Front. Optoelectron. China 2009, 2(3): 279–284
DOI 10.1007/s12200-009-0018-8

RESEARCH ARTICLE

Novel algorithm for synthesis of fiber gratings

Bo LV (✉), Ming CHEN, Dan LU, Taorong GONG, Tangjun LI, Shuisheng JIAN

Key Laboratory of All Optical Network and Advanced Telecommunication Network of EMC, Beijing Jiaotong University, Beijing 100044, China
Institute of Lightwave Technology, Beijing Jiaotong University, Beijing 100044, China

Abstract A novel algorithm for the synthesis of fiber gratings is presented. For the first time we propose an effective optimal approach to construct a coupling coefficient function by employing 4th-order Runge-Kutta (R-K) analysis method for calculating the reflection spectra of fiber gratings. The numerical results show that with this proposed method, some required optical filters have been yielded with better features compared with other methods such as Gel'Fand-Levitan-Marchenko (GLM) algorithm. In addition, the performance of different interpolation functions particularly utilized in our algorithm, including linear-type, spline-type, and Hermit-type, are discussed in detail.

Keywords gratings, inverse problem, synthesis algorithm, coupling coefficient function, interpolation functions, apodization

1 Introduction

It is known that high-speed optical fiber communication systems rely critically on the design of complex filters to perform various functions such as dispersion compensation [1], distributed feedback lasing [2], optical filtering [3], in codirectional couplers [4], etc. Several experimental techniques have been demonstrated to fabricate nonuniform gratings, permitting an accurate control of both the local grating pitch and the apodization profile along the structure [5–7]. These techniques give substantial flexibility to the grating design process. In this process, a synthesis is to determine a fiber grating index modulation profile corresponding to the given or wanted optical reflection spectrum.

To solve the problem, a variety of synthesis algorithms has been proposed [8–11]. The simplest approach, called Born approximation, exploits the approximate Fourier transform [12] relationships between the filter spectral response and the coupling function. This method is so fast but has limited use for the design of high-reflectivity gratings. The second group of inverse scattering methods is that of exact solutions to integral equations known as Gel'Fand-Levitan-Marchenko (GLM) integral equations. This solution can yield exact and smooth coupling coefficients but is restricted to only the rational function. Finally, there exists a third group of exact inverse scattering algorithms using genetic algorithm [13], by which the coupling coefficient function is sampled and optimized according to some weighting mechanism. This approach provides a general way but still has some drawbacks. The critical disadvantage is that calculation speed strictly depends on the sampling number of coupling coefficient function and sometimes it is difficult to converge in solving the equations. In this paper we present a novel approach to the solution of inverse problems, in particular the problem of synthesizing fiber gratings. In essence it provides a new way to construct a coupling coefficient function with some interpolation functions and reveals better performance than that of traditional GLM solutions.

The outline of this paper is as follows: Sect. 2 contains the principle of analytical method with Riccati equation, and in Sect. 3 the synthesis algorithm is presented in detail. Section 4 contains the numerical results and practical applications, and finally the main conclusions are summarized in Sect. 5.

2 Analysis method

In waveguides whose refractive index profile varies along the direction of propagation (z coordinate), scattering among the modes that travel in the waveguide produces power exchange that can be expressed as a set of coupled wave equations [14]. The resulting coupling potential can be modulated by a quasi-sinusoidal function, and we can write

Received December 12, 2008; accepted February 18, 2009

E-mail: fengdieer_13@126.com

$$k(z) = n_0 + \Delta n(z)\cos\left(\frac{2\pi}{\Lambda}z + \theta(z)\right), \tag{1}$$

where the functions $\triangle n(z)$ and $\theta(z)$ are slowly varying compared with the grating period Λ. In particular, the forward propagating wave v_1 and the backward propagating wave v_2 of the only fundamental mode are related by the coupled mode equations [14] under signal mode operation:

$$\begin{aligned}\frac{\partial v_1(z,\delta)}{\partial z} &= -\mathrm{i}\delta v_1 + q(z)v_2,\\ \frac{\partial v_2(z,\delta)}{\partial z} &= \mathrm{i}\delta v_2 + q(z)v_1.\end{aligned} \tag{2}$$

In Eq. (2), $q(z)$ is defined as the complex coupling coefficient:

$$q(z) = \frac{-\mathrm{i}\pi}{2n_0\Lambda}\Delta n(z)\exp[-\mathrm{i}\theta(z)], \tag{3}$$

and δ denotes the phase shift per unit length compared with the Bragg wavelength ($\lambda_B = 2n_0\Lambda$):

$$\delta = \beta - \beta_B = \frac{2\pi n_0}{\lambda} - \frac{\pi}{\Lambda}. \tag{4}$$

The local reflection coefficient can be further defined as

$$\rho(z,\delta) = \frac{v_2(z,\delta)}{v_1(z,\delta)}. \tag{5}$$

By calculating $\frac{\partial\rho}{\partial z}$ in Eq. (5) combining with Eq. (2), we obtain the well-known Riccati equation:

$$\frac{\partial\rho}{\partial z} = 2\mathrm{i}\delta\rho - q(z)\rho^2 + q(z). \tag{6}$$

This differential equation can be numerically solved for the reflection coefficient with 4th-order Runge-Kutta (R-K) algorithm with the boundary condition $\rho(L,\delta) = 0$. Accordingly, the approximate results can be solved for the reflection coefficient $r(\delta) = \rho(0,\delta)$ and furthermore the group delay function $\tau(\lambda)$ and corresponding dispersion function $D(\lambda)$ can be acquired as

$$\tau(\lambda) = -\frac{\lambda^2}{2\pi c}\frac{\mathrm{d}\theta_\rho}{\mathrm{d}\lambda}, \tag{7}$$

$$D(\lambda) = \frac{\mathrm{d}\theta_\rho}{\mathrm{d}\lambda}, \tag{8}$$

where θ_ρ stands for the phase of reflection coefficient $r(\delta)$.

In Ref. [13], a key issue about the numerical error lies on the number of samples of coupling coefficient function $q(z)$. If the number is small, there may be an unexpected error and sometimes it is rather difficult to obtain the proper reflection spectrum, whereas the large sampling number may bring a lot of running time and as a result, the process is indeed low-efficiency. To address this problem, we mainly make two modifications as proposed in our novel algorithm. One is the construction of coupling coefficient function $q(z)$ by employing sorts of interpolating functions, such as Hermit, cubic, spline, etc. The other improvement is to define the searching space and related searching rule to optimize the reflection coefficient close to the target spectrum. The specification of synthesis algorithms will be introduced in Sect. 3.

3 Synthesis algorithms

From Eq. (6), the objective is to search for the optimal coupling coefficient function $q(z)$ that can produce the best results as close as possible to the given reflection characters of fiber gratings. In this paper, we divide our synthesis algorithms into four steps as below.

1) Construction of searching matrix. We define the searching space $\boldsymbol{S}(z_1, z_2, ..., z_N)$, described by an $M \times N$ matrix as

$$\boldsymbol{S}(z_1,z_2,...,z_N) = (s(z_1), s(z_2), ..., s(z_N)) = \begin{pmatrix} q_1^1(z_1) & \cdots & q_1^N(z_N) \\ \vdots & \ddots & \vdots \\ q_M^1(z_1) & \cdots & q_M^N(z_N)\end{pmatrix}_{M\times N}, \tag{9}$$

where the element $q_j^i(z_i)$ denotes the jth random value selected from the ith column of matrix $\boldsymbol{S}$, $(q_1^i(z_i), q_2^i(z_i), \dots, q_j^i(z_i), q_{j+1}^i(z_i), \dots, q_M^i(z_i))$, at the length position z_i of grating.

2) Interpolations. Given a sample input vector $\boldsymbol{Q} = (q_{j_1}^1(z_1), q_{j_2}^2(z_2), \dots, q_{j_k}^k(z_k), \dots, q_{j_N}^N(z_N))$, we can produce the new testing coupling coefficient function $q(z)$ with certain interpolation function, of which the type could be Hermit, linear, or spline, etc. After this process, the exact testing $q(z)$ can be utilized into the Riccati equation and the corresponding reflection spectrum can be produced as well.

3) Evaluation function. There are several ways to measure the distance between the calculated reflection spectrum and the objective reflection spectrum. Assuming that $R = |r|^2$ refers to the reflectivity spectrum, several evaluation functions are introduced as

$$E_{\text{sum}} = \sum_j (R^p_{\text{target},j} - R^p_{\text{calc},j}), \tag{10}$$

$$E_p = \left[\sum_j (|R^p_{\text{target},j} - R^p_{\text{calc},j}|)^p\right]^{1/p}, \tag{11}$$

$$E_{\max} = \max_j (|R_{\text{target},j} - R_{\text{calc},j}|), \tag{12}$$

where R_{target} and R_{calc} refer to the target and calculation reflection of grating, respectively.

4) Reconstructions. If the evaluation value is unsatisfied, in general, it is necessary to select another input vector, $\boldsymbol{Q}' = (q^1_{j'_1}(z_1), q^2_{j'_2}(z_2), \dots, q^k_{j'_k}(z_k), \dots, q^N_{j'_N}(z_N))$, instead of the former vector $\boldsymbol{Q} = (q^1_{j_1}(z_1), q^2_{j_2}(z_2), \dots, q^k_{j_k}(z_k), \dots, q^N_{j_N}(z_N))$. We define this process as reconstructions.

The detailed synthesis algorithm is described as follows:

a. Begin;

b. Let $k = 1$;

c. Select an initial testing vector $\boldsymbol{Q} = (q^1_{j_1}(z_1), q^2_{j_2}(z_2), \dots, q^k_{j_k}(z_k), \dots, q^N_{j_N}(z_N))$ at random from the searching space $(s(z_1), s(z_2), \dots, s(z_N))$;

d. While ($s(z_k)$ is not empty)

e. Expand $\boldsymbol{Q}$ to $q(z)$ with a chosen interpolation function;

f. Solve the Riccati equation with 4th-order Runge-Kutta algorithm;

g. Compute the specialized evaluation function E;

h. $s(z_k) = s(z_k) - \{q^k_{j_1}(z_k)\}$;

i. Reconstruct the testing vector $\boldsymbol{Q}' = (q^1_{j'_1}(z_1), q^2_{j'_2}(z_2), \dots, q^k_{j'_k}(z_k), \dots, q^N_{j'_N}(z_N))$ and repeat steps e, f, g to get its corresponding E';

j. If $(E' < E)$ then update $\boldsymbol{Q}$ replacing by $\boldsymbol{Q}'$; else remain vector $\boldsymbol{Q}$;

k. $s(z_k) = s(z_k) - \{q^k_{j'_1}(z_k)\}$;

l. Repeat the steps i, j, k until $s(z_k) = \phi$;

m. Mark the optimal testing vector $\boldsymbol{Q} = (q^1_{j^*_1}(z_1), q^2_{j_2}(z_2), \dots, q^k_{j_k}(z_k), \dots, q^N_{j_N}(z_N))$;

n. $k = k + 1$;

o. Repeat the steps c to m until $k = M + 1$;

p. Find the final vector $\boldsymbol{Q}^* = (q^1_{j^*_1}(z_1), q^2_{j^*_2}(z_2), \dots, q^k_{j^*_k}(z_k), \dots, q^N_{j^*_N}(z_N))$ and approximate coupling coefficient function $q(z)$;

q. End.

4 Numerical examples and discussion

In the subsequent sections, the method here developed will be applied to several numerical examples in order to check its validity and grade of accurateness.

4.1 Bandpass filter

An interesting application of the synthesis algorithm is to design a fiber-based optical bandpass filter. In this paper, if we assume that the required bandwidth is 0.2 nm and the target bandpass filter is square-shaped, we can apply our synthesis algorithm to build the filters. The detailed stimulated parameters are presented in Table 1. It is noted that the testing coupling coefficient function can be generated from testing vector $\boldsymbol{Q}$ by means of different interpolation functions. Consequently, corresponding reflection functions can be computed as well. Figure 1 shows the reflectivity curves versus wavelength via several kinds of interpolation functions, e.g., linear, spline, and Hermit. Also, Figs. 1 and 2 show the reflectivity and related coupling coefficient from GLM algorithm [9] with six iterations.

The computing results in Fig. 1 show the reflection spectra. We can conclude that 1) the linear-type interpolation $q(z)$ yields the unqualified bandpass filter with less than 0.2 nm bandwidth whereas the spline-type interpolation one provides the contrary character, which is more than 0.2 nm bandwidth; 2) the Hermit-type interpolation $q(z)$ has better performance than the traditional coupling coefficient function from 6th-order GLM algorithm, which verified the effectiveness and somehow merit of our synthesis algorithm. By using our algorithm, the optical bandpass filter with almost reflectivity of 99% can be finally obtained.

4.2 Bandpass filter with low dispersion

Once the potential of this technique has been demonstrated, we can take a practical problem as an example, namely the fiber-based optical bandpass filter with low dispersion. In this case we assume the bandwidth of a rectangular bandpass filter is still 0.2 nm and also weight the evaluation with the dispersion factor in Eqs. (7) and (8). As a result, the calculated coupling coefficient is shown in Fig. 3, and the in-band dispersion and reflection spectra are illustrated in Figs. 4 and 5, respectively.

In Fig. 4, the spline-type interpolation $q(z)$ can yield the lowest in-band dispersion value. The linear-type interpolation one shows its worst curve of inside dispersion because the corresponding coupling coefficient function $q(z)$ shown in Fig. 3 is not so smooth which can seriously influence some sensitive characters of fiber gratings such as group delay, dispersion, dispersion slope, etc. On the other hand, there is a trade-off that the reflectivity spectrum is broadened and the outside ripple is higher than the results of pure bandpass-required filters.

Table 2 shows the simulated results with different interpolation functions such as spline, Hermit, and linear, also compared with GLM. The main properties of grating, here e.g., 3-dB bandwidth, separation degree, and max in-band dispersion, are listed in detail. It is concluded that Hermit type is evaluated as the best choice because the corresponding filter has 0.18-nm bandwidth which is

Table 1 Parameters in synthesis algorithm

scale of searching space $\boldsymbol{S}$	number of R-K	range of λ/nm	resolution of λ/nm	evaluation type
$N = 8$ $M = 32$	256	1549.7–1550.3	0.0012	$E_{\max}$

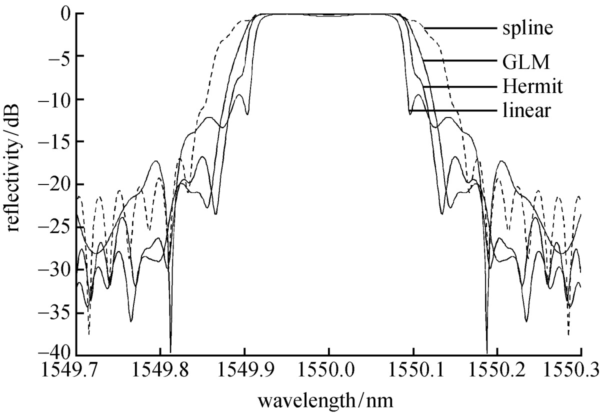

Fig. 1 Calculated reflection spectra with different interpolation functions and GLM

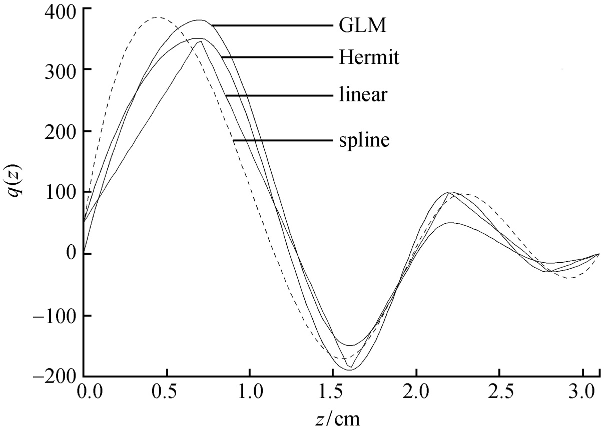

Fig. 2 Corresponding coupling coefficient function $q(z)$ for building bandpass filter

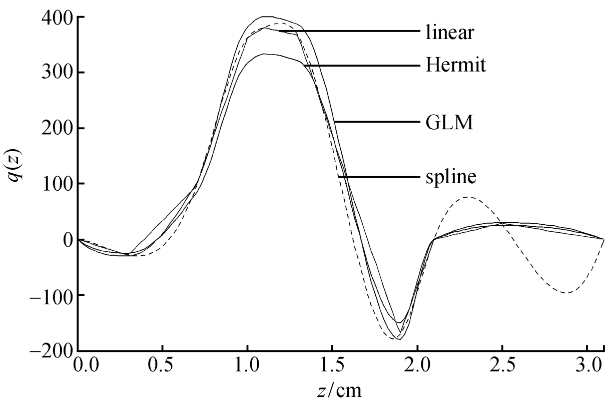

Fig. 3 Coupling coefficient function $q(z)$ for building bandpass filter with low dispersion

closest to the target bandpass filter; moreover, the separation degree is better than the GLM method, and the in-band dispersion is permitted as well. Note that the yielding dispersion is so flat within ± 0.05 nm, but there is still a trade-off between separation degree and flatness of

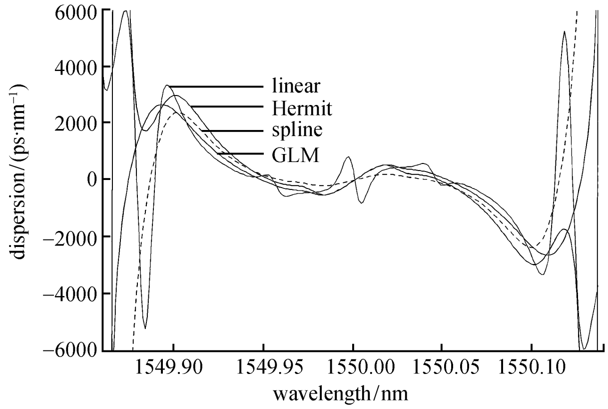

Fig. 4 In-band dispersion for building bandpass filter with low dispersion

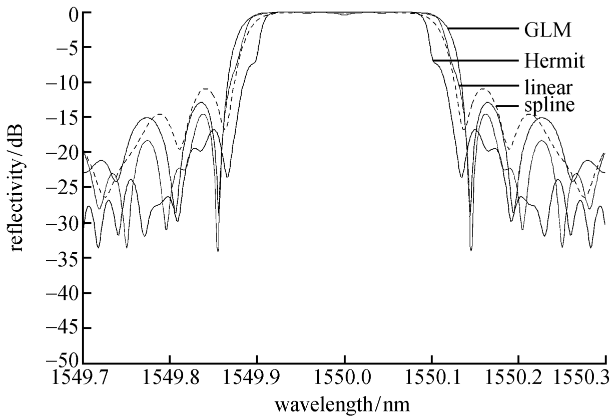

Fig. 5 Calculated reflection spectra for bandpass filter with low dispersion

dispersion. In order to balance the relationship, we can improve the evaluation functions containing both the reflection spectrum and its group delay or dispersion by introducing different weights.

4.3 Practical example of dispersion equalization

One practical example where the synthesis of a linear group delay function required is dispersion equalization, which is the key component for dispersion compensation in long-haul high-capacity fiber optical communication systems. In this paper, we assume that the first-order dispersion compensation parameter $D = 2550\ \mathrm{ps \cdot km^{-1} \cdot nm^{-1}}$ which can be equalized for about 150 km single mode fiber (SMF). The target dispersion filter we designed can be produced by the apodization function as the modulation scheme of refractive index's amplitude. The apodization function $f(z)$ is yielded from Eq. (3), which can be described as

$$\Delta n = \Delta n_0 f(z). \tag{13}$$

Table 2 Numerical results with different interpolation functions and GLM for low-dispersion bandpass filter

type of interpolation function	3-dB bandwidth/nm	separation degree/dB	max in-band dispersion/(ps·nm^{-1})
spline	0.22	11.0	2300
Hermit	0.18	17.5	2800
linear	0.24	14.5	3200
GLM	0.26	13.0	2500

In this paper, given that $\Delta n_0 = 10^{-4}$ and the length of grating fiber $L = 14$ cm, with our synthesis method the final apodization function as shown in Fig. 6(a) is obtained and the corresponding apodization board has been manufactured as well.

The chirped Bragg grating is fabricated with the ultra-violet exposure method by employing phase mask [15–18]. Figure 6(b) shows the practical reflection spectrum and group delay measured with CD400 provided by PerkinElmer company (modulated frequency is 70 MHz). The measured dispersion is approximately 2650 $\mathrm{ps \cdot km^{-1} \cdot nm^{-1}}$ and the 3-dB bandwidth of reflection spectrum is 0.33 nm which can be used for the 10 Gbit/s dispersion compensated system [19–21].

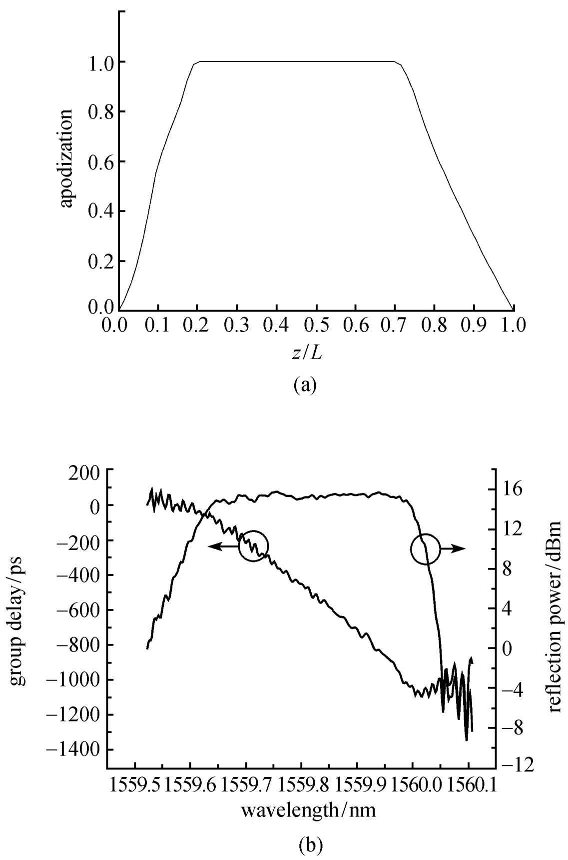

Fig. 6 Practical dispersion grating. (a) Manufactured apodization board with synthesis method; (b) measured reflection spectrum and group delay of chirped fiber Bragg grating

It is obvious that the main trouble with our method is the selection of the proper interpolation function. In the process of designing the bandpass optical filter only, the Hermit-type interpolation shows its best performance whereas the spline-type interpolation can be evaluated as the optimal choice for the design of bandpass filter with low dispersion. An example of fabricated dispersion equalization is presented to verify the effectiveness of synthesis method as well. Nevertheless, the distribution of searching space $\boldsymbol{S}$ could also significantly affect the running time and even the computing error. Generally speaking, it has proven to be successful for the synthesis of many sorts of transfer function, also preserving the phase response as shown in Sect. 4.2. We can freely design the reflectance so that particular requirements are satisfied, which are its unique merits compared with the GLM method.

5 Conclusion

A novel method to solve the synthesizing fiber gratings is proposed. This algorithm particularly defines the rule of optimization for coupling coefficient function by employing the 4th-order Runge-Kutta analysis method. Furthermore, the type of interpolation function, such as linear, spline, and Hermit, has been taken into account as well. The numerical examples presented show that this synthesis algorithm may reveal some merits compared with the GLM method. One obvious improvement in comparison with the GLM method is the preservation of the phase information. It is also worth mentioning that some issues, such as the selection of different interpolation function and the construction of well-balanced searching space $\boldsymbol{S}$, should be investigated in future research.

Acknowledgements This work was supported by the Natural Science Foundation of Beijing (Grant No. 4062027) and the National High Technology Research and Development Program (863) of China (No. 2007AA01Z258).

References

1. Ouellete F. Dispersion cancellation using linearly chirped Bragg grating filters in optical waveguides. Optics Letters, 1987, 12(10): 847–849
2. Sekartedjo K, Eda N, Furuya K, Suematsu Y, Koyama F, Tanbun-Ek

T. 1.5 μm phase-shifted DFB lasers for single-mode operation. Electronics Letters, 1984, 20(2): 80–81

3. Yu Q, Pan Z Q, Yan L S, Willner A E. Chromatic dispersion monitoring technique using sideband optical filtering and clock phase-shift detection. Journal of Lightwave Technolgy, 2002, 20 (12): 2267–2271
4. Bornholdt C, Kappe F, Müller R, Nolting H P, Reier F, Stenzel R, Venghaus H, Weinert C M. Meander coupler, a novel wavelength division multiplexer/demultiplexer. Applied Physics Letters, 1990, 57(24): 2517–2519
5. Loh W H, Cole M J, Zervas M N, Barcelos S, Laming R I. Complex grating structures with uniform phase masks based on the moving fiber-scanning beam technique. Optics Letters, 1995, 20(20): 2051–2053
6. Asseh A, Storoy H, Sahlgren B E, Sandgren S, Stubbe R A H. A writing technique for long fiber Bragg gratings with complex reflectivity profiles. Journal of Lightwave Technolgy, 1997, 15(8): 1419–1423
7. Paster D, Capmany J, Ortega D, Tatay V, Marti J. Design of apodized linearly chirped fiber gratings for dispersion compensation. Journal of Lightwave Technology, 1996, 14(11): 2581–2588
8. Winick K A, Roman J E. Design of corrugated waveguide filters by Fourier-transform techniques. IEEE Journal of Quantum Electronics, 1990, 26(11): 1918–1929
9. Peral E, Capmany J, Marti J. Iterative solution to the Gel'Fand-Levitan-Marchenko coupled equations and application to synthesis of fiber gratings. IEEE Journal of Quantum Electronics, 1996, 32 (12): 2078–2084
10. Feced R, Zervas M N, Muriel M A. An efficient inverse scattering algorithm for the design of nonuniform fiber Bragg gratings. IEEE Journal of Quantum Electronics, 1999, 35(8): 1105–1115
11. Brinkmeyer E. Simple algorithm for reconstructing fiber gratings from reflectometric data. Optics Letters, 1995, 20(8): 810–812
12. Dobrowolski J A, Lowe D. Optical thin film synthesis program based on the use of Fourier transforms. Applied Optics, 1978, 17 (19): 3039–3050
13. Skaar J, Risvik K M. A genetic algorithm for the inverse problem in synthesis of fiber gratings. Journal of Lightwave Technology, 1998, 16(10): 1928–1932
14. Erdogan T. Fiber grating spectra. Journal of Lightwave Technology, 1997, 15(8): 1277–1294
15. Ning T G, Liu Y, Tan Z W, Pei L, Jian S S. Impact on dispersion compensated system using chirped fiber Bragg grating with delay ripple. Acta Optica Sinica, 2003, 23(9): 1064–1067 (in Chinese)
16. Ning T G, Pei L, Liu Y, Tan Z W, Tong Z, Yan F P, Jian S S. Optimization of electron-beam-written step-chirped phase masks to improve quality of the chirped FBG. Journal of Optoelectronics · Laser, 2006, 17(12): 1409–1412 (in Chinese)
17. Tan Z W, Li B, Wang Y H, Ren W H, Liu Y, Ning T G, Jian S S. Background loss of the fiber induced by the ultraviolet light exposure. Chinese Journal of Lasers, 2007, 34(2): 239–242 (in Chinese)
18. Xie Z H, Ning T G, Pei L, Li T J, Jian S S. Fabrication of 13 cm linearly chirped fiber grating. Acta Optica Sinica, 2001, 21(11): 1381–1383 (in Chinese)
19. Tan Z W, Ning T G, Liu Y, Tong Z, Jian S S. Suppression of the interactions between fibre gratings used as dispersion compensators in dense wavelength-division multiplexing systems. Chinese physics, 2006, 15(8): 1819–1823
20. Jian S S, Zhao Y C, Wei D P. Dispersion compensation experimental on 19 ps pulse width over 133 km single mode fiber with tunable chirp and wavelength gratings. Science in China E: Technological Sciences, 1999, 42(2): 165–170
21. Liu X, Wei X, Gnauck A H, Doerr C R, Chandrasekhar S. Analysis of loss ripple and its application to the mitigation of optical filtering penalty. IEEE Photonics Technology Letters, 2005, 17(1): 82–84