharonov–Casher effect of atoms in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = \left(\frac{m}{\mathrm{i}h}\right)^{5/2} \frac{2\upsilon}{r_0 r \sqrt{T}} \exp\left\{\frac{\mathrm{i}}{\hbar}\left[\frac{m\upsilon^2}{2}T - \frac{mgT(z + |z_0|)}{2}\right.\right.$$
$$\left.\left. - \frac{mg^2 T^3}{12}\right]\right\} \cos\left[\frac{m\upsilon d\sin\theta}{2\hbar} + \frac{mgTd\sin\phi}{2\hbar}\right.$$
$$\left. - \frac{1}{2\hbar c^2}\oint(\mu\times\boldsymbol{E})\cdot\mathrm{d}\boldsymbol{r}\right]$$

(20)

where E is the electric field of the charged wire.

13 Kapitza–Dirac diffraction effect of atoms

In 1933, Kapitza and Dirac (KD) [12] predicted that electrons travelling through standing light wave would be diffracted. Since then, several experiments have been made to verify this effect, but all have failed [13,14]. However, the diffraction of atoms passing through standing wave laser field was verified [15–17]. Many theoretical studies on atoms were devoted to the KD effect. These theories are solved by either the Schrödinger equations [18] or the density matrix equations [19], but none of them give ex-

plicit formulas of diffraction fringes with relative physical parameters. Furthermore, to obtain the diffraction solutions of the equations, it is necessary to assume that the kinetic energy term in the equations should be neglected (the Raman–Nath approximation). For example, in the theoretical calculation of [15,16], the authors only gave the momentum probabilities of some special points [$P_{2n}=J_n^2(z)$] and did not give an analytically continuous momentum distribution of probability. In fact, the experimental curve of the momentum probability is continuously distributive [15].

With the abovementioned method, we can obtain analytically a wave function of the state describing the Kapitza–Dirac diffraction effect of atoms in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = \left(\frac{m}{\mathrm{i}h}\right)^{5/2} \frac{2\upsilon}{k r_0 r \sqrt{T}} \exp\left\{\frac{\mathrm{i}}{\hbar}\left[\frac{m\upsilon^2}{2}T - \frac{mgT(z + |z_0|)}{2}\right.\right.$$
$$\left.\left. - \frac{mg^2 T^3}{12} - \frac{V_0\tau}{2}\right]\right\} \sum_{n=-\infty}^{\infty}(-\mathrm{i})^n J_n\left(\frac{V_0\tau}{2\hbar}\right)$$
$$\times \frac{\sin[(\frac{m\upsilon\sin\theta}{\hbar k} + \frac{mgT}{\hbar k} - 2n)kd]}{\frac{m\upsilon\sin\theta}{\hbar k} + \frac{mgT}{\hbar k} - 2n}$$

(21)

where [2]

$$V = V_0 \cos^2 kz, \quad V_0 = \frac{\hbar\Omega^2}{\omega}$$

(22)

where the detuning is $\delta\omega = \omega_a - \omega$ (ω_a and ω are the atomic and field frequencies, respectively), k is the wave number of the standing wave laser, and $\Omega = \mu E_0/\hbar$ is the Rabi frequency, with μ being the dipole moment and E_0 the maximum field amplitude. J_n is the nth-order Bessel function.

Equation (21) is valid for the Kapitza–Dirac diffraction effect of electrons in the gravitational field, except for $V_0 = e^2 I/2\varepsilon_0 mc\omega$ [20]. The wave function and its corresponding probability in Eq. (21) have a continuous momentum distribution of probability.

In addition to the above discussions, we can also study the Aharonov–Bohm effect of charged particles in the gravitational field using the Feynman path integral theory.

14 Aharonov–Bohm effect of charged particles

It is shown in Fig. 2 that there is a tiny solenoid located between the two slits, designed so that a magnetic field perpendicular to the plane of the figure can be produced in its interior. With the abovementioned method, we can ob-

tain a wave function of the state describing the Aharonov–Bohm effect [21] of charged particles in the gravitational field:

$$\Psi(\boldsymbol{r}, T) = (\frac{m}{\mathrm{i}h})^{5/2} \frac{2av}{r_0 r \sqrt{T}} \exp\{\frac{\mathrm{i}}{\hbar}[\frac{mv^2}{2}T - \frac{mgT(z + |z_0|)}{2}$$

$$- \frac{mg^2 T^3}{12}]\} \frac{\sin[\frac{mva \sin\theta}{2\hbar} + \frac{mgTa}{2\hbar}]}{\frac{mva \sin\theta}{2\hbar} + \frac{mgTa}{2\hbar}}$$

$$\times \cos[\frac{mvd \sin\theta}{2\hbar} + \frac{mgTd}{2\hbar} - \frac{q\Phi}{2\hbar}] \tag{23}$$

where Φ is the magnetic flux: $\Phi = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{r}$.

Adjusting the magnetic flux to make the sum of the last two terms in the last factor of the above equation disappear, we obtain the following: for general magnetic flux,

$$g = \frac{q\Phi}{mTd} \tag{24}$$

and for quantum magnetic flux,

$$g = \frac{nq\Phi_0}{mTd}, \quad n = 0, 1, 2, \cdots \tag{25}$$

where q is the charge of the charged particle and $\Phi_0 = h/(2e)$, where e is the electron charge. The two above equations can be used to determine the acceleration of gravity.

15 Comparison between the abovementioned theories and the present experimental results

As stated above, we give wave functions of atoms in the abovementioned configurations. Thus, we get the probability of finding the atoms on the screen. It is the absolute square of each wave function, i.e., $|\Psi(\boldsymbol{r}, \boldsymbol{T})|^2$. Furthermore, we can obtain fringe position, fringe spacing, and fringe shift in terms of the maximum value of the fringe intensity in the abovementioned equations. After comparing the abovementioned theories with the present experimental results (comparison in detail will be given elsewhere), we find that our formulas are good agreement with the present experimental measurements.

For example, taking the data of the helium atom in [4]: λ=0.056 nm; d=8 μm; and f=64 cm, we obtain the fringe spacing $\Delta z = \lambda f/d$=4.5 μm, which consistent with the experimental result of 4.5±0.6 μm in [4]. For intensity distribution, we get probability $|\psi(p_z, T)|^2$ of the sodium atom in $g = 0$, see Fig. 4, where $P_z = mv\sin\theta$, $x = p_z/\hbar k$, $y = \Omega^2 \tau / 2\delta\omega$, and $\beta = d/\lambda$. We compare our theoritical curves (*solid lines*) with the experimental curves (*dot lines*) of Fig. 1(a) in [15], and find that they are in good agreement with each other.
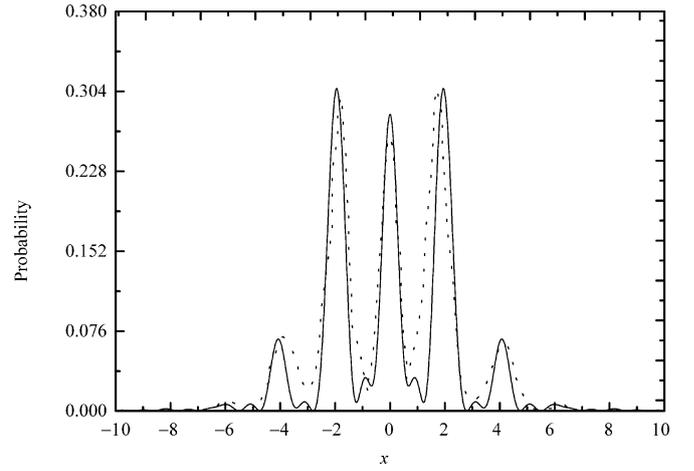

Fig. 4 Atomic diffraction pattern corresponding to the following data: y=1.52, β=0.64. The experimental curve is dotted line [15]

In summary, we can conclude the following:

1. Using Feynman's path integral theory, we present and establish a new theoretical formulation of atom optics and explicitly give wave functions describing the states of atoms through the abovementioned configurations.
2. Our formulas provide a unified explanation of the diffraction and interference patterns of atoms in the abovementioned configurations and are in good agreement with the experiments.
3. We show that the fringe spacing is rarely dependent on the acceleration of gravity, but the fringe shift is dependent on it.
4. For the KD effect, in our derivation of the theory, it is not necessary to assume that the kinetic energy term of particles in the Schrödinger equation should be neglected.
5. Our formulas can be applied to determine the particle acceleration due to gravity and to design cold atom interferometer in inertial navigation.

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