

# Off-axis three-mirror reflective zoom system based on freeform surface

Guijuan XIE, Jun CHANG (✉), Ke ZHANG, Jide ZHOU, Yajun NIU

School of Optoelectronics, Beijing Institute of Technology, Beijing 100081, China

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**Abstract** A reflective optical system is not affected by chromatic aberration, so it has a wide range of applications. Based on the design theory of reflective zoom system with three mirrors, this paper presents the simulation, optimization, and image quality evaluation of the traditional off-axis three-mirror zoom system and freeform off-axis three-mirror reflective zoom system. In these systems, the optical design was aided by software CODEV. Through the analysis of aberrations and structural performance for the traditional aspherical off-axis three-mirror system, the freeform surface was introduced to the tertiary mirror to improve the balance capacity for optical aberrations. This off-axis three-mirror reflective zoom system based on freeform surface could provide technical reference to the study of such systems.

**Keywords** off-axis system design, three-mirror reflective zoom system, freeform surface, optical design

## 1 Introduction

The reflective zoom optical system has many advantages, such as light weighted, free of chromatic aberration, and high thermal stability. These advantages would be evident in its wide applications. As the two-mirror reflective zoom system has less optimization variables, it cannot meet the requirements of the wide field of view and the large relative aperture, and the practical use is restricted. To expand its application scope, a third mirror has been introduced to the two-mirror reflective zoom system. The three-mirror coaxial reflective zoom system, in a small field of view, can be optimized through its own degrees of freedom. But in a large field of view, the central obscuration is so large that the energy entering the system is affected and the resolution is reduced. However, the off-axis three-mirror

reflective zoom system exists no central obscuration, and both the field of view and the resolution could be improved compared to the coaxial system.

Conventional optical systems that only use spherical and aspherical surfaces have been unable to resolve the contradictions between the increasing field of view and the miniaturization of volume of the optical system. The freeform surface optics was then introduced. Its asymmetric form is able to provide flexible space layout, and its multiple parameters can offer more degrees of freedom to enhance the aberrations balance, especially for the non-symmetric aberrations. The surface shape can significantly reduce off-axis aberrations of the optical system [1,2], and expanded the field of view. The research based on the freeform surface has become an important direction of a new generation of the high-performance space optical system [3].

The freeform surface has been widely applied in non-imaging systems since the 1990s, such as in the illumination system [4,5]. In recent years, great efforts have been taken in the field of freeform optics, and a breakthrough have already been made [6,7]. The progress in freeform optics has also demonstrated its possible applications in imaging systems, such as head mounted display (HMD) [8], and laid a solid technical foundation for the development of the off-axis optical system based on the freeform surface [3].

In this paper, the traditional off-axis three-mirror zoom system and the off-axis three-mirror reflective zoom system based on freeform surface have been designed and analyzed respectively. Their image quality has also been simulated and evaluated. Through the analysis of structural properties, the aberrations of the traditional off-axis three-mirror reflective zoom system, and the explanation of the specific role of freeform surface in off-axis aberrations balancing in the large field of view, we ultimately proposed the off-axis three-mirror reflective zoom system based on the tertiary freeform mirror with specific parameters.

## 2 Design of traditional off-axis reflective zoom system with three mirrors

Currently, the coaxial reflective zoom system design has been carried out effectively, such as the design of mechanically compensated reflective zoom system with three mirrors [9]. This system was based on zoom optical theory and Seidel aberration theory, and requirements of the system were met by adding some appropriate constraints and further optimization. On the basis of the coaxial system, the required off-axis reflective zoom system can be obtained by adding certain amount of eccentricity and tilt and by further optimization. This off-axis reflective zoom system was based on the vector aberration theory [10].

In this paper, we designed the off-axis reflective zoom system with an offset aperture stop and a biased input field. The layout of this optical system is shown in Figs. 1(a) and 1(b). The primary mirror was fixed, and the distance between the secondary mirror and the tertiary mirror was changed with zooming. The focal length of the system was 25 to 75 mm. The F/# was 10. The field of view was  $4^\circ \times 5^\circ$  to  $4^\circ \times 15^\circ$ . The total length of the optical system was shorter than 12 mm. In this system, there were eight variables to correct the five kinds of aberrations: spherical aberration, coma, astigmatism, field curvature and distortion, and its advanced aberration correction was also taken

into account. Specific configuration parameters are shown in Table 1. Figures 2(a) and 2(b) are the system modulation transfer function (MTF) curve values.

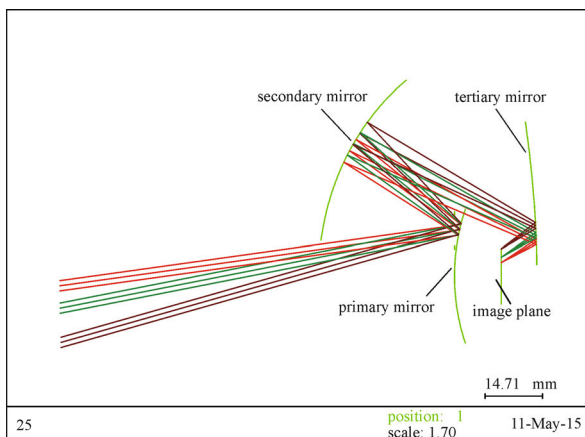
As is shown in Fig. 2, the astigmatism and coma aberration cannot be well corrected when only the secondary aspheric coefficients were set variable. To further improve the image quality and balance system aberrations, the three reflective mirrors were all set to be high-order aspheric surfaces and all their aspherical coefficients were set to be variables. The configuration parameters, finally obtained after optimization, are shown in Table 2. The MTF graphs of the system at the two focal lengths are shown in Figs. 3(a) and 3(b). As can be seen from the curves, the system MTF at each focal length can be close to the diffraction limit at 80 lp/mm.

## 3 Design and discussion of reflective zoom system based on freeform surface

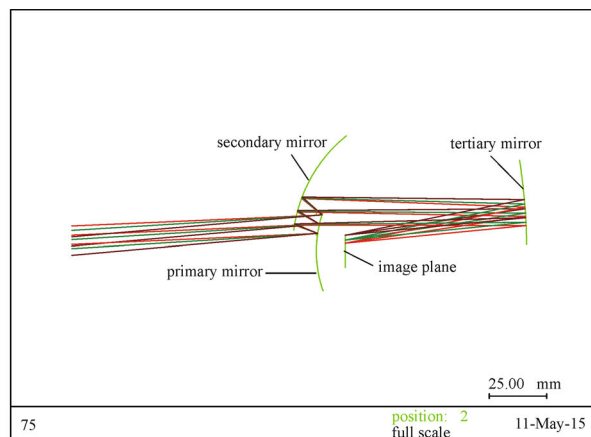
The parameters of the high-order off-axis aspheric reflective zoom system shown in Fig. 3 are depicted as below. The field of view (FOV) was  $6^\circ \times 23^\circ$  at the shorter focus ( $f = 25$  mm). The FOV was  $6^\circ \times 8^\circ$  at the longer focus ( $f = 75$  mm). Under both conditions, the system showed a good image quality with a resolution of 80 lp/mm. However, when the FOV increased, such as to  $6^\circ \times 23^\circ$  at the shorter focus (Fig. 4(a)), and to  $5^\circ \times 8^\circ$  at the

**Table 1** Off-axis three-mirror zoom optical system structure parameters

mirror	radius/mm	separation/mm		conic
		$f = 25$	$f = 75$	
primary mirror	49.0623	- 34.7492	- 10.0215	0.4864
secondary mirror	62.3761	55.8540	97.1031	0.1222
tertiary mirror	- 219.3012	- 9.1145	- 74.9965	- 11.31235

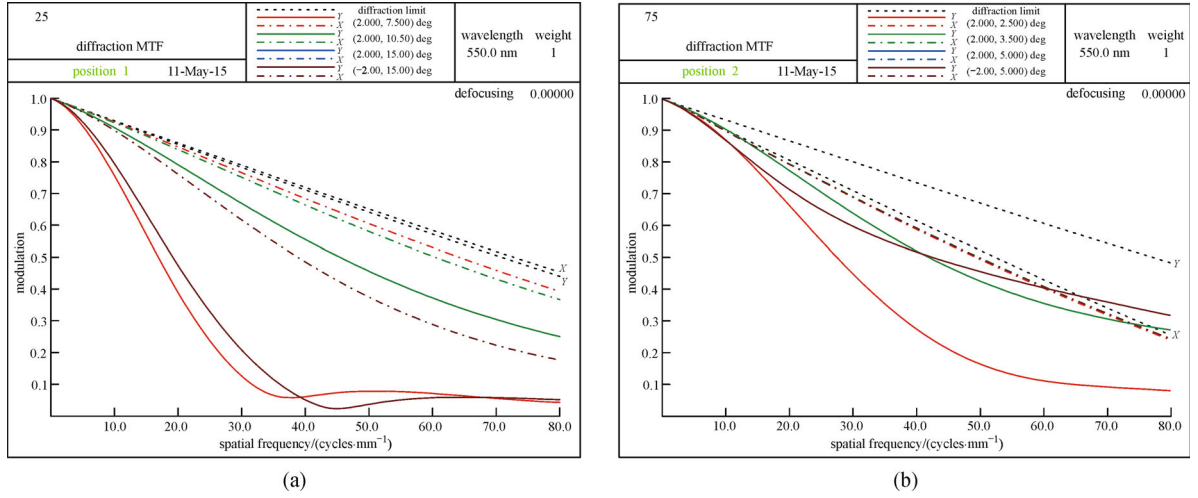


(a)



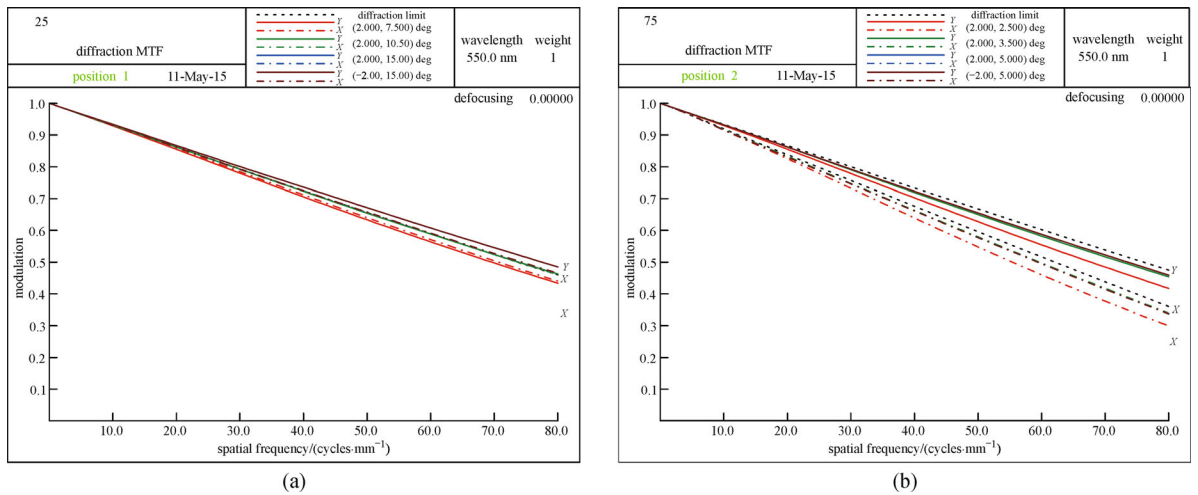
(b)

**Fig. 1** Off-axis optical system structural diagram. (a)  $f = 25$  mm; (b)  $f = 75$  mm


**Fig. 2** Three reflective zoom optical system MTF. (a)  $f = 25$  mm; (b)  $f = 75$  mm

**Table 2** High-order three off-axis aspheric mirrors focus optical system structure parameters

mirror	radius/mm	separation/mm		aspherical coefficients					
		$f = 25$	$f = 75$	conic	4th	6th	8th	10th	
primary mirror	38.3783	- 18.0437	- 6.5437	0.5517	- 5.9840E - 007	- 5.9022E - 010	0	0	
secondary mirror	43,4576	39.1537	79.4389	0.1363	- 5.0966E - 008	- 2.0755E - 011	0	0	
tertiary mirror	- 210.3284	- 11.3329	- 62.7483	- 0.1728	1.6011E - 007	1.8344E - 010	0	0	


**Fig. 3** MTF value after optimization of higher order aspherical coefficients. (a)  $f = 25$  mm; (b)  $f = 75$  mm

longer focus (Fig. 4(b)), the image quality significantly deteriorated at the shorter focus. When the FOV continued to increase, the related aberrations gradually became larger, such as coma, astigmatism and off-axis advanced aberrations. The symmetric high-order aspheric system can no longer balance the dramatic increase of advanced axis aberrations. Therefore, in order to obtain an off-axis reflective zoom system with good image quality and a wide FOV we need to introduce a new type of reflective surface

to add the optimization variables. In this paper, the freeform surface was adopted to design the off-axis reflective zoom system.

Free-form surface is a kind of special irregular and complex surface shape, which provides more degrees of freedom for the optical design. Representative mathematical models of freeform surface include: Zernike polynomials (Eq. (1)),  $XY$  polynomial (Eq. (2)), Gaussian polynomial (Eq. (3)) [3].

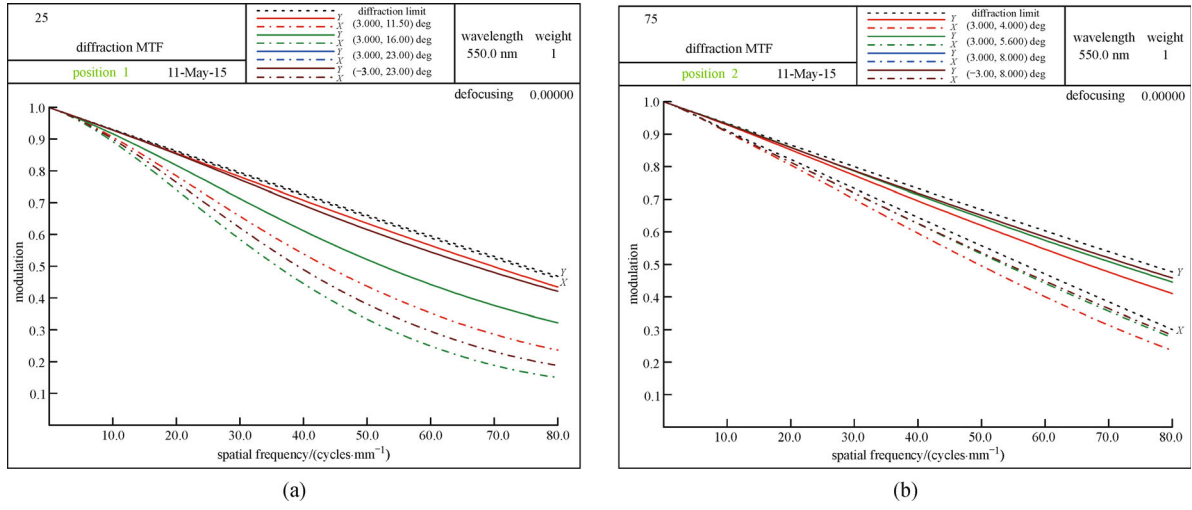


Fig. 4 MTF after the change of three anti-zoom optical system. (a)  $f = 25$  mm; (b)  $f = 75$  mm

$$z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} + \sum_{i=1}^8 \alpha_i r^{2i} + \sum_{i=1}^N A_i Z_i(\rho, \varphi), \quad (1)$$

$$z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} + \sum_{j=2}^{66} c_j x^m y^n, \quad (2)$$

$$z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1+k)c^2(x^2 + y^2)}} + \sum_i \phi_i(x, y) \omega_i. \quad (3)$$

In this paper, we used  $XY$  polynomial to describe the freeform surface, which was introduced to the tertiary

mirror. We completed the design of off-axis three-mirror reflective zoom system based on the freeform surface in the FOV of  $4^\circ \times 5^\circ$  to  $4^\circ \times 15^\circ$ ,  $6^\circ \times 8^\circ$  to  $6^\circ \times 23^\circ$ , and  $8^\circ \times 10^\circ$  to  $8^\circ \times 28^\circ$ , respectively. The MTF graphs of the system in these specific FOV at both focal lengths are shown in Figs. 5–7. The resolution was 80 lp/mm.

As is shown in Figs. 5–7, the imaging performance in the freeform off-axis three-mirror reflective zoom system is much better than that in the traditional off-axis three-mirror reflective zoom system, especially for the large field of view. This comparison shows the better off-axis aberration correction capability of freeform surfaces. The system structural parameters in the FOV of  $6^\circ \times 8^\circ$  to  $6^\circ \times 23^\circ$  are listed in Table 3. The  $XY$  polynomial describing the tertiary mirror with a freeform surface consists 20 terms, and their parameters are shown in Table 4.

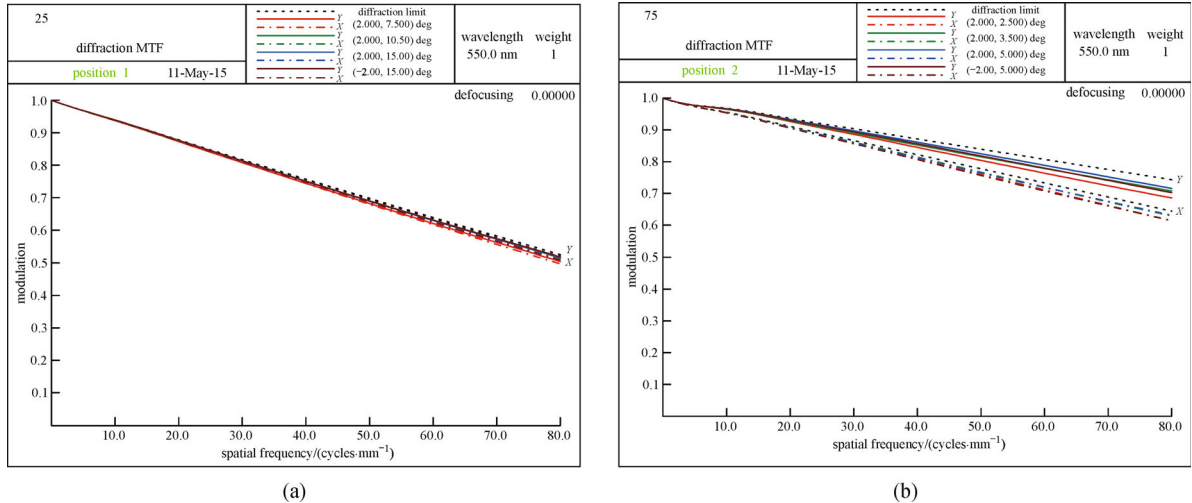


Fig. 5 MTF of FOV =  $4^\circ \times 5^\circ - 4^\circ \times 15^\circ$ . (a)  $f = 25$  mm; (b)  $f = 75$  mm

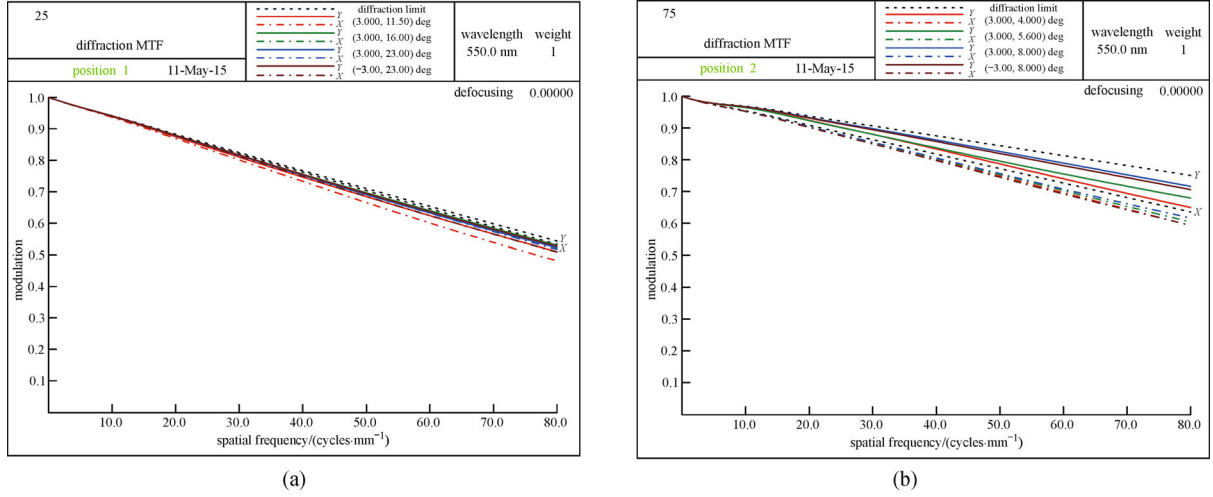


Fig. 6 MTF of FOV =  $6^\circ \times 8^\circ - 6^\circ \times 23^\circ$ . (a)  $f = 25$  mm; (b)  $f = 75$  mm

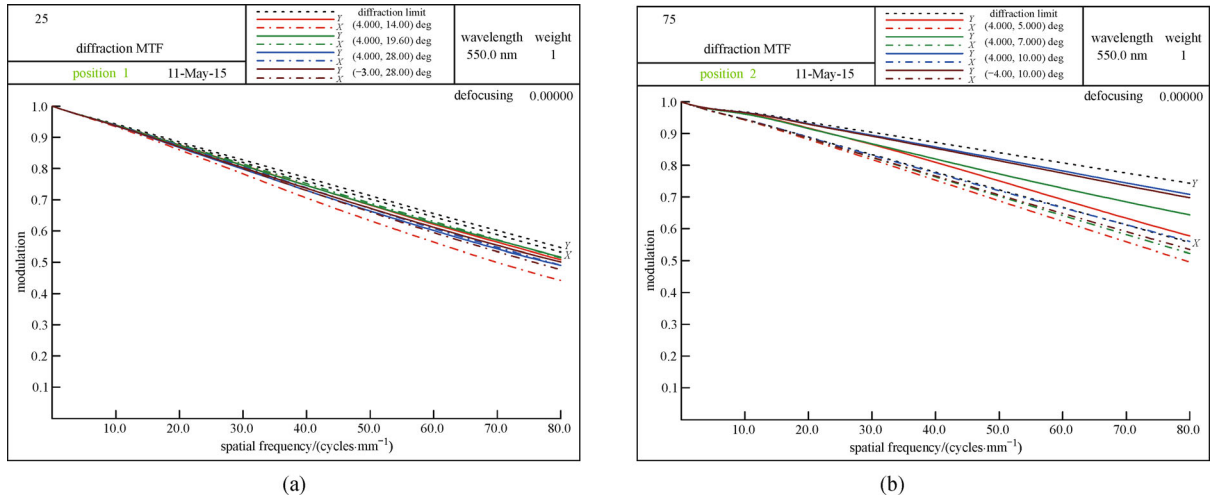


Fig. 7 MTF of FOV =  $8^\circ \times 10^\circ - 8^\circ \times 28^\circ$ . (a)  $f = 25$  mm; (b)  $f = 75$  mm

Table 3 Optical system structural parameters

mirror	radius/mm	separation/mm		conic
		$f = 25$	$f = 75$	
primary mirror	40.6620	- 27.0010	- 10.0121	06100
secondary mirror	53.1010	48.5302	60.7743	0.1022
tertiary mirror	- 341.9011	- 10.1226	- 38.5029	4.9509

## 4 Conclusions

In this paper, firstly, we carried out the design of the traditional off-axis three-mirror reflective zoom system based on the tertiary and high-order aspheric surface. It can be seen that this surface shape had a positive effect on balancing off-axis aberrations. But for the larger field of

view, it can hardly meet the requirements at the longer focal length (75 mm). Thus we proposed a method of introducing the freeform surface to the tertiary mirror to optimize the system. Secondly, we applied this system in three different fields of view,  $4^\circ \times 5^\circ$  to  $4^\circ \times 15^\circ$ ,  $6^\circ \times 8^\circ$  to  $6^\circ \times 23^\circ$ , and  $8^\circ \times 10^\circ$  to  $8^\circ \times 28^\circ$  at both focal lengths of 25 and 75 mm. Finally, we gave the results of the image

**Table 4**  $XY$  polynomial coefficients

polynomial	coefficients	polynomial	coefficients
$X$	-0021	$X^3Y$	-6.7310E-009
$Y$	0.0372	$X^2Y^2$	-5.3161E-007
$X^2$	-0.0034	$XY^3$	5.1208E-007
$XY$	0.0002	$Y^4$	1.8509E-006
$Y^2$	-0.0041	$X^5$	-1.5608E-010
$X^3$	3.6674E-008	$X^4Y$	-2.8325E-009
$X^2Y$	8.2996E-006	$X^3Y^2$	-1.0654E-010
$XY^2$	-1.5412E-005	$X^2Y^3$	2.8539E-008
$Y^3$	5.2632E-005	$XY^4$	-6.3283E-009
$X^4$	2.8398E-007	$Y^5$	3.2166E-008

quality evaluation and the freeform surface structural parameters in the FOV of  $6^\circ \times 8^\circ$  to  $6^\circ \times 23^\circ$ . We could see the potential that the off-axis three-mirror reflective zoom system based on the freeform surface can meet the requirements for superior image quality in the large field of view and in the wide imaging spectrum.

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**Guijuan Xie** obtained her bachelor's degree in optical information science and technology from Anhui University in 2013, and she has been a M.S. candidate at School of Optoelectronics, Beijing Institute of Technology since 2013. Her current research is optical design.



**Jun Chang** is a Professor and Ph.D. Supervisor of School of Optoelectronics at Beijing Institute of Technology. He received his Ph.D. degree in optics from Changchun University of Science and Technology in 2002. He has 13 years experience in reflective zoom optical system, reflective zoom optical system based on active optics, the high precision detection of the offset state of micro system assembly, the design of the universal compensator for testing aspheric surfaces and short baseline distance and long detecting distance measurement with binocular vision based on OpenCV. He is the author of more than 60 journal papers. His current interests include optical design and measurement, and electro-optical countermeasures.



**Ke Zhang** obtained her bachelor's degree from Department of Physics, Changchun University of Science and Technology in 2006, and M.S. degree in 2009. Since 2013, she has been a Ph.D. candidate at School of Optoelectronics, Beijing Institute of Technology. Meanwhile, she is an engineer at China North Vehicle Research Institute, mainly responsible for optical structure design of aiming system and photoelectric countermeasure system. Her current research is optical design.



**Jide Zhou** obtained his bachelor's degree in optical information science and technology from Huadong East China Jiaotong University in 2013, and he has been a M.S. candidate at School of Optoelectronics, Beijing Institute of Technology since 2013. His current research is optical design.



**Yajun Niu** obtained his bachelor's degree in optoelectronic information engineering from Beijing Institute of Technology in 2012, his M.S. degree in optical engineering from Beijing Institute of Technology in 2014. Since 2014, he has been a Ph.D. candidate in optical engineering at Beijing Institute of Technology, where his research interests include optical design, testing, and super-resolution imaging.