

Optical constants study of YAG:Ce phosphor layer blended with SiO₂ particles by Mie theory for white light-emitting diode package

Run HU^{1,2}, Xiaobing LUO (✉)^{1,2}, Huai ZHENG¹, Sheng LIU^{2,3}

1 School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

2 Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, China

3 School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

© Higher Education Press and Springer-Verlag Berlin Heidelberg 2012

Abstract Optical constants, including scattering coefficient, absorption coefficient, asymmetry parameter and reduced scattering coefficient, of cerium-doped yttrium aluminium garnets (YAG:Ce) phosphor blended with SiO₂ particle for white light-emitting diode (LED) packages were calculated based on Mie theory in this study. Calculation processes were presented in detail. Variations of the optical constants with the changes of phosphor weight fraction, dopant weight fraction, phosphor particle radius and SiO₂ particle radius, were shown and analyzed separately. It was found that the asymmetry parameter is the intrinsic characteristic of the particles, and the increase of the phosphor weight fraction (or concentration) will lead to the increase of the optical constants. It was also discovered that the increase of the dopant weight fraction will enhance the scattering coefficient, but result in the decreases of the reduced scattering coefficient and the absorption coefficient.

Keywords Mie theory, phosphor, optical constant, light-emitting diode (LED)

1 Introduction

Compared with traditional light sources, white light source based on light-emitting diodes (LEDs) have a considerable impact on issues such as energy consumption, environment protection and even the health of individuals for their extraordinary features. Owing to the achievements of semiconductor materials and packaging approaches, there are three dominant ways to produce white light based on

LEDs currently, i.e., 1) a blue LED die with yellow phosphor; 2) an ultraviolet LED die with blue and yellow phosphors (or red, green and blue phosphors); 3) a device with red, green and blue LEDs together [1–3]. Among these methods, the former two methods both are involved with phosphor materials, which can be stimulated by short-wavelength light from the LED dies and emit long-wavelength light. The white light can be obtained by the mixture of the light from the LED dies and fluorescent light from the phosphor layer. Cerium-doped yttrium aluminium garnets (YAG:Ce) phosphor is one kind of the widely used phosphors due to its high quantum efficiency, good resolution, good thermal conductivity, wide waveband, and especially strong yellow emission under the excitation of blue light (450–470 nm) [4–7]. The YAG:Ce phosphors are usually prepared by solid-state reaction method, which will cause impurities and defects. It is well known that these surface defects of phosphors induce low luminescence efficiency and strong absorption of blue light, therefore coating inorganic materials is a good way to improve the performance of YAG:Ce phosphors. SiO₂ particles are one kind of the suitable coating materials, which could enhance the light scattering and weaken the blue light absorption of the YAG:Ce phosphor [5,8].

To characterize the optical properties of YAG:Ce phosphors, many researchers have performed a lot of studies by experiments or Monte Carlo simulations. Fujita et al. [4] revealed the interdependence among the YAG:Ce particle size, luminous efficacy and the light scattering coefficient. They also found that the light scattering coefficient decreased with the increase of the phosphor particle size. Liaparinos et al. [9] studied X-ray and light transport within granular phosphor materials by a developed computational model using Monte Carlo methods. Waters [10] found the emission intensity of light from the

phosphor powders exhibited strong dependence on the optical constants, especially the absorption and scattering coefficients. Liu et al. [11–13] simulated the phosphor layer with different concentrations, thicknesses, locations within the LED packages, and found that variation of the phosphor layer had a large impact on the optical performance of LEDs. Zhu et al. [6,14] tested the optical properties of YAG:Ce phosphor, and based on the discoveries in the experiments, they proposed a novel packaging approaches. Nowadays, it is a consensus that the optical properties of the phosphors play an important role in determining the optical performance of LEDs, such as luminous efficiency, correlated color temperature and spatial color uniformity [15–19]. However, the optical properties of the phosphors, in the final analysis, depend on the optical constants, including the scattering coefficient, absorption coefficient, asymmetry parameter, etc. In previous studies of the phosphor materials, values of the light constants were obtained by fitting to the experimental or simulation data. Due to the differences of the experimental or simulation techniques and fitting approaches, the optical constants of specific phosphor materials were varied within a wide range of values [9]. Therefore, it is of great importance to obtain the precise optical constants of the phosphor materials [20].

In this study, a numerical model was demonstrated to calculate the optical constants of YAG:Ce phosphors blended with SiO₂ particles based on Mie scattering theory. The interdependence among the phosphor particle radius, scattering coefficient, absorption coefficient, asymmetry parameter, weight fraction of phosphor and the SiO₂ dopant weight fraction were investigated. The detailed analyses and predictions were presented.

2 Calculation of optical constants

When a particle is illuminated by a beam of light, the amount and angular distribution of the light scattered by the particle, as well as the absorbed amount, depend on the intrinsic characteristics of the particle. In this study, the Mie theory was applied to calculate the amount of the energy of scattering and absorption light for it is valid for all possible ratios of particle radius to the wavelength. According to Mie theory, extinction efficiency Q_{ext} , scattering efficiency Q_{sca} , absorption efficiency Q_{abs} , and asymmetry parameter g are normally calculated by the following equations [21–23]:

$$Q_{\text{ext}} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \text{Re}(a_n + b_n), \quad (1)$$

$$Q_{\text{sca}} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) (|a_n|^2 + |b_n|^2), \quad (2)$$

$$Q_{\text{abs}} = Q_{\text{ext}} - Q_{\text{sca}}, \quad (3)$$

$$g = \frac{4}{x^2} \left\{ \sum_{n=1}^{\infty} \frac{n(n+2)}{n+1} \text{Re}(a_n a_{n+1}^* + b_n b_{n+1}^*) + \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \text{Re}(a_n b_n^*) \right\}, \quad (4)$$

where a_n and b_n are the expansion coefficients with even symmetry and odd symmetry, respectively, which can be determined by

$$a_n = \frac{m^2 j_n(mx) [x j_n(x)]' - j_n(x) [m x j_n(mx)]'}{m^2 j_n(mx) [x h_n^{(1)}(x)]' - h_n^{(1)}(x) [m x j_n(mx)]'}, \quad (5)$$

$$b_n = \frac{j_n(mx) [x j_n(x)]' - j_n(x) [m x j_n(mx)]'}{j_n(mx) [x h_n^{(1)}(x)]' - h_n^{(1)}(x) [m x j_n(mx)]'}, \quad (6)$$

where $j_n(x)$ and $h_n^{(1)}(x)$ are the Bessel function and first kind of Hankel function, respectively; x is the size parameter, which is calculated by Eq. (7); m is the complex refractive index of the particle relative to the ambient medium. In this study, the ambient medium is silicone gel, thus the complex refractive index of phosphor (m_p) and SiO₂ (m_s) are calculated by Eq. (8).

$$x = kr = \frac{2\pi n_{\text{sil}}}{\lambda} r, \quad (7)$$

$$m_p = n_p/n_{\text{sil}}, \quad m_s = n_s/n_{\text{sil}}, \quad (8)$$

where k is wave number, r is sphere radius, and λ is the wavelength. n_p , n_s and n_{sil} are the refractive indices of phosphor, SiO₂, and silicone gel, respectively. The radii of phosphor range from 0.7 to 15 μm , and the radii of the dopant SiO₂ usually range from 0.5 to 1 μm . The refractive index of SiO₂ is 1.44, while it is 1.53 for the silicone gel. In this study, it's assumed that the SiO₂ particles had no absorption thus the imaginary part of the refractive index was ignored. Since the YAG:Ce phosphor has properties of scattering and absorption, the refractive index of phosphor has a complex form as shown in Eq. (9) [7]

$$n_p = n_p' - n_p'', \quad (9)$$

where n_p' and n_p'' are the real and imaginary part of the refractive indices of the phosphor particle. For phosphor particles, n_p' and n_p'' can be calculated as Eqs. (10) and (11), respectively [7].

$$n_p' = \sqrt{2.08745 + 1.2081\lambda^2/(\lambda^2 - 0.02119) + 17.2049\lambda^2/(\lambda^2 - 1404.45)}, \quad (10)$$

$$n_p'' = \alpha/(2k), \quad (11)$$

where α is the absorption coefficient of phosphor crystal.

In the LED packages, the YAG:Ce phosphor and SiO₂ particles are mixed uniformly before being embedded into the silicone gel to obtain the phosphor layer. In the previous studies, the phosphor concentration was found important and usually shared the same unit with density, i.e., g/cm³. Since the volume of the phosphor mixture is difficult to measure, the phosphor concentration, however, is not easy to obtain in the packaging industry. Therefore, we choose the weight fraction instead of the concentration to benefit the industry. Assume that the weight of the YAG:Ce phosphor, SiO₂ particle, and the silicone gel are m_p , m_s and m_{sil} , respectively, and then the phosphor weight fraction wf_1 and the dopant weight fraction wf_2 are defined as

$$wf_1 = \frac{m_p + m_s}{m_p + m_s + m_{sil}}, \quad (12)$$

$$wf_2 = \frac{m_s}{m_p + m_s}. \quad (13)$$

Therefore, the volume fractions of the phosphor and SiO₂ are calculated by

$$vf_1 = \frac{m_p/\rho_p}{m_p/\rho_p + m_s/\rho_s + m_{sil}/\rho_{sil}}, \quad (14)$$

$$vf_2 = \frac{m_s/\rho_s}{m_p/\rho_p + m_s/\rho_s + m_{sil}/\rho_{sil}}, \quad (15)$$

where ρ_p , ρ_s and ρ_{sil} are the density of phosphor, SiO₂ and silicone gel, respectively. In this study, ρ_p , ρ_s and ρ_{sil} are 4.6, 2.2, and 1.1 g/cm³, respectively. Assume that the particle is sphere with a radius of r , its volume V therefore is $4\pi r^3/3$. The optical constants of phosphor, including the scattering coefficient μ_{sca_p} and the absorption coefficient μ_{abs_p} can be calculated by

$$\mu_{sca_p} = Q_{sca} A v f_1 / V = 3 Q_{sca} v f_1 / (4r), \quad (16)$$

$$\mu_{abs_p} = Q_{abs} A v f_1 / V = 3 Q_{abs} v f_1 / (4r), \quad (17)$$

where A is the cross area of the particle ($=\pi r^2$). The scattering coefficient μ_{sca_s} and the absorption coefficient μ_{abs_s} of SiO₂ particle can also be calculated by replacing the volume fraction vf_1 with vf_2 .

For the phosphors, reduced scattering coefficient δ_{sca} is an important parameter and frequently used, which is a lumped property incorporating the scattering coefficient and the asymmetry parameter as Eq. (18) [7,21]

$$\delta_{sca_p} = \mu_{sca_p}(1 - g_p), \quad (18)$$

where g_p is the average cosine of the scattering angle that

characterizes the asymmetry of the scattering function, and it can be calculated by Eq. (4). The reduced scattering coefficient δ_{sca_s} of the SiO₂ particles can also be obtained similarly. For the mixture media of phosphors and silicone gel, the total scattering coefficient μ_{sca} , total absorption coefficient μ_{abs} and total reduced scattering coefficient δ_{sca} can be calculated as

$$\mu_{sca} = \mu_{sca_p} + \mu_{sca_s}, \quad (19)$$

$$\mu_{abs} = \mu_{abs_p} + \mu_{abs_s}, \quad (20)$$

$$\delta_{sca} = \delta_{sca_p} + \delta_{sca_s}. \quad (21)$$

3 Results and discussion

When the phosphor weight fraction wf_1 was 0.2 without SiO₂ dopant, the scattering coefficient μ_{sca_p} , the asymmetry parameter g_p , the reduced scattering coefficient δ_{sca_p} and the absorption coefficient μ_{abs_p} for blue light ($\lambda = 465$ nm) and yellow light ($\lambda = 550$ nm) were shown in Figs. 1 and 2, respectively. For the blue light, the absorption coefficient α of phosphor crystals varied from 8 to 20 mm⁻¹, while α varied from 0.1 to 0.5 mm⁻¹ for the yellow light [7]. It is seen that for the blue light, with the increase of phosphor particle radius, the μ_{sca_p} and δ_{sca_p} decreased rapidly, the g_p increased slightly, and the μ_{abs_p} decreased a little. For the yellow light, the μ_{sca_p} , δ_{sca_p} and g_p showed similar trend of change, while the μ_{abs_p} were almost kept the same. It is also noticed that the crystal absorption coefficient α had a large impact on the absorption coefficient μ_{abs_p} , and also influenced the g_p when the phosphor particle radius was big, but it had no impact on the scattering coefficients. The reasons may include: 1) when the phosphor particle radius was small, the number of the particle in unit volume was much large, and the probability that light was scattered by the particle was therefore enhanced greatly; on the contrary, when the phosphor particle radius increased, the light scattered effect was weakened; 2) when the phosphor radius increased, the effect of particle size was gradually enhanced and the scattered light gradually became non-uniform, thus the asymmetry parameter increased. Since light is a kind of electromagnetic wave, we can understand these phenomena by electromagnetic field theory [21,23]. When the particle is illuminated by light, the total scattered field is obtained by superposing the scattered wavelets on the incident wave. When the particle radius is small, all the secondary wavelets are approximately in phase and there were no much variation of scattering with direction. As the particle radius increases, the number of possibilities for enhancement and cancellation of the scattered wavelets increase correspondingly. The larger the particle is, the

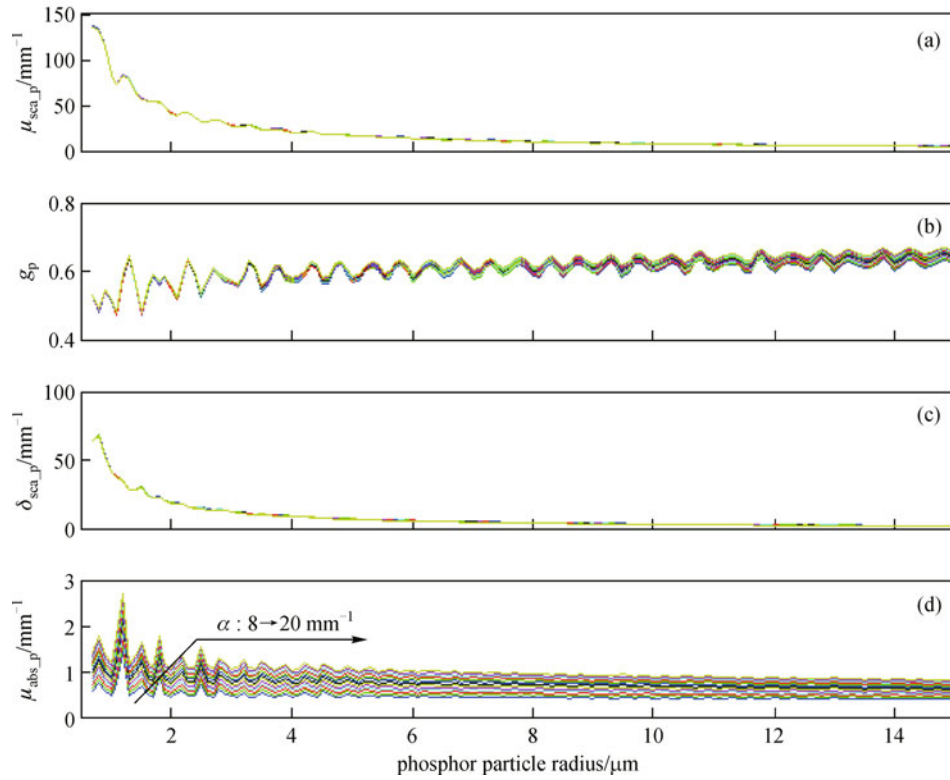


Fig. 1 Variation of optical constants of YAG:Ce phosphor without SiO₂ dopant for blue light with the increase of absorption coefficient of phosphor crystal α

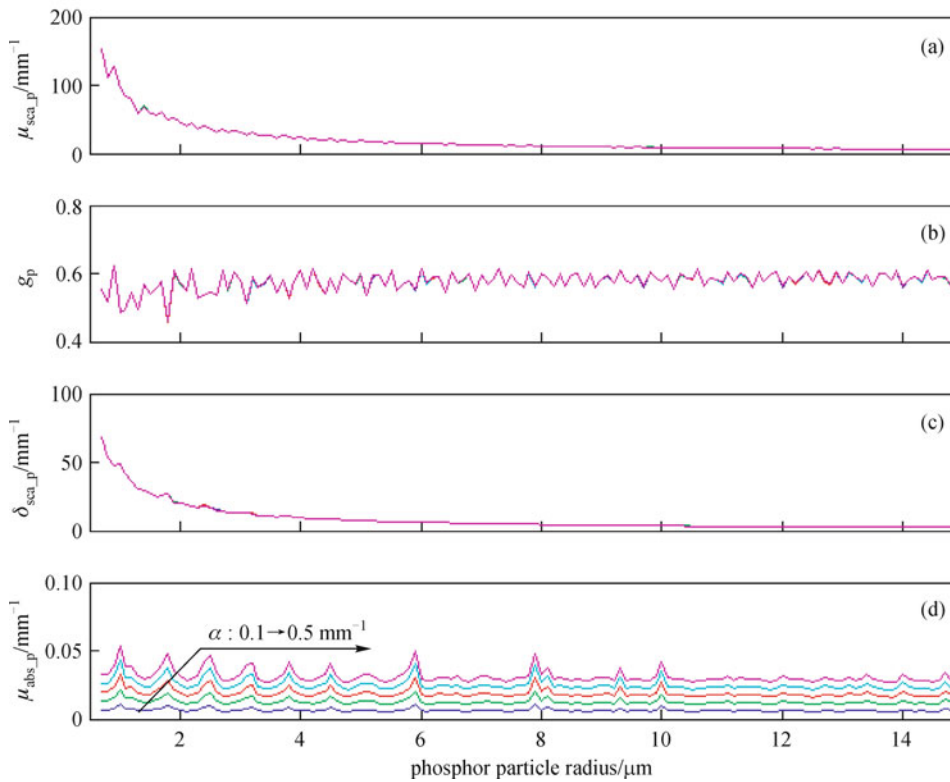


Fig. 2 Variation of optical constants of YAG:Ce phosphor without SiO₂ dopant for yellow light with the increase of absorption coefficient of phosphor crystal α

more peaks and valleys of the superposing wave in the scattering pattern are. Therefore the larger the variation of scattering in directions is and the larger asymmetry parameter is. Large asymmetry parameter means strong forward scattering [24,25]. We can predict that small phosphor particle can enhance the isotropy of the light performance, while large phosphor particle can alleviate the back-scattering but reduce the scattering coefficient. Thus, the asymmetry parameter and the scattering coefficient are a pair of paradox, which are dependent on the phosphor radius, and these will determine the final optical performance of phosphor layer.

The variation of the optical constants of phosphor without SiO₂ dopant with the increase of phosphor weight fraction wf_1 for the blue light ($\lambda = 465$ nm) are illustrated in Fig. 3. It is observed that when the phosphor particle radius was less than 4 μm , the μ_{sca_p} and δ_{sca_p} increased with the increase of wf_1 . However, the μ_{sca_p} and δ_{sca_p} almost kept the same when the phosphor particle radius was large. It is also found that with the increase of wf_1 , the μ_{abs_p} increased. The wf_1 had no impact on the asymmetry parameter g_p . There are three reasons for these results, which are described as follows: 1) when the radius of phosphor particle increased, the scattering effect was weakened from Fig. 1(a); 2) when the weight fraction of phosphor wf_1 increased, the phosphor concentration increased proportionally, thus the number of phosphor

particles in unit volume increased and the scattering effect was enhanced; 3) when the number of phosphor particles in unit volume increased, the probability of light collided with the particles increased and the absorption effect was strengthened dominantly.

When the YAG:Ce phosphor was blended with SiO₂ particles, the optical constants of YAG:Ce phosphor and SiO₂ for blue light ($\lambda = 465$ nm) with a constant wf_1 ($= 0.3$) and the increase of the dopant weight fraction wf_2 are pictured in Figs. 4 and 5, respectively. According to Eq. (13), the SiO₂ weight fraction increased with the increase of wf_2 , thus the weight fraction of YAG:Ce phosphor decreased and the μ_{sca_p} , δ_{sca_p} and μ_{abs_p} reduced, while the μ_{sca_s} and δ_{sca_s} raised. It can be found that the asymmetry parameter g_p and g_s did not change with the variation of wf_2 , and the g_s was much larger than that of the phosphor. It is also noticed in Fig. 5 that with the increase of the SiO₂ particle radius, the μ_{sca_s} and g_s increased, but the δ_{sca_s} decreased slightly. The asymmetry parameter g_s of SiO₂ was almost approximated to 1, which means the dopant SiO₂ particle can enhance the forward scattering effect greatly. Finally, it's found that the absorption coefficient μ_{abs_s} was kept zero, which agreed with our assumption of the SiO₂ particle.

Figure 6 shows the variation of the μ_{sca} , δ_{sca} and μ_{abs} of the phosphor layer, where the phosphor weight fraction wf_1 was 0.3, the SiO₂ particle radius was 0.5 μm , and the

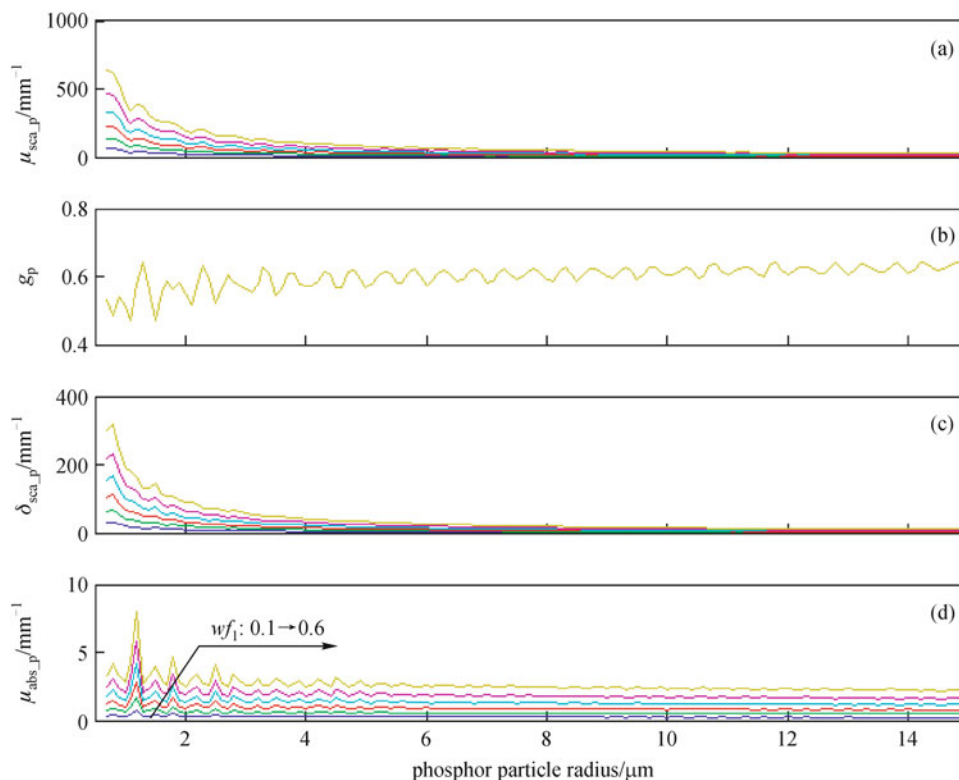


Fig. 3 Variation of the optical constants of YAG:Ce phosphor without SiO₂ dopant with the increase of phosphor weight fraction wf_1 for blue light

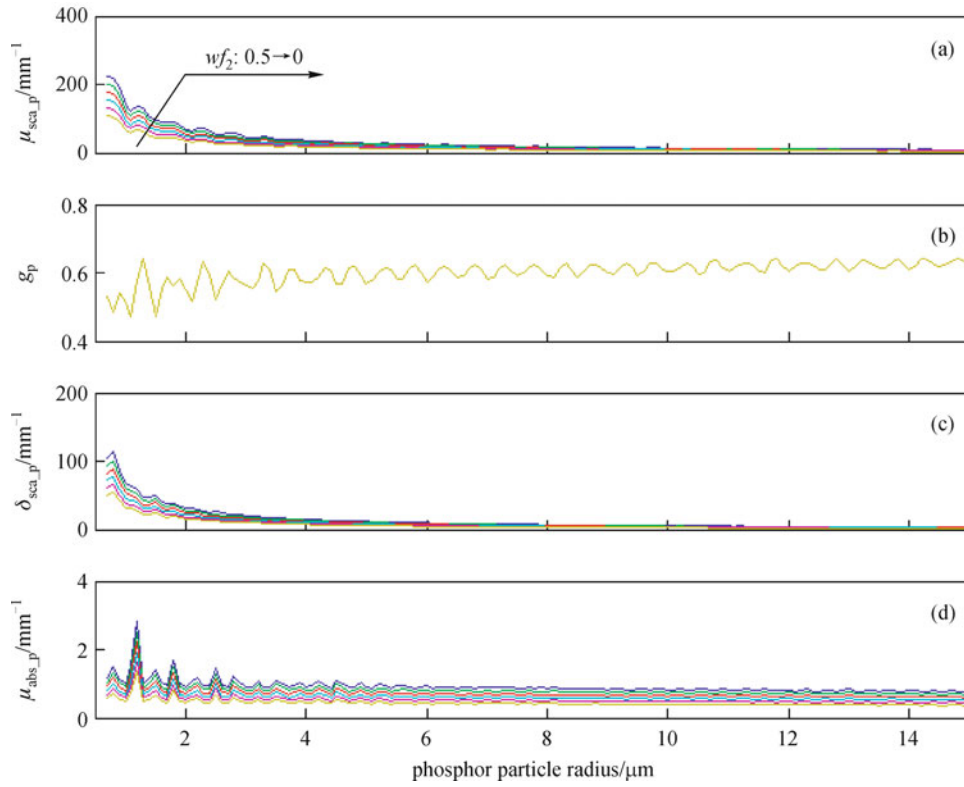


Fig. 4 Variation of the optical constants of YAG:Ce phosphor with the increase of dopant weight fraction w_{f_2} for blue light

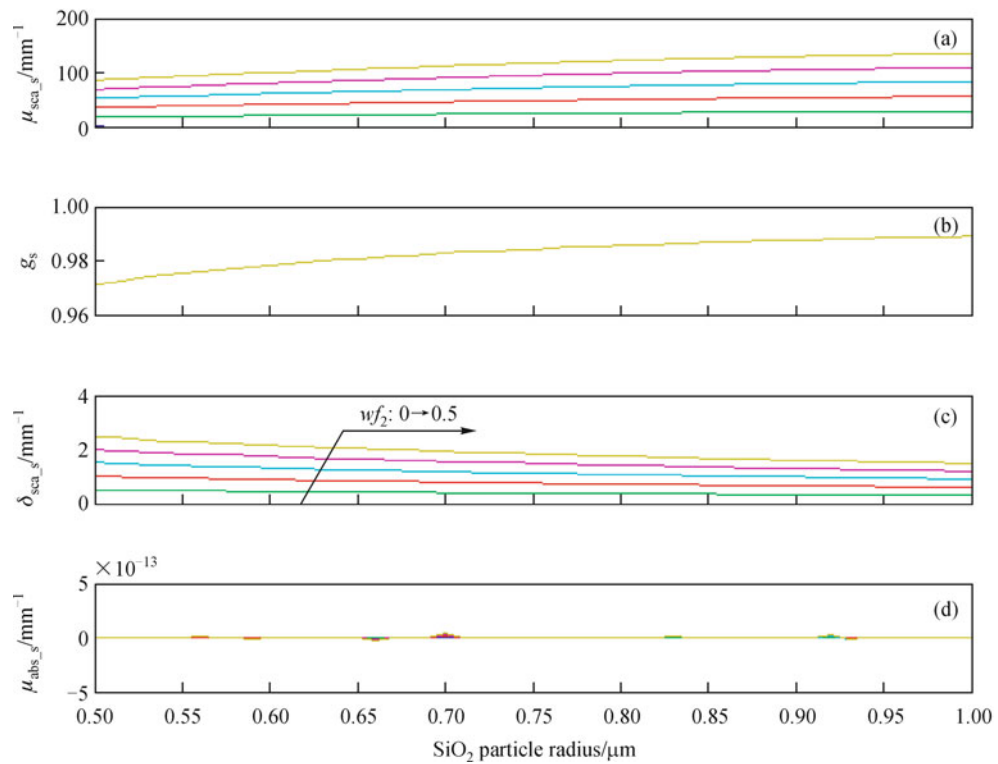


Fig. 5 Variation of the optical constants of SiO₂ particles with the increase of dopant weight fraction w_{f_2} for blue light

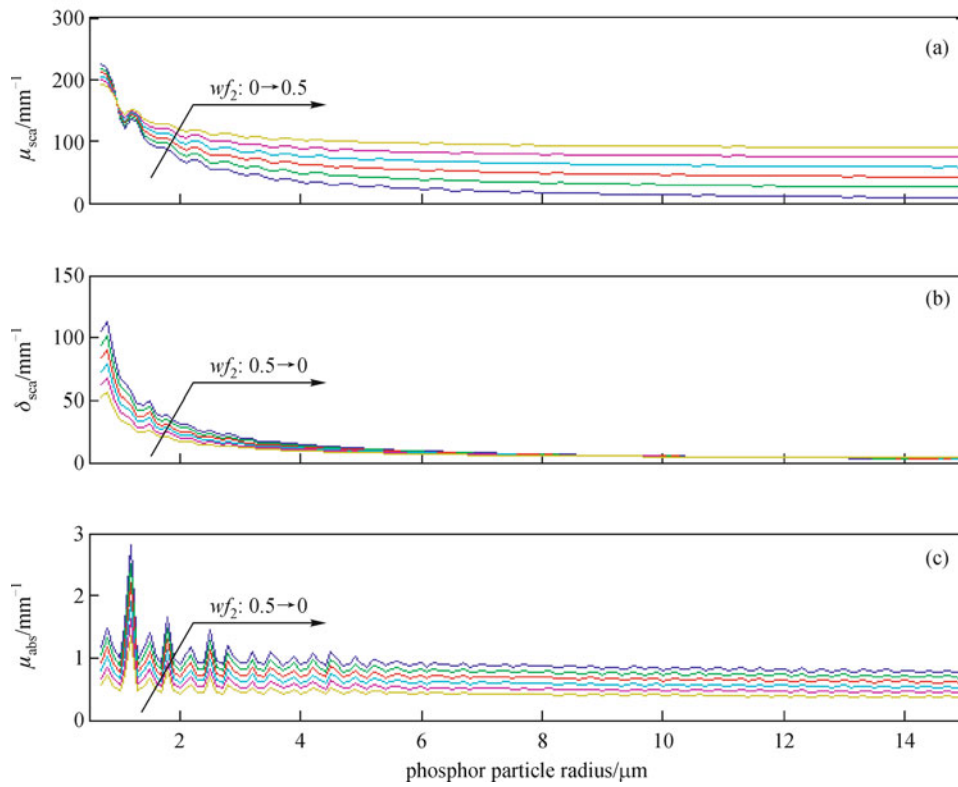


Fig. 6 Variation of the optical constants of phosphor layer with the increase of dopant weight fraction wf_2 and phosphor particle radius for blue light

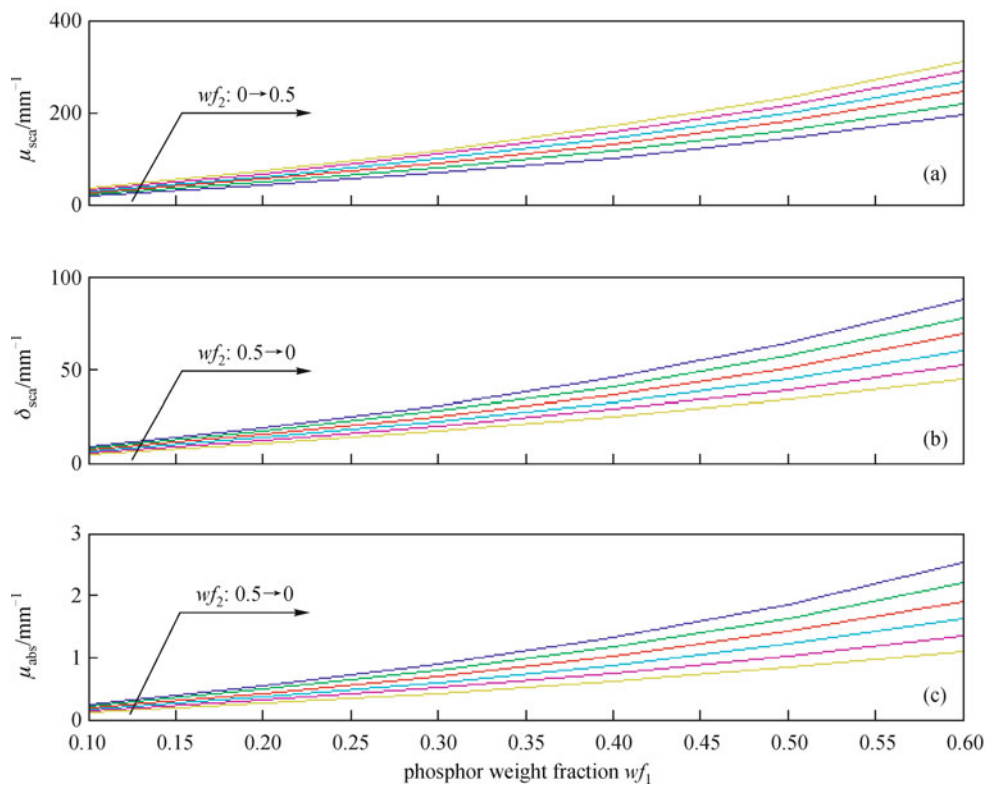


Fig. 7 Variation of the optical constants of phosphor layer with the increase of phosphor weight fraction wf_1 and dopant weight fraction wf_2 for blue light

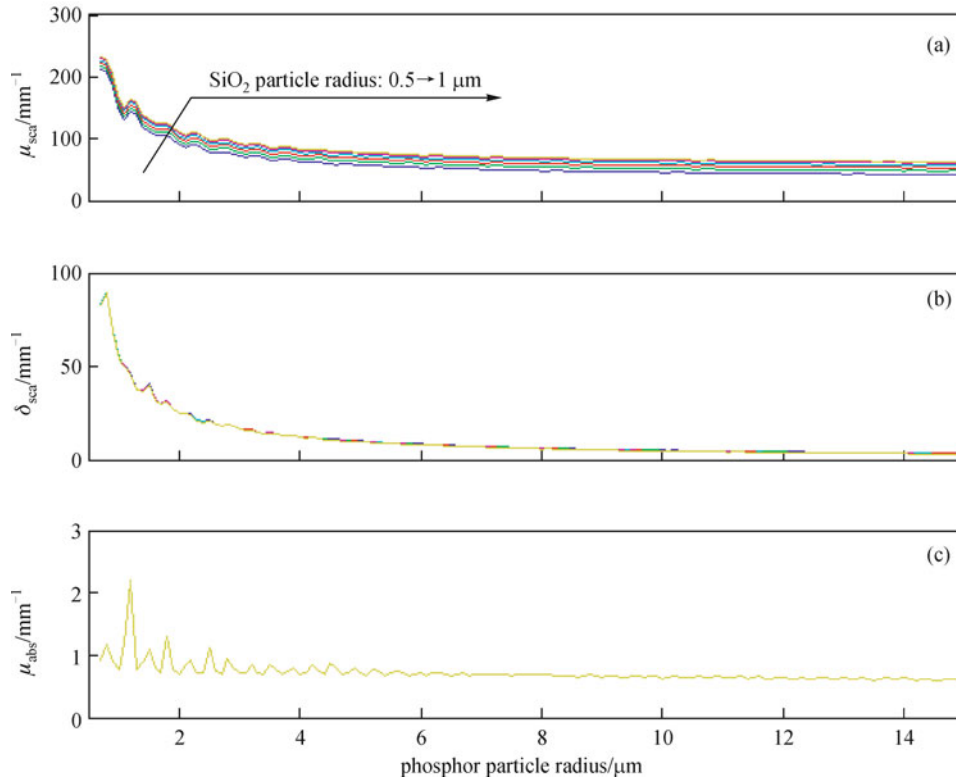


Fig. 8 Variation of the optical constants of phosphor layer with the increase of dopant SiO₂ particle radius for blue light

absorption coefficient of phosphor crystal α was 10 mm^{-1} . It is seen that with the increase of wf_2 , the μ_{sca} increased, while δ_{sca} and μ_{abs} decreased. The variation of the μ_{sca} , δ_{sca} and μ_{abs} of the phosphor layer with a constant phosphor particle radius ($= 2 \text{ }\mu\text{m}$), a constant SiO₂ particle radius ($= 0.5 \text{ }\mu\text{m}$), and a constant absorption coefficient of phosphor crystal α ($= 10 \text{ mm}^{-1}$) for blue light ($\lambda = 465 \text{ nm}$) were shown in Fig. 7. It is seen that with the increase of wf_1 , the μ_{sca} , δ_{sca} and μ_{abs} of the phosphor layer increased; with the increase of wf_2 , the μ_{sca} increased while δ_{sca} and μ_{abs} decreased. The reasons behind Figs. 6 and 7 may be found from Figs. 4 and 5. From Fig. 4, it's known that the optical constants of YAG:Ce phosphor were inversely proportional to the wf_2 , while the optical constants of SiO₂ particle were proportional to the wf_2 . By consideration of the order of these optical constants, the phenomena in Figs. 6 and 7 can be understood.

We also figured out the relationship between the optical constants of phosphor layer with the radii of SiO₂ and YAG:Ce phosphor particle. The weight fraction wf_1 and wf_2 were 0.3 and 0.2, respectively. The absorption coefficient of phosphor crystal α for blue light ($\lambda = 465 \text{ nm}$) was 10 mm^{-1} . As shown in Fig. 8, the increase of SiO₂ particle radius led to the enhancement of the μ_{sca} , while it had little influence on the δ_{sca} and μ_{abs} .

4 Conclusions

In this study, a numerical model of YAG:Ce phosphor blended with SiO₂ particles was developed based on Mie theory. The optical constants of the YAG:Ce phosphor, SiO₂ particle and the mixed phosphor layer, including the scattering coefficient, the absorption coefficient, the asymmetry parameter and the reduced scattering coefficient, were presented and discussed. It is found that: 1) the absorption coefficient of phosphor crystal mainly influences the absorption coefficient, and it has little impact on the scattering coefficient; 2) the asymmetry parameter is the intrinsic characteristic of the particles, and had nothing with the weight fraction; 3) the increase of the phosphor weight fraction (or concentration) will lead to the increase of the optical constants; 4) the increase of the dopant weight fraction will enhance the scattering coefficient, but lead to the decrease of the reduced scattering coefficient and the absorption coefficient. It is worthy to figure out the precise values of these optical constants, which are significant for the analyses of the optical properties of the materials or the optical performance for the devices. According to the optical constants, the specific phosphor materials for better optical performance of LED packages can be synthesized.

Acknowledgements This work was supported in part by the Major State Basic Research Development Program of China (No. 2011CB013105), and in part by National High Technology Research and Development Program of China (No. 2011AA03A109).

References

- Pimputkar S, Speck J S, Denbaars S P, Nakamura S. Prospects for LED lighting. *Nature Photonics*, 2009, 3(4): 180–182
- Tonzani S. Lighting technology: Time to change the bulb. *Nature*, 2009, 459(7245): 312–314
- Hu R, Luo X B, Liu S. Study on the optical properties of conformal coating light-emitting diode by Monte Carlo simulation. *IEEE Photonics Technology Letters*, 2011, 23(22): 1673–1675
- Fujita S, Umayahara Y, Tanabe S. Influence of light scattering on luminous efficacy in Ce:YAG glass-ceramic phosphor. *Journal of the Ceramic Society of Japan*, 2010, 118(2): 128–131
- Chao K F, Xiao Z G, Zhang H W, Niu W B, Xia W, Sang S Y, Xin Y, Li Q, Wu S L, Zhang S F. The surface coating of YAG:Ce phosphors with ZnO nanoparticles for white LED. *Materials Science Forum*, 2011, 675–677: 1283–1286
- Zhu Y T, Narendran N, Gu Y M. Investigation of the optical properties of YAG:Ce phosphor. In: *Proceedings of the Society for Photo-Instrumentation Engineers*, 2006, 6337: 63370S
- Liu Z Y, Liu S, Wang K, Luo X B. Measurement and numerical studies of optical properties of YAG:Ce phosphor for white light-emitting diode packaging. *Applied Optics*, 2010, 49(2): 247–257
- Cui H T, Zhang Y W, Hong G Y. The surface coating technology of phosphor. *J Functional Mater*, 2001, 32(6): 564–567 (In Chinese)
- Liaparinis P F, Kandarakis I S, Cavouras D A, Delis H B, Panayiotakis G S. Modeling granular phosphor screens by Monte Carlo methods. *Medical Physics*, 2006, 33(12): 4502–4514
- Waters D N. Intensity of emitted light from powder phosphors: the effects of light absorption and scattering in the powder matrix. *Journal of Materials Science Materials in Electronics*, 1996, 7(2): 61–65
- Liu Z Y, Liu S, Wang K, Luo X B. Optical analysis of color distribution in white LEDs with various packaging methods. *IEEE Photonics Technol*, 2008, 20(24): 2027–2029
- Liu Z Y, Liu S, Wang K, Luo X B. Optical analysis of phosphor's location for high-power light-emitting diodes. *IEEE Transactions on Device and Materials Reliability*, 2009, 9(1): 65–73
- Liu Z Y, Liu S, Wang K, Luo X B. Studies on optical consistency of white LEDs affected by phosphor thickness and concentration using optical simulation. *IEEE Transactions on Components and Packaging Technologies*, 2010, 33(4): 680–687
- Zhu Y T, Narendran N. Investigation of remote-phosphor white light-emitting diodes with multi-phosphor layers. *Japanese Journal of Applied Physics*, 2010, 49(10): 100203
- Zhang D W, Li B C. A multi-layer phosphor package of white-light-emitting diodes with high efficiency. *Optik (Stuttgart)*, 2010, 121(24): 2224–2226
- Moon K M, An S H, Kim H K, Chae J H, Park Y J. Phosphor concentration and geometry for high power white light emitting diode. In: *Proceedings of the Society for Photo-Instrumentation Engineers*, 2010, 7617: 76171Y
- You J P, Tran N T, Shi F G. Light extraction enhanced white light-emitting diodes with multi-layered phosphor configuration. *Optics Express*, 2010, 18(5): 5055–5060
- Keppens A, Denijs S, Wouters S, Ryckaert W R, Deconinck G, Hanselaer P. Modelling the spatical colour distribution of phosphor-white high power light-emitting diodes. In: *Proceedings of the Society for Photo-Instrumentation Engineers*, 2010, 7717: 77170J
- Sommer C, Reil F, Krenn J R, Hartmann P, Pachler P, Hoschopf H, Wenzl F P. The impact of light scattering on the radiant flux of phosphor-converted high power white light-emitting diodes. *Journal of Lightwave Technology*, 2011, 29(15): 2285–2291
- Yen W M, Shionoya S, Yamamoto H. *Phosphor Handbook*. 2nd ed. London: Taylor & Francis Group, 2007, 11–19
- Bohren C F, Huffman D R. *Absorption and Scattering of Light by Small Particles*. New York: John Wiley & Sons, 1983, 90–120
- Du H. Mie-scattering calculation. *Applied Optics*, 2004, 43(9): 1951–1956
- Mishchenko M I, Hovenier J W, Travis L D. *Light scattering by nonspherical particles: theory, measurements and applications*. New York: Academic Press, 1999, 10–52
- Acquista C, Cohen A, Cooney J A, Wimp J. Asymptotic behavior of the efficiencies in Mie Scattering. *Journal of the Optical Society of America*, 1980, 70(8): 1023–1025
- Sudiarta I W, Chylek P. Mie scattering efficiency of a large spherical particle embedded in an absorbing medium. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 2001, 70(4–6): 709–714