

A modified dual-wavelength matrix calculation method

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Abstract Fiber grating is an optical passive device which has been greatly developed in recent years. The largest application for fiber Bragg grating (FBG) is the fiber sensor. Cross-sensitivity of fiber grating sensor is the most important problem which has restricted the development of the fiber sensor. In this paper, we explain how the cross-sensitivity problem is produced based on the basic principle, and we suggest a modification for the traditional dual-wavelength matrix calculation method, which is used to solve the cross-sensitivity problem. The modified calculation method has a higher accuracy than the traditional one.

Keywords fiber Bragg grating (FBG), cross-sensitivity, fiber sensor, dual-wavelength matrix

1 Introduction

Fiber grating is an optical passive component that has been developed rapidly in recent years. With mature manufacturing technology and extensive application, fiber grating can be widely used in the future. A periodic refractive index distributing in dimension, where the photosensitive characteristic of the fiber material is used, can control the propagation of light in the fiber grating. These unique characteristics can make it possible for the application of fiber grating in the fiber communication net and sensor net fields. Cross-sensitivity of fiber grating sensor is an important problem which restricts the development of the fiber sensor. In the factual application, the wavelength of fiber Bragg grating (FBG) can be affected by stress and temperature, which means the fiber grating sensor has a cross-sensitivity problem between stress and temperature.

The dual-wavelength matrix calculation method [1] is currently the most popular method to solve the cross-sensitivity problem.

2 Theory derivation

Here is the basic theory of a sensor made by FBG.

When a broadband light comes into FBG, only a narrowband signal wavelength close to Bragg wavelength can be reflected, and the other broadband wavelength will be transmitted. The Bragg wavelength is [2]

$$\lambda_B = 2n_{\text{eff}}\Lambda, \quad (1)$$

where Λ is the grating period, and n_{eff} is the effective index of the fiber core.

When the stress or the temperature changes, Λ and n_{eff} are affected, which will make the Bragg wavelength offset. We can explain the relationship between the Bragg wavelength and stress and temperature by the equation below:

$$\lambda_B = \lambda_B(\varepsilon, T). \quad (2)$$

By making dualistic Taylor series expansion in Eq. (2), we can get an expression of λ_B as follows:

$$\begin{aligned} \lambda_B(\varepsilon, T) = & \lambda_B(\varepsilon_0, T_0) + \Delta\varepsilon \left(\frac{\partial \lambda_B}{\partial \varepsilon} \right)_{(\varepsilon_0, T_0)} + \Delta T \left(\frac{\partial \lambda_B}{\partial T} \right)_{(\varepsilon_0, T_0)} \\ & + \frac{1}{2!} \left[(\Delta\varepsilon)^2 \left(\frac{\partial^2 \lambda_B}{\partial \varepsilon^2} \right)_{(\varepsilon_0, T_0)} + (\Delta T)^2 \left(\frac{\partial^2 \lambda_B}{\partial T^2} \right)_{(\varepsilon_0, T_0)} \right. \\ & \left. + 2\Delta\varepsilon\Delta T \left(\frac{\partial^2 \lambda_B}{\partial \varepsilon \partial T} \right)_{(\varepsilon_0, T_0)} \right] + \dots \\ & + \frac{1}{n!} \left[\Delta\varepsilon \left(\frac{\partial}{\partial \varepsilon} \right)_{(\varepsilon_0, T_0)} + \Delta T \left(\frac{\partial}{\partial T} \right)_{(\varepsilon_0, T_0)} \right]^n \lambda_B, \end{aligned} \quad (3)$$

where $\Delta\varepsilon$ and ΔT are the change of stress and temperature to reference estate (ε_0, T_0) . Therefore, the change of Bragg wavelength which is caused by stress and temperature is

$$\begin{aligned} \Delta\lambda_B &= \lambda_B(\varepsilon, T) - \lambda_B(\varepsilon_0, T_0) \\ &= \Delta\varepsilon \left(\frac{\partial\lambda_B}{\partial\varepsilon} \right)_{(\varepsilon_0, T_0)} + \Delta T \left(\frac{\partial\lambda_B}{\partial T} \right)_{(\varepsilon_0, T_0)} \\ &\quad + \frac{1}{2!} \left[(\Delta\varepsilon)^2 \left(\frac{\partial^2\lambda_B}{\partial\varepsilon^2} \right)_{(\varepsilon_0, T_0)} + (\Delta T)^2 \left(\frac{\partial^2\lambda_B}{\partial T^2} \right)_{(\varepsilon_0, T_0)} \right. \\ &\quad \left. + 2\Delta\varepsilon\Delta T \left(\frac{\partial^2\lambda_B}{\partial\varepsilon\partial T} \right)_{(\varepsilon_0, T_0)} \right] + \dots \\ &\quad + \frac{1}{n!} \left[\Delta\varepsilon \left(\frac{\partial}{\partial\varepsilon} \right)_{(\varepsilon_0, T_0)} + \Delta T \left(\frac{\partial}{\partial T} \right)_{(\varepsilon_0, T_0)} \right]^n \lambda_B. \end{aligned} \quad (4)$$

When the high level in Eq. (4) is ignored, the formula of $\Delta\lambda$ can be obtained as follows:

$$\Delta\lambda_B = \Delta\varepsilon \left(\frac{\partial\lambda_B}{\partial\varepsilon} \right)_{(\varepsilon_0, T_0)} + \Delta T \left(\frac{\partial\lambda_B}{\partial T} \right)_{(\varepsilon_0, T_0)}. \quad (5)$$

The first item at the right of Eq. (5) is the wavelength excursion caused by stress, and the second item at the right of Eq. (5) is the wavelength excursion caused by temperature. On the other hand, wavelength excursion caused by stress can be expressed as [3]

$$\Delta\lambda_B = \lambda_B(1 - P_\varepsilon)\Delta\varepsilon, \quad (6)$$

where

$$P_\varepsilon = \frac{n_{\text{eff}}^2}{2} [p_{12} - \nu(p_{11} + p_{12})]$$

is the Elasto-optical coefficient, p_{11} and p_{12} are the weights of Elasto-optical tensor, and ν is the Poisson coefficient.

Wavelength excursion caused by temperature can also be expressed as follows:

$$\Delta\lambda_B = \lambda_B(\alpha + \beta)\Delta T, \quad (7)$$

where α is the coefficient of thermal expansion, and β is the thermo-optic coefficient.

Contrasting Eqs. (6) and (7) to Eq. (5), we can get the following equations:

$$k_\varepsilon = \frac{\partial\lambda_B}{\partial\varepsilon} = \lambda_B(1 - P_\varepsilon), \quad (8)$$

$$k_T = \frac{\partial\lambda_B}{\partial T} = \lambda_B(\alpha + \beta), \quad (9)$$

where k_ε is the stress sensitivity coefficient, and k_T is the temperature sensitivity coefficient.

When the fiber grating is used in sense measurement [4–8], the accurate measurement result cannot be achieved because the single FBG cannot distinguish whether the change of the Bragg wavelength is caused by stress or temperature. A traditional method used to solve this problem is the dual-wavelength matrix calculation method.

The dual-wavelength matrix calculation method can solve the problem like this: by using two FBG with different Bragg wavelengths in the system, the change of Bragg wavelength matrix can be received as follows:

$$\begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \end{bmatrix} = \begin{bmatrix} k_{1\varepsilon} & k_{1T} \\ k_{2\varepsilon} & k_{2T} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon \\ \Delta T \end{bmatrix}, \quad (10)$$

where $k_{1\varepsilon}$, k_{1T} are the stress sensitivity coefficient and the temperature sensitivity coefficient for one FBG each; and $k_{2\varepsilon}$, k_{2T} are the stress sensitivity coefficient and the temperature sensitivity coefficient for another. By using Eq. (10), the stress change and temperature change can be distinguished concretely.

Equation (10) is the final expression for a traditional dual-wavelength matrix calculation method. The high level is ignored in Eq. (5). If the high level hyper-two is ignored, and the two-level items are used in Eq. (4) as fix items, a modified dual-wavelength matrix calculation method equation can be obtained below:

$$\begin{aligned} \begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \end{bmatrix} &= \begin{bmatrix} k_{1\varepsilon} & k_{1T} \\ k_{2\varepsilon} & k_{2T} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon \\ \Delta T \end{bmatrix} \\ &\quad + \begin{bmatrix} k_{1\varepsilon^2} & k_{1T^2} \\ k_{2\varepsilon^2} & k_{2T^2} \end{bmatrix} \begin{bmatrix} (\Delta\varepsilon)^2 \\ (\Delta T)^2 \end{bmatrix} \\ &\quad + \begin{bmatrix} k_{1\varepsilon T} \\ k_{2\varepsilon T} \end{bmatrix} \Delta\varepsilon\Delta T, \end{aligned} \quad (11)$$

where $k_{1\varepsilon^2}$, k_{1T^2} , and $k_{1\varepsilon T}$ are the two-level stress sensitivity coefficient, the two-level temperature sensitivity coefficient, and the stress-temperature cross-sensitivity coefficient for one FBG; $k_{2\varepsilon^2}$, k_{2T^2} , and $k_{2\varepsilon T}$ are the two-level stress sensitivity coefficient, the two-level temperature sensitivity coefficient, and the stress-temperature cross-sensitivity coefficient for another. By using Eqs. (1), (4), (8), and (9), k_{ε^2} , k_{T^2} , and $k_{\varepsilon T}$ can be calculated as below:

$$\begin{aligned} k_{\varepsilon^2} &= \frac{1}{2} \left(\frac{\partial^2\lambda_B}{\partial\varepsilon^2} \right) \\ &= \frac{1}{2} \frac{\partial[\lambda_B(1 - P_\varepsilon)]}{\partial\varepsilon} \\ &= \frac{1}{2} k_\varepsilon(1 - p_\varepsilon) - \frac{1}{2} \lambda_B \frac{\partial P_\varepsilon}{\partial\varepsilon} \\ &= \frac{1}{2} \lambda_B(1 - p_\varepsilon)^2 - 2P_\varepsilon \Lambda \frac{\partial n_{\text{eff}}}{\partial\varepsilon}, \end{aligned} \quad (12)$$

$$\begin{aligned}
k_{T^2} &= \frac{1}{2} \left(\frac{\partial^2 \lambda_B}{\partial T^2} \right) \\
&= \frac{1}{2} \frac{\partial [\lambda_B(\alpha + \beta)]}{\partial T} \\
&= \frac{1}{2} k_T(\alpha + \beta) + \frac{1}{2} \lambda_B \frac{\partial(\alpha + \beta)}{\partial T} \\
&= \frac{1}{2} \lambda_B(\alpha + \beta)^2 + \frac{1}{2} \lambda_B \left(\frac{\partial \alpha}{\partial T} + \frac{\partial \beta}{\partial T} \right), \quad (13)
\end{aligned}$$

$$\begin{aligned}
k_{\varepsilon T} &= \frac{\partial^2 \lambda_B}{\partial \varepsilon \partial T} \\
&= \frac{\partial k_\varepsilon}{\partial T} \\
&= \frac{\partial [\lambda_B(1 - P_\varepsilon)]}{\partial T} \\
&= \lambda_B(\alpha + \beta)(1 - P_\varepsilon). \quad (14)
\end{aligned}$$

Obviously, Eq. (11), as the final expression for a modified dual-wavelength matrix calculation method, has more precision than Eq. (10), because the two-level items in Eq. (4) are considered.

3 Simulation result and analysis

Now the reflectivity of the FBG can be calculated by traditional dual-wavelength matrix method and modified dual-wavelength matrix method, respectively. In normal germanium-doped silica fiber,

$$\begin{aligned}
p_{11} &= 0.113, & p_{12} &= 0.252, \\
\nu &= 0.16, & n_{\text{eff}} &= 1.482, \\
\alpha &= 0.55 \times 10^{-6}, & \beta &= 8.6 \times 10^{-6}.
\end{aligned}$$

Suppose the original Bragg wavelength of FBG is $\lambda_B = 1550$ nm, the stress change is $\Delta\varepsilon = 500 \times 10^{-6}\varepsilon$, temperature change is $\Delta T = 20^\circ\text{C}$, the simulated result is shown in Fig. 1.

Figure 1(a) is the original reflectivity of FBG; and Fig. 1(b) is the reflectivity calculated by using traditional dual-wavelength matrix method and modified dual-wavelength matrix method, respectively, with the change of stress and temperature. We can find that in Fig. 1(b), the two reflectivity calculated by two methods are almost the same.

If we suppose the stress changing factor is $\Delta\varepsilon = 1500 \times 10^{-6}\varepsilon$, and the temperature change factor is $\Delta T = 60^\circ\text{C}$, the simulated result is shown in Fig. 2.

In Fig. 2(b), we can find that the reflectivity difference which is calculated by two methods is obvious. Comparing Fig. 1 with Fig. 2, we can make a conclusion: when the stress and temperature conditions are changed little, the reflectivity calculated by the modified method is almost the same as the traditional method; when the stress and temperature conditions are changed more, the reflectivity calculated by the modified method is more accurate than the traditional method.

4 Conclusion

In this paper, we modified the traditional dual-wavelength matrix calculation method by using the two-level items in

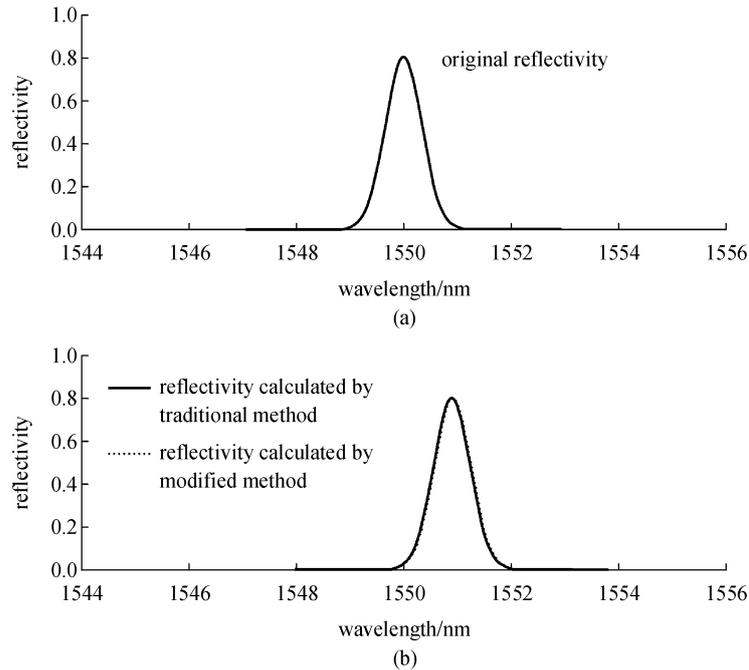


Fig. 1 Reflectivity of FBG with small change condition. (a) Original reflectivity of FBG; (b) traditional dual-wavelength matrix method versus modified dual-wavelength matrix method

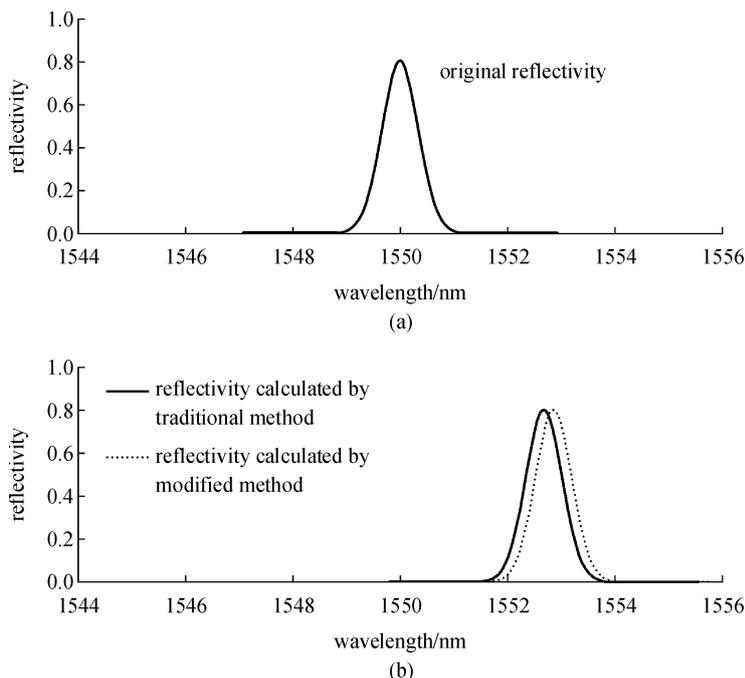


Fig. 2 Reflectivity of FBG with big change condition. (a) Original reflectivity of FBG; (b) traditional dual-wavelength matrix method versus modified dual-wavelength matrix method

Eq. (4) as fix items. The accuracy is improved greatly as shown in Figs. 1 and 2. With the modified method, we can get better results when the stress and temperature conditions are changed more, which can be useful for design in the future.

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