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RESEARCH ARTICLE

Position dependent circuit model for thin avalanche photodiodes

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Abstract This paper presents a circuit model for thin avalanche photodiodes (APDs). In this model, the nonuniformity of the electric filed in the multiplication region is modeled using a stepwise method. The model also tries to take the effects of carrier's position dependent properties, like carrier's dead length and the history of carrier's previous ionization into account by developing an effective electric field in the multiplication region. The output photocurrent and multiplication gain obtained from the proposed model for different lengths of the multiplication region achieve a good agreement in comparison with available experimental data. In addition, calculated excess noise factor reveals the model ability for noise and sensitivity analysis.

Keywords avalanche photodiode (APD), circuit modeling, multiplication gain, nonuniform electric field, excess noise factor

1 Introduction

Avalanche photodiodes (APDs) have wide application in optical communication systems due to their built-in gain, and it can yield high sensitivity for the receive side [1]. Many efforts have been made for circuit model of APDs, since such circuit model can simplify design of a whole transceiver in a single environment [2]. Either rate-equations [3], or current continuity equations [4], can be considered as the basis for APD's circuit model. However, APD's circuit model by these two equations usually ignore position dependent factors, such as nonuniformity of electric filed, carrier dead length and previous ionization history, and this makes inaccurate for modeling thin APDs, which attract many attentions due to their superior noise performance. There were also some efforts to take the effect of dead length into account [5] and nonuniform electric profile as well as carrier dead length [6]. However, both dismiss field-dependent carrier's previous ionization history. In this paper, we introduced a circuit model based on position dependent multiplication gain calculation, in which carrier's dead length and the history of carrier's previous ionization were included by an effective electric field for electrons and holes.

2 Theory

A schematic diagram of a typical p-i-n APD (PIN-APD) photodetector is shown in Fig. 1. Light is incident on the n-side of the junction. A nonuniform electric field profile resulting from a reverse bias voltage and its stepwise approximation are also illustrated in Fig. 1. According to one-dimensional Poisson's equation and using depletion approximation, the electric field profile in the lightly doped i-region can be written as

$$F(x) = qN_\mathrm{p}x_\mathrm{p}/\varepsilon_0\varepsilon_\mathrm{s} - qN_\mathrm{i}x/\varepsilon_0\varepsilon_\mathrm{s}, \qquad (1)$$

where q is the electron charge, ε_0 and ε_s are the vacuum permittivity and the semiconductor relative permittivity respectively, N_p and N_i are the doping concentration in the p- and i-region respectively, and x_p is the depletion region width in the p-region obtained as given in Ref. [6].

The position-dependent gain defined as the ratio of the number of final electron-hole pairs to that created initially in the i-region, is given by [7]

$$M = \frac{\exp\left(-\int_0^W \left(\alpha(x)-\beta(x)\right)\mathrm{d}x\right)}{1-\int_0^W \alpha(x)\exp\left(-\int_x^W \left(\alpha(x')-\beta(x')\right)\mathrm{d}x'\right)\mathrm{d}x}, \qquad (2)$$

where W represents the i-region thickness, $\alpha(x)$ and $\beta(x)$ are the electron and hole ionization coefficients respectively [8]:

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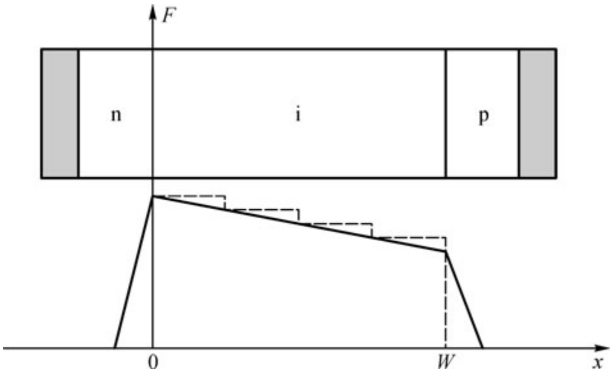

Fig. 1 Schematic diagram with electric field profile of a p-i-n avalanche photodiode (PIN-APD)

$$\alpha(x) = a_{\mathrm{n}}\exp\left(-b_{\mathrm{n}}/F\right)^{c_{\mathrm{n}}}, \tag{3}$$

$$\beta(x) = a_{\mathrm{p}}\exp\left(-b_{\mathrm{p}}/F\right)^{c_{\mathrm{p}}}, \tag{4}$$

where F ($\mathrm{V \cdot cm^{-1}}$) is the position-dependent electric field, a_{n} ($\mathrm{cm^{-1}}$) and a_{p} ($\mathrm{cm^{-1}}$), b_{n} ($\mathrm{V \cdot cm^{-1}}$) and b_{p} ($\mathrm{V \cdot cm^{-1}}$), c_{n} and c_{p} are all material constants.

To begin the circuit modeling procedure, we define $N(x)$ as a position dependent integration term and then approximate it using trapezoidal rule:

$$\begin{aligned} N(x) &= \int_x^W [\alpha(x') - \beta(x')]\mathrm{d}x' \\ &\approx \frac{W-x}{2m}\Bigg\{[\alpha(x) - \beta(x)] \\ &\quad + 2\sum_{k=1}^{m-1}\left(\alpha\left(k\frac{W-x}{m}\right) - \beta\left(k\frac{W-x}{m}\right)\right) \\ &\quad + [\alpha(W) - \beta(W)]\Bigg\}, \end{aligned} \tag{5}$$

where, it is assumed that the i-region from x to W is subdivided into m equal segments. As shown in Fig. 2(a), each $\alpha - \beta$ term in Eq. (5) can be modeled with a current source [e.g., $I(x) = \alpha(x) - \beta(x)$], all passing through a resistor R_{N} with a value of $(W-x)/2m$, where $N(x)$ appears as a node voltage. Observing Eq. (2), for calculating M, we need the exponential of $N(x)$, where D_{N} in Fig. 2(a) carries out the exponential function as below:

$$I_{\mathrm{N}}(x) = I_{\mathrm{S}}\exp[V_{\mathrm{N}}(x)/nV_{\mathrm{T}}] = \exp[-N(x)], \tag{6}$$

where I_0 is given a value of 1 A and the effect of nV_{T} is removed by modifying the value of R_{N} to

$$R_{\mathrm{N}} = nV_{\mathrm{T}}(W-x)/2m.$$

Based on the explained method, the integration term in the denominator of Eq. (2) can be approximated as

$$\begin{aligned} V_{\mathrm{D}} &= \int_0^W \alpha(x)\exp\left(\int_x^W \left(\alpha(x') - \beta(x')\right)\mathrm{d}x'\right)\mathrm{d}x \\ &= \int_0^W \alpha(x)\exp[N(x)]\mathrm{d}x \\ &\approx \frac{W}{2m'}\Bigg\{\alpha(0)\exp[N(0)] \\ &\quad + 2\sum_{k'=1}^{m'-1}\alpha(k'W/m')\exp[N(k'W/m')] \\ &\quad + \alpha(W)\exp[N(W)]\Bigg\}, \end{aligned} \tag{7}$$

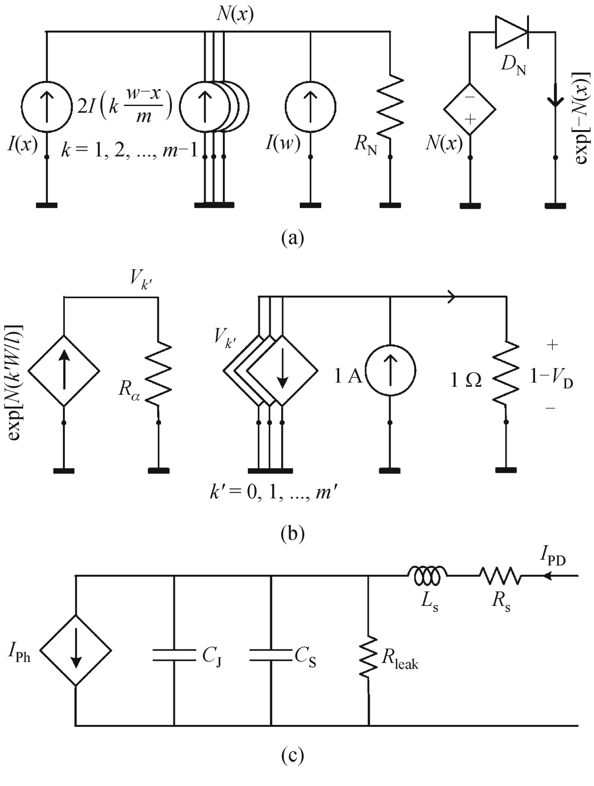

Fig. 2 Equivalent circuit model for avalanche photodiodes based on multiplication gain calculation. (a) Calculating $N(x)$ and $\exp[-N(x)]$; (b) calculating V_{D}; (c) output terminal of the device

where, it is assumed that the i-region thickness is subdivided to m' equal segments where each segment again divided to m segment (required to calculate $N(x)$ in each segment). As shown in Fig. 2(b), each $\exp[N(x)]$ term can be realized as a current using a circuit like that given in Fig. 2, requiring $m' + 1$ circuit totally. Meanwhile, a multiplication factor of $\alpha(x)$ is needed for each term, which defining $x = k'W/m'$ can be realized by $R_\alpha = W\alpha(x)/2m'$ for $k' = 0$ and m', and twice of that for $k' = 1$ to $m' - 1$.

Now, according to Eq. (2), we have

$$M = \exp[-N(0)]/(1-V_{\mathrm{D}}).$$

Once M is known, the multiplied photocurrent response can be obtained as (Ref. [9])

$$I_{\mathrm{Ph}} = q\eta\frac{P_{\mathrm{opt}}}{h\nu}M, \tag{8}$$

where P_{opt} is the incident optical power and η is the quantum efficiency defined as (Ref. [9])

$$\eta = \eta_{\mathrm{i}}(1-R)\mathrm{e}^{-\alpha_{\mathrm{p}}d}\left(1-\frac{\mathrm{e}^{-\alpha_{\mathrm{i}}W}}{1+\alpha_{\mathrm{i}}L_{\mathrm{p}}}\right), \tag{9}$$

where η_{i} is the internal quantum efficiency, R is the facet reflectivity of the p-region, α_{p} and α_{i} are the absorption coefficient of the p- and i-region, respectively, d is the width of p-region and L_{p} is the diffusion length for holes. Figure 2(c) resembles the output terminal of the device. In this figure,

$$I_{\mathrm{PD}} = I_{\mathrm{Ph}} + (C_{\mathrm{J}} + C_{\mathrm{S}})\frac{\mathrm{d}V_{\mathrm{A}}}{\mathrm{d}t} + \frac{V_{\mathrm{A}}}{R_{\mathrm{leak}}}, \tag{10}$$

where I_{PD} is the total output current of the device, V_{A} is the bias voltage over the junction, R_{leak} is the parallel parasitic resistance of the APD, and C_{S} and $C_{\mathrm{J}} = \varepsilon_0\varepsilon_{\mathrm{s}}A/(x_{\mathrm{n}} + x_{\mathrm{p}} + W)$ are the parasitic and junction capacitances.

3 Results and discussion

To reach to a more accurate circuit model for thin APDs, we should consider the carrier's dead length as well as the history of carrier's previous ionization, both depend on the electric field. These effects can be taken into account by defining an effective electric field for electrons and holes [7,10] as

$$F_{\mathrm{eff,e}}(x) = \int_x^{x'=x-l_{\mathrm{e}}} \mathrm{d}x'' F(x'')f_{\mathrm{ce}}(x''), \tag{11}$$

$$F_{\mathrm{eff,h}}(x) = \int_x^{x'=x+l_{\mathrm{h}}} \mathrm{d}x'' F(x'')f_{\mathrm{ch}}(x''), \tag{12}$$

where l_{e} and l_{h} are electron's and hole's dead length respectively, and $f_{\mathrm{ce}}(x)$ and $f_{\mathrm{ch}}(x)$ are correlation functions defined as (Refs. [7,10])

$$\begin{aligned} f_{\mathrm{ce,h}}(x) &= \frac{2}{\sqrt{\pi}\lambda_{\mathrm{e,h}}}\exp\left(-\frac{(x'-x)^2}{\lambda_{\mathrm{e,h}}^2}\right) \\ &\approx \frac{2}{\sqrt{\pi}\lambda_{\mathrm{e,h}}}\exp\left(-\frac{l_{\mathrm{e,h}}^2}{\lambda_{\mathrm{e,h}}^2}\right). \end{aligned} \tag{13}$$

In these equations, λ_{e} and λ_{h} are correlation length for electrons and holes respectively obtained as (Ref. [7])

$$\lambda_{\mathrm{e,h}} \approx l_{\mathrm{e,h}} = V_{\mathrm{de,h}}/F(x), \tag{14}$$

where V_{de} and V_{dh} are voltage drops across the electron's and hole's dead length, respectively.

To verify the model capability for predicting M, a homojunction GaAs PIN-APD has been simulated for different i-region thickness and the results have been compared with experimental data presented in Ref. [7]. In this model, universal width-independent parameters given in Ref. [8] are utilized for ionization coefficients. The parameters used in simulation are tabulated in Table 1. The results are shown in Fig. 3. This comparison shows that our model resembles the experimental data in an acceptable manner especially the value of breakdown voltage. However, near breakdown the model underestimates the gain. This is mainly due to ignorance of field in cladding layers, and possibly the problem of inaccurate estimation of field in GaAs devices.

To justify the model accuracy more, as a second example, an InAlAs-APD with an area of about 31415 μm^2 is modeled and simulated. As shown in Fig. 4, the results for photocurrent are in excellent agreement with the

Table 1 Parameters used in simulations

symbol	value		unit
	GaAs	InAlAs	
A	31415	31415	μm^2
α_{n}	6.01×10^6	4.17×10^6	cm^{-1}
b_{n}	2.39×10^6	2.09×10^6	V·cm^{-1}
c_{n}	0.9×10^6	1.2×10^6	
α_{p}	3.59×10^6	2.65×10^6	cm^{-1}
b_{p}	2.26×10^6	2.79×10^6	V·cm^{-1}
c_{p}	0.92×10^6	1.07×10^6	
N_{i}	1×10^{16}	1×10^{16}	cm^{-3}
N_{n}	5×10^{18}	2×10^{18}	cm^{-3}
N_{p}	5×10^{18}	2×10^{18}	cm^{-3}
W_{i}	100–800	100–799	nm
W_{n}	1000	200	nm
W_{p}	1000	300	nm
R_{s}	5	20	Ω
L_{s}	0.3	0.3	nH
R_{leak}	10^{10}	10^{10}	Ω
C_{s}	0.2	0.25	pF

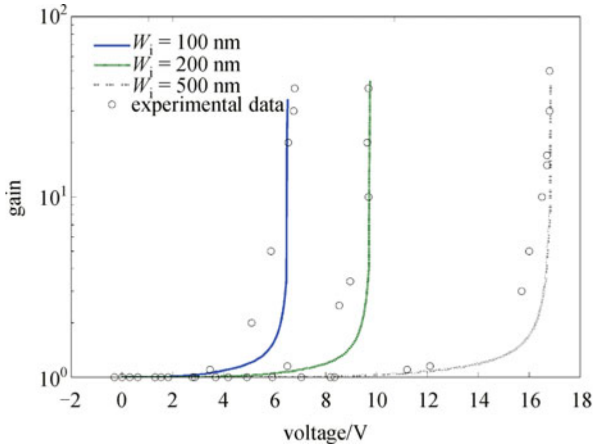

Fig. 3 Multiplication gain as function of reverse bias voltage for homojunction GaAs APDs with i-region of W_i = 100, 200 and 500 nm (solid-, dashed-, and doted-lines) compared to experimental data reported in Ref. [7]

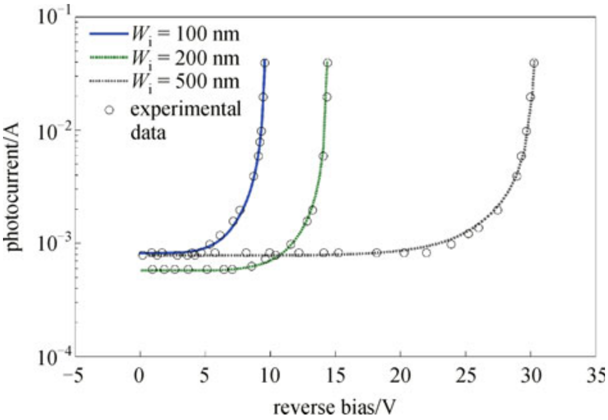

Fig. 4 Photocurrent as function of reverse bias voltage for homojunction InAlAs APDs with i-region of W_i = 100, 200 and 500 nm (solid-, dashed-, and doted-lines) compared to experimental data presented in Ref. [11]

experimental data reported in Ref. [11], as the model provides almost accurate estimation for current levels and breakdown voltages. This result indicates that accounting for carrier's dead length and previous ionization history is a key factor in circuit modeling of thin APDs.

Figure 5 represents excess noise factor versus gain for a GaAs APDs with different thickness of i-region based on the following relation assuming only electron injection at $x = W$ [7],

$$F(M) = kM + (1-k)\left(2-\frac{1}{M}\right). \tag{15}$$

Simulation results carried out for W_i = 100, 200, 500 and 800 nm reveal an acceptable agreement with experimental data afforded by Ref. [8]. In Fig. 5, however, for most of widths of i-region, the noise has been lightly underestimated that can be referred to underestimation of gain in these GaAs devices. To certify the model ability for estimation of excess noise factor, it is tested with another InAlAs APD, for W_i = 190, 363 and 799 nm. As exhibited in Fig. 6, results of simulated excess noise factor are close to the empirical data [8] for low gain and high gain situations. Without developing an effective electric field in the multiplication region, the predicted excess noise factor will be overestimated. In fact, considering the dead length,

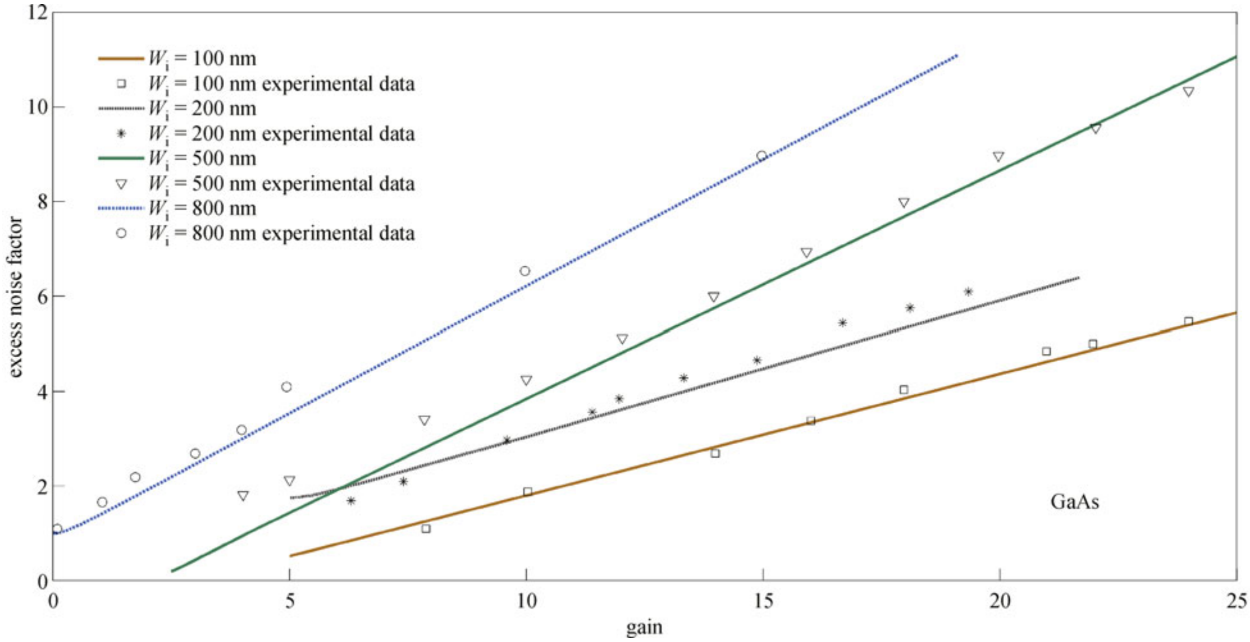

Fig. 5 Comparison of simulation and experimental results of excess noise factor of a GaAs APDs [6] for different thickness of i-region; W_i = 100, 200, 500 and 800 nm

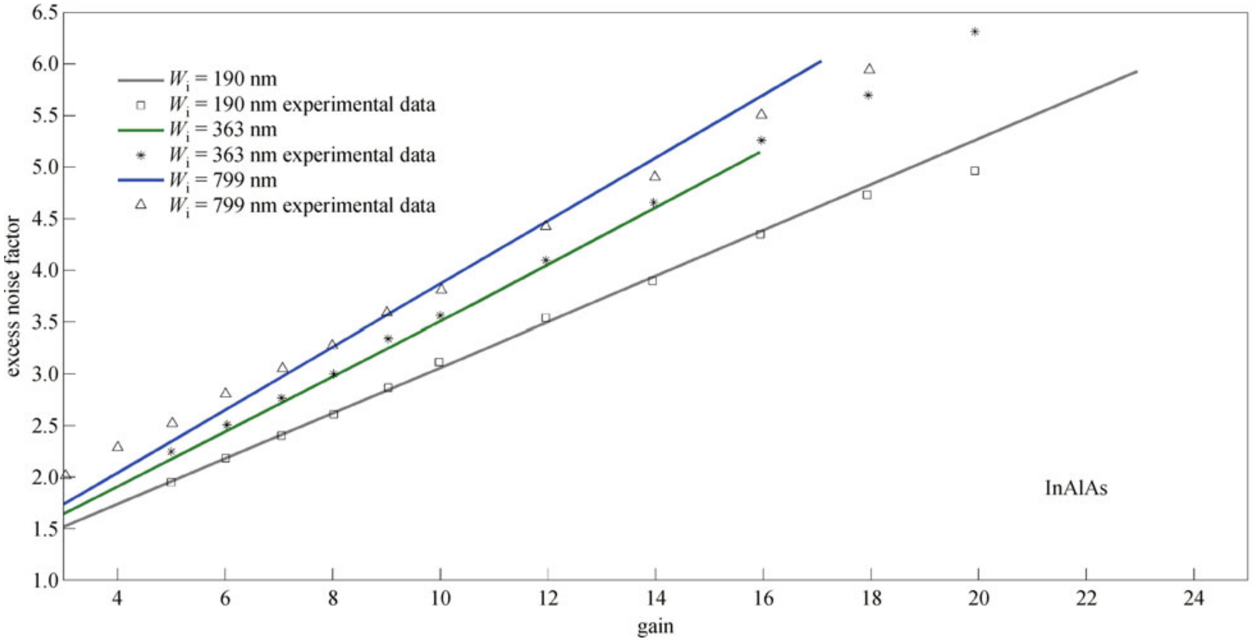

Fig. 6 Comparison of simulation and experimental results of excess noise factor of an InAlAs APDs [8] for different thickness of i-region; W_i = 190, 363 and 799 nm

in which carriers gain sufficient energy to have a non-negligible ionization probability helps the model to predict the multiplication gain and resulting excess noise factor accurately.

4 Conclusions

This paper has proposed a new and simple circuit model for thin APDs suitable for performance evaluation of optical systems. The model can be used for excess noise factor calculation. The main contribution of this work is in consideration of nonuniform electric field profile, carrier's dead length and previous ionization history, simultaneously. The model can be simply extended to other types of photodetectors like separate absorption, grading, charge, and multiplication APD (SAGCM-APD). Simulation indicates that accounting for nonlocal effects using a modified electric field results in a better estimation of the APD characteristics as the model provides excellent agreement with published experimental data.

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