

A new construction method of LDPC codes for optical transmission systems

Jianguo YUAN (✉), Yuexing JIA, Wenjuan BI, Liang XU

Key Laboratory of Optical Fiber Communications Technology, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

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Abstract Based on the construction method of systematically constructed Gallager (SCG)(4, k) code, a new improved construction method of low density parity check (LDPC) code is proposed. Compared with the construction method of SCG(4, k) codes improved before, the proposed construction method has some advantages, such as saving storage space and reducing computation complexity in the hardware implementation. And then LDPC(5929, 5624) code with 5.42% redundancy is constructed by the proposed method. The simulation results and analysis show that the constructed LDPC(5929, 5624) code has better error-correction performance, lower redundancy and lower decoding complexity than those of a classic Reed-Solomon (RS)(255, 239) code. Therefore, LDPC(5929, 5624) code, constructed by the proposed construction method on LDPC codes, can better suitable for optical transmission systems.

Keywords low density parity check (LDPC) codes, net coding gain (NCG), construction method, optical transmission systems

1 Introduction

With the increasing development of optical transmission systems toward longer distance, larger capacity and higher bit rate, forward error correction (FEC) becomes the first choice to improve the systems performance because the FEC can gain a many larger transmission distance and make the system more robust under worse conditions [1–5]. Therefore, the FEC technique has been used to compensate for the optical transmission quality degradation from noise and pulse distortion in optical transmission systems. The low density parity check (LDPC) code proposed by Gallager in 1962 is a kind of the linear block code based on sparse parity-check

matrices [6]. The irregular LDPC code presented by Luby in 1977 turns out to be the most approximate to the Shannon limit [7,8], the performance of which is not only better than that of the regular ones, but also superior than that of Turbo codes.

As a channel with high signal noise ratio (SNR), optical fiber channel generally requires both the output error probability and redundancy to be quite low [2,9,10]. Owing to that, in this paper by analyzing the optical transmission systems as well as carrying out research on the construction algorithm of LDPC codes, a novel construction method of the regular LDPC codes and a LDPC(5929, 5624) code with low redundancy are proposed here. In addition, the simulation and the comparison analysis of the performance are also performed.

2 Construction methods of LDPC codes

The construction of parity-check matrices of LDPC codes cannot only determine their encode, but also play a crucial role in their decode. In particular, the various structures of LDPC codes result in sharp diversity on their implementation performance and their encoding/decoding complexity. Therefore, one of the research hot spots in recent years [2,7–10] has been concentrated on how to construct LDPC code types with excellent performance and low encoding/decoding complexity.

2.1 Construction principles of LDPC code

The main construction principles [2–5,10] of LDPC code types for optical transmission systems can be acquired as follows:

- 1) There ought to be low error floor or no error floor at best;
- 2) The net coding gain (NCG) should be high and the redundancy of the code type should be low;
- 3) The codeword length cannot be too long, the time

delay arisen from encoding/decoding should not be too much, and the software/hardware implementation should be favorable, and so on.

In the meantime, in consideration of the analysis of the relative theories of high-bit-rate LDPC codes and the special characteristics of regular LDPC codes themselves, this paper also obey another two construction principles [2,10] as follows:

1) The LDPC codes constructed should have no four-cycle to suffice the requirements of Steiner limit, making the decoding of the LPDC code go with better decoding constringency;

2) The LDPC codes constructed should have lower density, that is, in the parity-check matrix of LDPC codes the number of ones should be absolutely less than that of zeros. In this way, there will be less calculation each time when iteratively decoding and the decoding complexity will be reduced.

2.2 Randomly construction method

There are a lot of random construction schemes, which are exemplified by several classical ones, including the method proposed by Gallager and Mackay [8,11]. The LDPC code constructed by Gallager [11] has the following characteristics: in the parity-check matrix \mathbf{H} s of LDPC codes the numbers of ones of rows and columns (i, j) are all fixed and j is not less than three; the elements in the \mathbf{H} are randomly generated, together with all the numbers of ones in rows and columns evenly distributed; the parity-check matrix is vertically divided into j horizontal submatrices of the same size, each of which has its per column only containing a 1, and for the first submatrix ones in its i th row are only distributed in columns from $(i-1)k + 1$ to ik , and the rest of the submatrices are just randomly transformation from the first one; the class of this code type is a set randomly ranged with equal probability by the bottom $j-1$ columns of the matrix \mathbf{H} . Gallager ultimately reached the following conclusion: on the condition of the certain column heavy j , considerably low SNR and rather long code length, error probability of the optimum decoder decreases with code length rising. However, the defect in Gallager's method is the possible appearance of four-cycle in the parity-check matrix of the LDPC code thus leading to descending of decoding performance.

As the parity-check matrix constructed via the construction method proposed by Gallager are generated randomly, owing to which the simple encoding as well as convenient decoding are not available to our application environment, especially in magnetic recording and optical communications, and other high-speed applications, this algorithm is unsuitable for the optical fiber communication system.

2.3 Systematically construction method

Gallager constructed LDPC codes randomly while Hösli

and Erik used the construction methods of random LPDC codes presented by Gallager for reference, and proposed a method [12] to construct a kind of regular LPDC codes, which were called SCG(j, k) (systematically constructed Gallager (j, k) codes), where $j \in \{2,3,4\}$ stands for the number of submatrices of the parity-check matrix and the column weight, and k is the fixed row weight of the parity-check matrix. SCG(j, k) codes can be determined uniquely by j and k , and for each SCG(j, k) code, the row number M and the column number N of the parity-check can be denoted by the following formula:

$$M = jk, \quad N = k^2.$$

The minimum distance with no four-cycle [12] can be given by SCG(j, k) codes (when $j = 2,3$, k can be either an odd or an even, and when $j = 4$, k can only be an odd), this benefits for the research and analysis as well as the implementation of the hardware [12]. Each column in the j submatrices of the parity-check matrix of SCG(j, k) codes contains only one 1. For the first submatrix of SCG($4, k$) code ones in its i th row are only distributed in columns from $(i-1)k + 1$ to ik , the second is the matrix is made up of a series of unit matrix and the first square matrix in the third one is a unit matrix with the back square matrix is a square which transforms from the front square with its column having one cyclic-shift, the first square matrix of the fourth is the mirror-image flip of the first square in the third in vertical center axis, similarly, the rest of the square is the mirror-image flip of the corresponding square in the third in same axis. SCG($2, k$) code consists of two front submatrix taken from SCG($4, k$) code while SCG($3, k$) code is made up of the first three submatrix taken from SCG($4, k$) code. Figure 1 shows an example of SCG($4, 5$).

3 Novel construction approach of LDPC codes

From the property [12] of SCG(j, k) code it can be seen that with the increment of the grouping length N the sparseness of the parity-check matrix of SCG(j, k) code will augment, which is convenient for reducing the decoding complexity in the case of long codeword length. For SCG($4, k$) code in which k is an odd and $k > 4$, the characteristics of the minimum distance is quite good, and it is easy to implement. Hence, SCG($4, k$) code is more preferred in practical applications. Based on this, this paper on the foundation of systematically construction method raised by Hösli and Erik proposes a novel construction algorithm, which can expressed as follows: the fourth submatrix of SCG($4, k$) code is a new form of the third in its reverse order. Then we will describe the scheme in detail.

As described in Section 2.3, SCG($4, k$) is constructed on the basis of SCG($3, k$), that is, the fourth submatrix of SCG($4, k$) is transformed from the third one. Different submatrix is constructed in various transform methods,

which is also directly determining the hardware store space and computational complexity in the future hardware application. Here, in this article we take the first three submatrix of the original SCG(4, k) code as those of the newly constructed parity-check matrix, and mark squares of the third submatrix from left to right in a order of standard serial number for: $0, 1, 2, \dots, k-3, k-2, k-1$. And the fourth submatrix is the reverse transformation of the third, namely the squares in the fourth submatrix is ranged in a order from left to right for the serial number: $k-1, k-2, k-3, \dots, 2, 1, 0$. Furthermore, label discrepancies between the third submatrix and the fourth: $-(k-1), -(k-3), -(k-5), \dots, -2, 0, 2, \dots, k-5, k-3, k-1$, which may be: $1, \dots, 2, 0, 2, \dots, k-3, k-1$ after the mold k . It's clearly seen that all the numbers on the left side of the symmetry axis that zero is taken as are odd when k ($k > 4$) is odd, while all the numbers on the right are even after the mold k . Take the SCG(4, 5) as an example. Squares in the third parity-check submatrix of the SCG(4, 5) code are arranged in a order from left to right: $0, 1, 2, 3, 4$, then that of the fourth submatrix is: $4, 3, 2, 1, 0$. The label difference between the squares in the third submatrix and that in the fourth one are: $-4, -2, 0, 2, 4$, take the value after the mold 5 for: $1, 3, 0, 2, 4$, obviously their value is different. Figure 2 depicts an example of SCG(4, 5) structured in the novel construction method.

From Fig. 2, we can intuitively find no four-cycle. In addition, the related Matlab programming codes can also better verify the conclusion that there indeed exist no four-cycle in the parity-check matrix (especially applicable for k at a larger odd number.

4 Simulation results and performance analysis of novel LDPC code type

Since the encoding/decoding implementation complexity of the LDPC code on Galois Fields (GF)(2) is much simpler than that of employing the iterative concatenated code and Block Turbo Code (BTC) investigated, we choose GF(2), binary phase shift keying (BPSK) modulation and additive white Gaussian noise channel (AWGN) as the simulation environment where the Matlab programming codes of the parity-check matrix of SCG(4, k) code are also composed according to our novel construction approach. Meanwhile, in consideration of the demand of low redundancy in optical transmission system, the construction of LDPC codes, when k equals 77, is emphasized. From the simulation results, it's clear that the LDPC code possess rank of 305, which not belongs a full one according to the property of SCG(4, k). That is to say, while there are three rows redundant in its parity-check matrix, the LDPC code has the information bit length of 5624 instead of 5621. Similarly, the corresponding LDPC code ought to be LDPC(5929, 5624) rather than LDPC(5929, 5621). Furthermore, the parity-check matrix of the LDPC code constructed using this algorithm introduces no four-cycle, which also matches the construction principles of LDPC code types for optical transmission systems.

Through simulation of the LDPC(5929, 5624) in the AWGN channel, we can get the relevant curve of bit error rate (BER) and bit SNR (E_b/N_0). Figure 3 shows the simulation results.

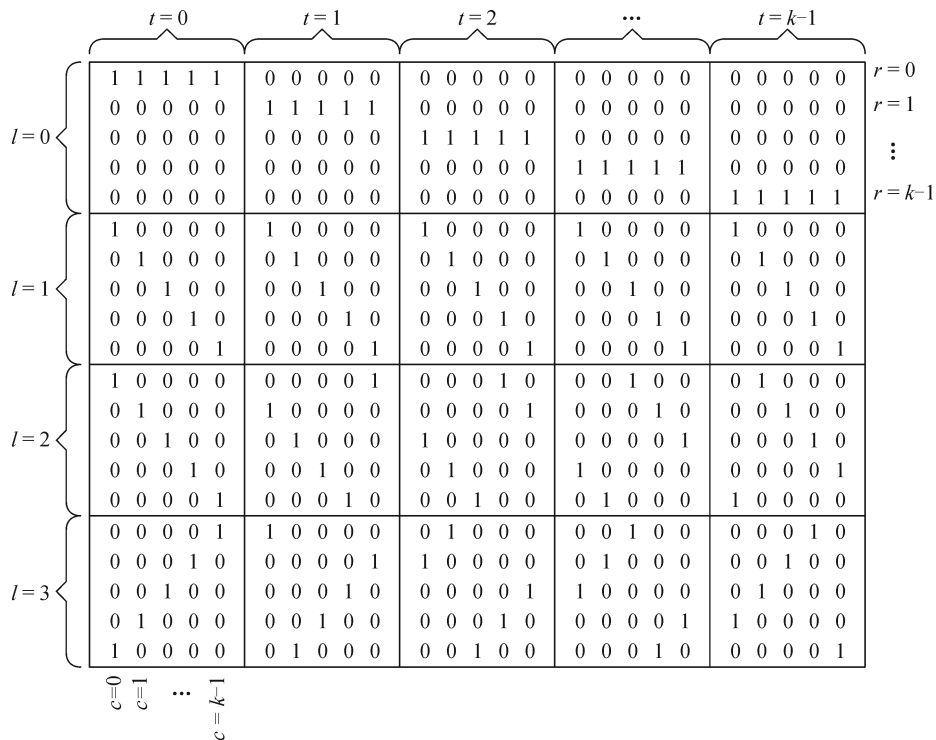


Fig. 1 Check matrix of SCG(4, 5) code constructed by Hösli and Erik

	$t=0$	$t=1$	$t=2$	\dots	$t=k-1$	
$l=0$	1 1 1 1 1	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	$r=0$
	0 0 0 0 0	1 1 1 1 1	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	$r=1$
	0 0 0 0 0	0 0 0 0 0	1 1 1 1 1	0 0 0 0 0	0 0 0 0 0	\vdots
	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	1 1 1 1 1	0 0 0 0 0	
	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	1 1 1 1 1	$r=k-1$
$l=1$	1 0 0 0 0	1 0 0 0 0	1 0 0 0 0	1 0 0 0 0	1 0 0 0 0	
	0 1 0 0 0	0 1 0 0 0	0 1 0 0 0	0 1 0 0 0	0 1 0 0 0	
	0 0 1 0 0	0 0 1 0 0	0 0 1 0 0	0 0 1 0 0	0 0 1 0 0	
	0 0 0 1 0	0 0 0 1 0	0 0 0 1 0	0 0 0 1 0	0 0 0 1 0	
	0 0 0 0 1	0 0 0 0 1	0 0 0 0 1	0 0 0 0 1	0 0 0 0 1	
$l=2$	1 0 0 0 0	0 0 0 0 1	0 0 0 1 0	0 0 1 0 0	0 1 0 0 0	
	0 1 0 0 0	1 0 0 0 0	0 0 0 0 1	0 0 0 1 0	0 0 1 0 0	
	0 0 1 0 0	0 1 0 0 0	1 0 0 0 0	0 0 0 0 1	0 0 0 1 0	
	0 0 0 1 0	0 0 1 0 0	0 1 0 0 0	1 0 0 0 0	0 0 0 0 1	
	0 0 0 0 1	0 0 0 1 0	0 0 1 0 0	0 1 0 0 0	1 0 0 0 0	
$l=3$	0 1 0 0 0	0 0 1 0 0	0 0 0 1 0	0 0 0 0 1	1 0 0 0 0	
	0 0 1 0 0	0 0 0 1 0	0 0 0 0 1	1 0 0 0 0	0 1 0 0 0	
	0 0 0 1 0	0 0 0 0 1	1 0 0 0 0	0 1 0 0 0	0 0 1 0 0	
	0 0 0 0 1	1 0 0 0 0	0 1 0 0 0	0 0 1 0 0	0 0 0 1 0	
	1 0 0 0 0	0 1 0 0 0	0 0 1 0 0	0 0 0 1 0	0 0 0 0 1	
	$c=0$	$c=1$	\dots	$c=k-1$		

Fig. 2 New check matrix of SCG(4, 5) code constructed in this paper

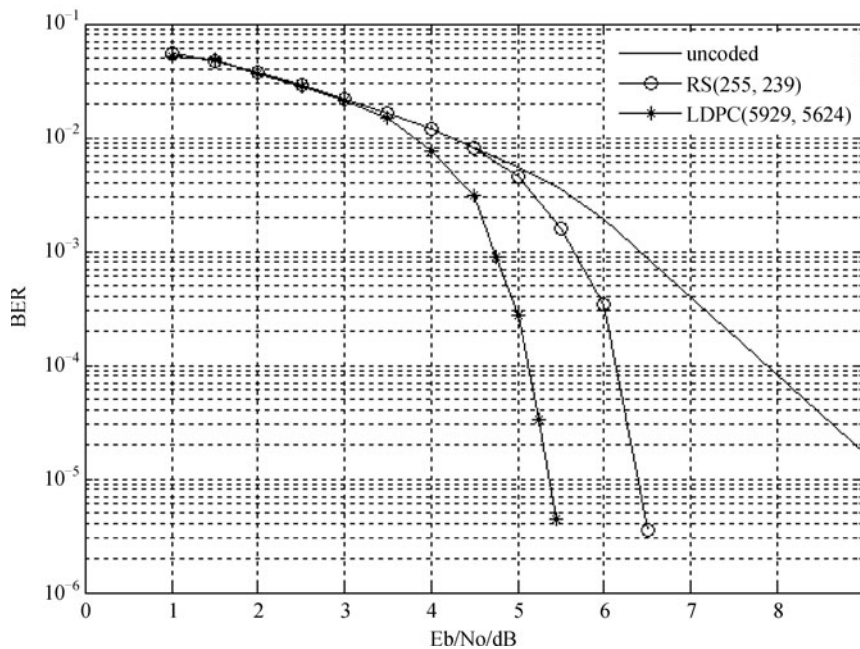


Fig. 3 Simulation result diagram of novel LDPC(5929, 5624) code

In Fig. 3, analysis shows that the error correction performance of new constructed LDPC(5929, 5624) is obviously superior to the RS(255, 239). The NCG of LDPC(5929, 5624) is about 5.04 dB at BER of 10^{-6} , which is 1.60 dB higher than that of classical RS(255, 239) code;

From the outer derivation to the curve in Fig. 3, there is an estimation that the NCG can achieve 7.18 dB at BER of 10^{-12} , 1.57 dB higher than that of RS(255, 239) code. Hence, the novel LDPC(5929, 5624) code with better error-correction performance and the low redundancy low

of only 5.42% can better suitable for optical transmission systems.

To get sufficient contrast analysis, here we make a thorough comparison involving the LDPC(5929, 5624), the RS(255, 239) in ITU-T G 975 [13] and the two code types in ITU-T G 975.1 [14] which are simultaneously listed in Table 1, where the NCG performance and redundancy at BER of 10^{-6} and 10^{-12} are respectively revealed.

Table 1 Comparison of error-correction performance between LDPC (5929, 5624) code and other code types

code type	redundancy /%	NCG at BER of 10^{-6} /dB	NCG at BER of 10^{-12} /dB
RS(255, 239)	6.69	3.44	5.61
BCH(3860, 3824) + BCH(2040, 1930)	6.69	4.70	7.98
RS(255, 239) + CSOC ($k_0/n_0 = 6/7, J = 8$)	24.48	4.81	7.95
LDPC(5929, 5624)	5.42	5.04	7.18

From Table 1, it can be seen that the redundancy of LDPC(5929, 5624) code is lower than those of RS(255, 239) code and BCH(3860, 3824) + BCH(2040, 1930) code. The NCG of LDPC(5929, 5624) is about 5.04 dB at BER of 10^{-6} , which is 1.60 dB higher than that of classical RS(255, 239) code and about 7.18 dB at BER of 10^{-12} , which is 1.57 dB higher than that of RS(255, 239) code. Therefore, the error correction performance of the novel LDPC code, compared with RS(255, 239) code and BCH(3860, 3824) + BCH(2040, 1930) code, is obviously better.

Furthermore, it's very to draw the conclusion here after a careful comparison between Figs. 1 and 2. That is to say, the parity-check matrix of the fourth submatrix of SCG(4, k) code proposed in this paper consists of the reversion arrangements of the phalanxes of the third submatrix, while in the parity-check matrix of SCG(4, k) code proposed by Hösli and Erik when k is an odd, the phalanxes of the third and fourth submatrices are not identical, so in the future hardware implementation, the construction method can save hardware storage space and reduce the calculation complexity. Moreover, the decoding of the LDPC codes in the hardware can parallelly be implemented, so the decoding velocities of the two novel LDPC codes are very rapid, the complexities of implementing the two novel LDPC codes, compared with the BCH(3860, 3824) + BCH(2040, 1930) and RS(255, 239) + CSOC($k_0/n_0 = 6/7, J = 8$) two concatenated codes in ITU-T G.975.1, are relatively lower, and the storing space can be saved and the computing complexity can be reduced in implementing the hardware in future. So the LDPC (5929, 5624) can be used as candidate super-FEC code

type in optical transmission system and can better suit for high-speed ultra long-haul optical transmission systems.

5 Conclusions

An improved novel construction method of LDPC codes, based on the construction method of SCG(4, k), is proposed in this paper, where the fourth submatrix is constructed via the transformation from phalanxes of the third one in reverse order, in this way the storage space is saved and the calculation complexity is also reduced. The simulation analysis shows that the LDPC(5929, 5624) code constructed by the proposed method in this paper, compared with the classical RS(255, 239) code to be widely used in optical transmission system, has better error correction performances. As a result, the novel LDPC code has the advantages, such as better error correction performances, lower redundancy and lower decoding complexity to meet the requirements of optical transmission systems. So the LDPC(5929, 5624) can be used as candidate super-FEC code type in optical transmission system and can better suit for high-speed ultra long-haul optical transmission systems.

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