Supplementary Material

Finite element modeling of electromagnetic properties in photonic bianisotropic structures

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A Finite element formulation in bianisotropic medium

The left side and the right side term of Eq. (3) are

$$\begin{aligned}\text{Left} &= \int \mathrm{d}V \left\{\left((\nabla\times -\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{eh}})\left[\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}(\nabla\times +\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{he}})\right]\boldsymbol{E}\right)\cdot\boldsymbol{F} - k_0^2\bar{\boldsymbol{\varepsilon}}_{\boldsymbol{r}}\boldsymbol{E}\cdot\boldsymbol{F}\right\}\\ &= \int \mathrm{d}V \begin{pmatrix}\nabla\times\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\nabla\times\boldsymbol{E}\cdot\boldsymbol{F} + \nabla\times\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{he}}\boldsymbol{E}\cdot\boldsymbol{F} - k_0^2\bar{\boldsymbol{\varepsilon}}_{\boldsymbol{r}}\boldsymbol{E}\cdot\boldsymbol{F}\\ -\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{eh}}\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\nabla\times\boldsymbol{E}\cdot\boldsymbol{F} - \mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{eh}}\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{he}}\boldsymbol{E}\cdot\boldsymbol{F}\end{pmatrix},\end{aligned}$$

$$\begin{aligned}\text{Right} &= \int \mathrm{d}V \left\{\left((\nabla\times -\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{eh}})\left[\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}(\nabla\times +\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{he}})\right]\boldsymbol{F}\right)\cdot\boldsymbol{E} - k_0^2\bar{\boldsymbol{\varepsilon}}_{\boldsymbol{r}}\boldsymbol{F}\cdot\boldsymbol{E}\right\}\\ &= \int \mathrm{d}V \begin{pmatrix}\nabla\times\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\boldsymbol{\nabla}\times\boldsymbol{F}\cdot\boldsymbol{E} + \nabla\times\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{he}}\boldsymbol{F}\cdot\boldsymbol{E} - k_0^2\bar{\boldsymbol{\varepsilon}}_{\boldsymbol{r}}\boldsymbol{F}\cdot\boldsymbol{E}\\ -\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{eh}}\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\nabla\times\boldsymbol{F}\cdot\boldsymbol{E} - \mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{eh}}\frac{1}{\bar{\boldsymbol{\mu}}_{\boldsymbol{r}}}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{\boldsymbol{he}}\boldsymbol{F}\cdot\boldsymbol{E}\end{pmatrix}.\end{aligned}$$

Thus, the difference of left and right terms is

$$\text{Left} - \text{Right} = \int \mathrm{d}V \left\{\begin{aligned} &\left(-\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\nabla\times\boldsymbol{E}\cdot\boldsymbol{F} - \nabla\times\frac{1}{\bar{\boldsymbol{\mu}}_r}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{he}\boldsymbol{F}\cdot\boldsymbol{E}\right)\\ &+\left(\nabla\times\frac{1}{\bar{\boldsymbol{\mu}}_r}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{he}\boldsymbol{E}\cdot\boldsymbol{F} + \mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\nabla\times\boldsymbol{F}\cdot\boldsymbol{E}\right)\\ &+\left(-\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{he}\boldsymbol{E}\cdot\boldsymbol{F} + \mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{he}\boldsymbol{F}\cdot\boldsymbol{E}\right)\end{aligned}\right\}, \tag{S1}$$

the third term in the integral has

$$\begin{aligned}&-\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{he}\boldsymbol{E}\cdot\boldsymbol{F} + \mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\mathrm{i}k_0\bar{\boldsymbol{\chi}}^r_{he}\boldsymbol{F}\cdot\boldsymbol{E}\\ &= -k_0^2\left(\boldsymbol{F}\cdot\left[\left(\bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\bar{\boldsymbol{\chi}}^r_{he}\right)^{\mathrm{T}} - \bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\bar{\boldsymbol{\chi}}^r_{he}\right]\boldsymbol{E}\right)\\ &= -k_0^2\left(\boldsymbol{F}\cdot\left[(-\bar{\boldsymbol{\chi}}^r_{eh})^{\mathrm{T}}\frac{1}{\bar{\boldsymbol{\mu}}_r^{\mathrm{T}}}(-\bar{\boldsymbol{\chi}}^r_{he})^{\mathrm{T}} - \bar{\boldsymbol{\chi}}^r_{eh}\frac{1}{\bar{\boldsymbol{\mu}}_r}\bar{\boldsymbol{\chi}}^r_{he}\right]\boldsymbol{E}\right) = 0,\end{aligned}$$

and the second term in the integral has

$$\int \mathrm{d}V \left(\nabla \times \frac{1}{\bar{\boldsymbol{\mu}}_r} \mathrm{i}k_0 \bar{\boldsymbol{\chi}}_{he}^r \boldsymbol{E} \cdot \boldsymbol{F} + \mathrm{i}k_0 \bar{\boldsymbol{\chi}}_{eh}^r \frac{1}{\bar{\boldsymbol{\mu}}_r} \nabla \times \boldsymbol{F} \cdot \boldsymbol{E} \right)$$
$$= \mathrm{i}k_0 \int \mathrm{d}V \left[-\nabla \times \boldsymbol{F} \cdot \frac{1}{\bar{\boldsymbol{\mu}}_r} \bar{\boldsymbol{\chi}}_{he}^r \boldsymbol{E} + \nabla \times \frac{1}{\bar{\boldsymbol{\mu}}_r} \bar{\boldsymbol{\chi}}_{he}^r \boldsymbol{E} \cdot \boldsymbol{F} \right]$$
$$= \mathrm{i}k_0 \int \mathrm{d}V \left[\nabla \times \left(\frac{1}{\bar{\boldsymbol{\mu}}_r} \bar{\boldsymbol{\chi}}_{he}^r \right) \boldsymbol{E} \cdot \boldsymbol{F} - \nabla \times \boldsymbol{F} \cdot \left(\frac{1}{\bar{\boldsymbol{\mu}}_r} \bar{\boldsymbol{\chi}}_{he}^r \right) \boldsymbol{E} \right]$$
$$= \mathrm{i}k_0 \int \mathrm{d}V \nabla \cdot \left[\left(\frac{1}{\bar{\boldsymbol{\mu}}_r} \bar{\boldsymbol{\chi}}_{he}^r \right) \boldsymbol{E} \times \boldsymbol{F} \right] = \oint \mathrm{d}S \left[\left(\frac{1}{\bar{\boldsymbol{\mu}}_r} \bar{\boldsymbol{\chi}}_{he}^r \right) \boldsymbol{E} \times \boldsymbol{F} \right] = 0,$$

in the similar way, the first terms in integral is also zero,

$$\int \mathrm{d}V \left(-\mathrm{i}k_0 \bar{\boldsymbol{\chi}}_{eh}^r \frac{1}{\bar{\boldsymbol{\mu}}_r} \nabla \times \boldsymbol{E} \cdot \boldsymbol{F} - \nabla \times \frac{1}{\bar{\boldsymbol{\mu}}_r} \mathrm{i}k_0 \bar{\boldsymbol{\chi}}_{he}^r \boldsymbol{F} \cdot \boldsymbol{E} \right) = 0.$$

B Semi-analysis method for reflection and transmission of light in bianisotropic slab

The curl terms in Eq. (1) can be replaced by matrix $\bar{\boldsymbol{M}} = \begin{pmatrix} 0 & \mathrm{i}k_z & -\mathrm{i}k_y \\ -\mathrm{i}k_z & 0 & \mathrm{i}k_x \\ \mathrm{i}k_y & -\mathrm{i}k_x & 0 \end{pmatrix}$ in an uniform bianisotropic media, we then can got a 6×6 matrix form equation

$$\begin{pmatrix} 0 & \bar{\boldsymbol{M}} \\ -\bar{\boldsymbol{M}} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{e} \\ \boldsymbol{h} \end{pmatrix} = \mathrm{i}k_0 \begin{pmatrix} \bar{\boldsymbol{\varepsilon}}_r & \bar{\boldsymbol{\chi}}_{eh}^r \\ \bar{\boldsymbol{\chi}}_{he}^r & \bar{\boldsymbol{\mu}}_r \end{pmatrix} \begin{pmatrix} \boldsymbol{e} \\ \boldsymbol{h} \end{pmatrix}, \tag{S2}$$

where $k_i, i = x, y, z$ is wave vector of light propagating in this uniform medium. Assume the effective refraction index of light in medium is n_{eff}, wave vector can be rewritten as $k_x = k_0 n_{\text{eff}} \cos\theta \sin\phi$, $k_y = k_0 n_{\text{eff}} \sin\theta \sin\phi$ and $k_z = k_0 n_{\text{eff}} \cos\phi$, where θ and ϕ are Euler angles. We then get a general eigen-problem equation

$$\begin{pmatrix} 0 & \bar{\boldsymbol{R}} \\ -\bar{\boldsymbol{R}} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{e} \\ \boldsymbol{h} \end{pmatrix} = \frac{1}{n_{\text{eff}}} \begin{pmatrix} \bar{\boldsymbol{\varepsilon}}_r & \bar{\boldsymbol{\chi}}_{eh}^r \\ \bar{\boldsymbol{\chi}}_{he}^r & \bar{\boldsymbol{\mu}}_r \end{pmatrix} \begin{pmatrix} \boldsymbol{e} \\ \boldsymbol{h} \end{pmatrix}, \tag{S3}$$

where $\bar{\boldsymbol{R}} = \begin{pmatrix} 0 & \cos\phi & -\sin\theta\sin\phi \\ -\cos\phi & 0 & \cos\theta\sin\phi \\ \sin\theta\sin\phi & -\cos\theta\sin\phi & 0 \end{pmatrix}$. As a result, with certain direction of light in the medium we can get n_{eff} and electromagnetic field as the eigenvalue and eigenvector from the last eigen-problem. Then, we finally get the reflection and transmission spectrum semi-analytically from transfer matrix method by using the continuity of wave vector and electromagnetic field on boundary between air and bianisotropic slab. For example, in Section 3, light propagates along positive and negative y direction, and the boundaries are vertical with y axis; thus the x and z components of the electric and magnetic field are continuing. Therefore, on the first boundary, we have

$$\begin{pmatrix} F_{in}^x(1) & F_{in}^z(1) & F_{re}^x(1) & F_{re}^z(1) \\ F_{in}^x(3) & F_{in}^z(3) & F_{re}^x(3) & F_{re}^z(3) \\ F_{in}^x(4) & F_{in}^z(4) & F_{re}^x(4) & F_{re}^z(4) \\ F_{in}^x(6) & F_{in}^z(6) & F_{re}^x(6) & F_{re}^z(6) \end{pmatrix} \begin{pmatrix} a_{in}^x \\ a_{in}^z \\ a_{re}^x \\ a_{re}^z \end{pmatrix} = \begin{pmatrix} F_+^1(1) & F_+^2(1) & F_-^1(1) & F_-^2(1) \\ F_+^1(3) & F_+^2(3) & F_-^1(3) & F_-^2(3) \\ F_+^1(4) & F_+^2(4) & F_-^1(4) & F_-^2(4) \\ F_+^1(6) & F_+^2(6) & F_-^1(6) & F_-^2(6) \end{pmatrix} \begin{pmatrix} a_+^1 \\ a_+^2 \\ a_-^1 \\ a_-^2 \end{pmatrix},$$

where F represent eigen-function of light in air or bianisotropic slab from Eq. (S3), a represent corresponding amplitude, subscript in and re represent the positive and negative propagating light in air, subscript $+$ and $-$ represent the positive and negative propagating light in the bianisotropic slab, superscript x and z represent the polarization of light in air, superscript 1 and 2 represent two modes in bianisotropic slab with different refractive index n_1 and n_2. On the second boundary and thickness of slab L, we then have

$$\begin{pmatrix} F_+^1(1) & F_+^2(1) & F_-^1(1) & F_-^2(1) \\ F_+^1(3) & F_+^2(3) & F_-^1(3) & F_-^2(3) \\ F_+^1(4) & F_+^2(4) & F_-^1(4) & F_-^2(4) \\ F_+^1(6) & F_+^2(6) & F_-^1(6) & F_-^2(6) \end{pmatrix} \begin{pmatrix} a_+^1 e^{-ik_0 n_1 L} \\ a_+^2 e^{-ik_0 n_2 L} \\ a_-^1 e^{ik_0 n_1 L} \\ a_-^2 e^{ik_0 n_2 L} \end{pmatrix} = \begin{pmatrix} F_{in}^x(1) & F_{in}^z(1) \\ F_{in}^x(3) & F_{in}^z(3) \\ F_{in}^x(4) & F_{in}^z(4) \\ F_{in}^x(6) & F_{in}^z(6) \end{pmatrix} \begin{pmatrix} a_{tr}^x \\ a_{tr}^z \end{pmatrix}, \quad \text{(S4)}$$

where a_{tr} is amplitude of transmission light on second boundary. We could then get the amplitude of reflection and transmission light with certain incident light from the last two equations as transfer matrix method and transmissivity is square of corresponding amplitude.

C Wave function in bianisotropic waveguide in Section 5

For the slab waveguide in Section 5, we can reduce three-dimension to two-dimension as x dependence is constant. As a result, the free-source Maxwell's equation with the bianisotropic term can be reduced to two equations,

$$\begin{aligned} \mathrm{i}k_0\varepsilon_r e_z - k_0\chi_{33}h_z + \frac{\partial}{\partial y}h_x - \frac{\partial}{\partial x}h_y &= 0, \\ k_0\chi_{33}e_z - \frac{\partial}{\partial y}e_x + \frac{\partial}{\partial x}e_y + \mathrm{i}k_0\mu_r h_z &= 0, \end{aligned} \quad \text{(S5)}$$

where k_0 is wave number in vacuum, e_n and h_n $(n = x, y, z)$ are three components of electric and magnetic fields. Since $e_x = \frac{1}{\mathrm{i}k_0\varepsilon_r}\frac{\partial}{\partial y}h_z$, $e_y = -\frac{1}{\mathrm{i}k_0\varepsilon_r}\frac{\partial}{\partial x}h_z$, $h_x = -\frac{1}{\mathrm{i}k_0\mu_r}\frac{\partial}{\partial y}e_z$ and $h_y = \frac{1}{\mathrm{i}k_0\mu_r}\frac{\partial}{\partial x}e_z$, we rewrite the last two equations as

$$\begin{aligned} \left(\frac{\partial}{\partial x}\frac{1}{\mu_r}\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\mu_r}\frac{\partial}{\partial y} + k_0^2\varepsilon_r\right)e_z + \mathrm{i}k_0^2\chi_{33}h_z &= 0, \\ \left(\frac{\partial}{\partial x}\frac{1}{\varepsilon_r}\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\varepsilon_r}\frac{\partial}{\partial y} + k_0^2\mu_r\right)\mathrm{i}h_z + k_0^2\chi_{33}e_z &= 0. \end{aligned} \quad \text{(S6)}$$

When the differential of ε_r and μ_r can be ignored, i.e., ε_r and μ_r are constants, we could combine last two equation as

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2\sqrt{\varepsilon_r\mu_r}\left(\sqrt{\varepsilon_r\mu_r} \pm \chi_{33}\right)\right]\left(e_z \pm \mathrm{i}\sqrt{\frac{\mu_r}{\varepsilon_r}}h_z\right) = 0. \quad \text{(S7)}$$

Since $e_y = -\frac{1}{\mathrm{i}k_0\varepsilon_r}\frac{\partial}{\partial x}h_z = \frac{\beta}{k_0\varepsilon_r}h_z$, where β is propagation constant, we then have two wave equation for transverse electric fields with opposite ellipse (sometimes can be circle) polarization by replacing h_z with e_y

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2\sqrt{\varepsilon_r\mu_r}\left(\sqrt{\varepsilon_r\mu_r} \pm \chi_{33}\right)\right]\left(e_z \pm \mathrm{i}\frac{k_0\sqrt{\varepsilon_r\mu_r}}{\beta}e_y\right) = 0. \quad \text{(S8)}$$

Besides that, since β reverse sign when light propagates along $-x$ direction, the polarization will change helicity between two opposite propagating modes with same absolute β. The above conclusion may be not exact for waveguide, but the positive and negative propagating modes still have opposite ellipse polarization.