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RESEARCH ARTICLE

Analysis of mode-field half widths of two kinds of single-mode waveguides

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Abstract The mode-field half widths of two kinds of single-mode waveguides are investigated. Based on the maximal matching efficiency method, the relationship between the mode-field half width and normalized frequency is analyzed. Furthermore, two equations of mode-field half widths as a function of normalized frequency are proposed through mathematical modeling and the curve fitting method. Numerical calculations indicate that they are accurate within a parameter range.

Keywords mode-field half width, Gaussian approximation, normalized frequency, optical waveguide, matching efficiency

1 Introduction

For the symmetrical step refractive index single-mode optical waveguide, Gaussian approximation is widely applied because the concept of the Gaussian beam and the computing mean of Fourier transformation for the Gaussian function are classical scientific ideas [–6]. The key here is how to ascertain its mode-field half width. Moreover, as one of the most important parameters for single-mode optical waveguide, the mode-field half width is closely related to splice loss, micro-bending loss, two waveguides, source-to-fiber coupling efficiency, and so on [7,8]. Thus, research on mode-field half widths is significant for practical applications.

For an arbitrary planar single-mode waveguide, Liang et al. proposed two mode-field half width definitions based on the near- and far-field second moments [8,9]. Two well-known mode-field radius definitions were proposed by Petermann [10,11] based on the near-field second moment and the far-field second moment for a circular symmetric single-mode fiber. Our research results indicate that the two kinds of mode-field half widths are not optimal if they are used as mode-field half widths of the Gaussian field.

Owing to the special nature of normalized frequency, the relationship between beam parameters and normalized frequency is studied [12,13]. Especially, two empirical formulas of mode-field half widths as a function of normalized frequency, initially presented by Marcuse, are used widely [7,14,15]. However, research indicates that his formula for the mode-field half width of a planar symmetric step index single-mode waveguide has a big deviation compared to the theoretical value.

In this paper, we analyze the characteristic of the matching efficiency between the eigen mode-field of a single-mode waveguide and its Gaussian approximation by numerical calculation. The numerical relationship between the mode-field half width and normalized frequency is discussed according to the maximal matching efficiency method. For the convenience of analysis and calculation, the condition of the mode-field half width as a function of normalized frequency is established through mathematical modeling, and two expressions are presented based on the curve fitting method.

2 Mode-field half width of a planar symmetric step index single-mode waveguide

For a planar symmetric step refractive index single-mode waveguide, its fundamental mode, TE_0 mode, is described as follows [16]:

$$\Psi_{\text{eigen}}(x) = \cos\frac{Ux}{a}, \quad |x| \leqslant a, \tag{1}$$

$$\Psi_{\text{eigen}}(x) = \cos U \exp\left[-W\left(\left|\frac{x}{a}\right| - 1\right)\right], \quad |x| > a. \tag{2}$$

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For the convenience of analysis but without discounting validity, the $\exp[i(\beta z - \omega t)]$ item in the above formula is omitted, where

$$U = a[(k_0 n_1)^2 - \beta^2]^{1/2},$$

$$W = a[\beta^2 - (k_0 n_2)^2]^{1/2}$$

are the normalized standing wave parameter and the normalized evanescent wave parameter, respectively; $k_0 = 2\pi/\lambda$ is the wave number in vacuum, β is the propagation constant, λ is the wavelength of an electromagnetic wave in vacuum, a is the half thickness of the core layer, and n_1 and n_2 are the refractive indices of core and cladding, respectively.

For the sake of convenience, the field distribution of the TE_0 mode is often approximated by the Gaussian shape

$$\Psi(x) = \exp\left(-\frac{x^2}{\omega_j^2}\right), \tag{3}$$

where ω_j is mode-field half width, the suffix j refers to SM, DO, M and G, respectively, and they denote the relationship to the definitions of the mode-field half width.

There are two kinds of definitions of mode-field half width, that is, the second moment mode-field half width ω_{SM} and the differential operator mode-field half width ω_{DO}[17]:

$$\omega_{SM} = a\left[2 + \frac{2}{W} + \frac{2}{W^2} - \frac{2}{U^2} - \frac{2W}{3(1+W)}\right]^{1/2}, \tag{4}$$

$$\omega_{DO} = a\left[\frac{1+W}{U^2 W}\right]^{1/2}. \tag{5}$$

Marcuse presented a formula for the mode-field half width as a function of normalized frequency. Let us call it Marcuse's mode-field half width ω_M[7]:

$$\omega_M = a\left[0.00460313 + \frac{1.22083}{\sqrt{V}} + \frac{1.53597}{V^3} - \frac{0.146705}{V^5}\right]. \tag{6}$$

For example, we set $n_1 = 1.4493$, $n_2 = 1.4440$, $\lambda = 1.55$ μm and $a = 3$ μm, then we can get that $V = 1.5053$, $U = 0.9163$, $W = 1.1942$, $\omega_{SM} = 4.5815$ μm, $\omega_{DO} = 4.4380$ μm and $\omega_M = 4.2929$ μm.

Numerical data show that they have some differences. It is difficult for us to decide which of them should be the mode-field half width so that the Gaussian field can be closer to its principal field. Furthermore, our research indicates that they are not optimal based on matching efficiency.

The matching efficiency can be evaluated through an overlap integral [18]:

$$\eta = \frac{\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_1(x,y)\Psi_2(x,y)\mathrm{d}x\mathrm{d}y\right]^2}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_1^2(x,y)\mathrm{d}x\mathrm{d}y \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_2^2(x,y)\mathrm{d}x\mathrm{d}y}, \tag{7}$$

where $\Psi_1(x)$ and $\Psi_2(x)$ refer to the eigen mode-field and its Gaussian field, respectively.

For a planar symmetric step refractive index single-mode waveguide, the two-dimension in Eq. (7) can be simplified into one-dimension. Substituting Eqs. (1)–(3) into Eq. (7), we can obtain matching efficiency η between $\Psi_{eigen}(x)$ and $\Psi(x)$, displayed in Fig. 1. It shows that the matching efficiency η varies with the increasing mode-field radius ω, and when $\omega = \omega_G = 4.5000$ μm, the matching efficiency reaches the maximum, $\eta_{max} = 99.66\%$. Comparing $\omega_{SM} = 4.5815$ μm, $\omega_{DO} = 4.4380$ μm and $\omega_M = 4.2929$ μm, it is clear that they have a certain deviation from ω_G, and Marcuse's mode-field half width ω_M has a large difference with ω_G. So only the mode-field half width ω_G obtained by the maximal matching efficiency method is optimal.

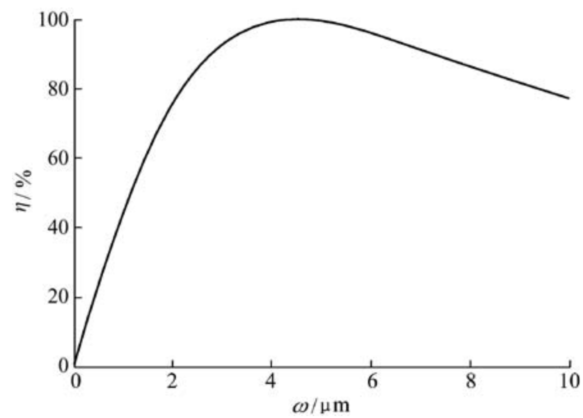

Fig. 1 Characteristic curve of matching efficiency

Through a similar procedure, a relational curve between mode-field half width ω_G and normalized frequency V is obtained from the maximal matching efficiency method as the solid line shown in Fig. 2, and it can be seen that the mode-field half width ω_G decreases with increasing normalized frequency V.

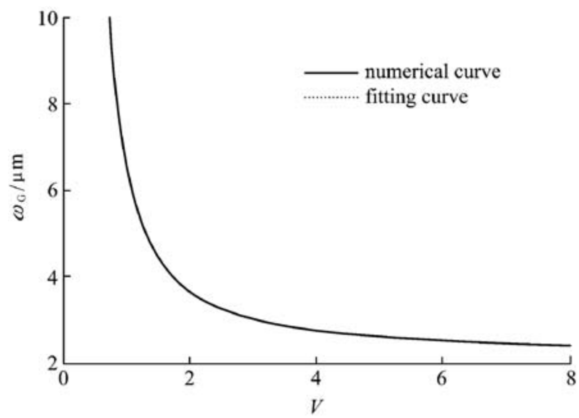

Fig. 2 Relational curve of mode-field half width

For the convenience of calculation and analysis, we would like to establish an equation between the mode-field half width and the normalized frequency. Through mathematical modeling and combining with the curve fitting method, an equation of the mode-field half width as a function of normalized frequency is obtained as follows:

$$\omega_G = 5.021\exp(0.6396V^{-1.256}) - 2.85. \quad (8)$$

From Eq. (8), the dotted line is displayed in Fig. 2 and it is clear that it is in excellent agreement with the solid line. Furthermore, we calculated the relative error between the numerical data and that from Eq. (8) to explain the accuracy of Eq. (8). The relative error based on its definition can be expressed as follows:

$$\alpha = \frac{B-A}{A} \times 100\%, \quad (9)$$

where A is the numerical data from the maximal matching method and B is the calculated result by Eq. (8).

The numerical calculation has shown that the relative error undulates along with then increase in normalized frequency, but α varies within $\pm 1\%$ as V changes from 0.74 to 12. Therefore, Eq. (8) is accurate within a reasonable parameter range.

For comparison purposes, the matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_G)$ and $\eta(\Psi_{\text{eigen}}, \Psi_M)$ are calculated, where $\Psi_G(x) = \exp(-x^2/\omega_G^2)$ and $\Psi_M(x) = \exp(-x^2/\omega_M^2)$. The results are displayed in Fig. 3.

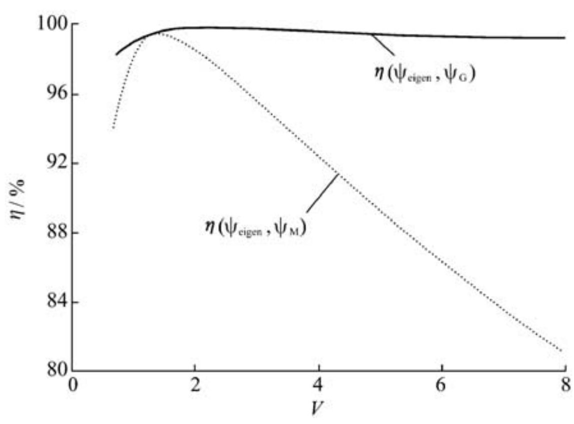

Fig. 3 Comparative curves of two kinds of matching efficiencies

As shown above, we can see that the two lines have a large difference. The calculative results indicate that except for a small range $1.17 < V < 1.88$, $\eta(\Psi_{\text{eigen}}, \Psi_M) \geqslant 99\%$, matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_M)$ is not good, however, matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_G)$ satisfies compared with $\eta(\Psi_{\text{eigen}}, \Psi_M)$, and it always satisfies $\eta(\Psi_{\text{eigen}}, \Psi_G) \geqslant 99\%$ within the range $1.04 < V < 21.9$. Thus, ω_G is more accurate than ω_M if they are used for the mode-field half width in the Gaussian approximate.

3 Mode-field radius of a circular-symmetric step index single-mode fiber

As for the circular-symmetric step index optical fiber, under weakly guiding approximation, the $\exp[i(\beta z - \omega t)]$ item in the following formula is omitted, and its fundamental mode, LP_{01} mode, can be expressed as follows [16]:

$$\Psi_{\text{eigen}}(r) = J_0\left(U\frac{r}{a}\right), \quad r \leqslant a, \quad (10)$$

$$\Psi_{\text{eigen}}(r) = \frac{J_0(U)}{K_0(W)} K_0\left(W\frac{r}{a}\right), \quad r > a. \quad (11)$$

Similarly, the field distribution of the LP_{01} mode is often expressed approximately by the Gaussian field as

$$\Psi(r) = \exp\left(-\frac{r^2}{\omega_j^2}\right), \quad (12)$$

where ω_j is mode-field radius, the suffix j is SM, DO, M and G, respectively, and they denote the relationship to the definitions of the mode-field radius.

The problem is how to obtain an accurate mode-field radius. There is a second moment mode-field radius ω_{SM} and a differential operator mode-field radius ω_{DO} [19] also, where

$$\omega_{SM} = \frac{\sqrt{6}a}{3}\left[1 - \frac{2}{U^2} + \frac{2}{W^2} + \frac{2J_0(U)}{UJ_1(U)}\right]^{1/2}, \quad (13)$$

$$\omega_{DO} = \frac{\sqrt{2}aJ_1(U)}{WJ_0(U)}. \quad (14)$$

In the same way, Marcuse presented a formula for the mode-field radius as a function of the normalized frequency ω_M[14]:

$$\omega_M = a\left[0.65 + 1.619V^{-3/2} + 2.879V^{-6}\right]. \quad (15)$$

Taking an example, we assume that the basic parameters of an optical fiber are as follows: the refractive index of the core layer $n_1 = 1.4569$, the refractive index of the cladding layer $n_2 = 1.4440$, the core layer radius $a = 3$ μm, the wavelength in vacuum $\lambda = 1.55$ μm, the normalized frequency $V = 2.3539$, the normalized standing wave parameter $U = 1.6335$ and the normalized evanescent wave parameter $W = 1.6949$.

From Eqs. (13)–(15), we can also obtain the three kinds of mode-field radii $\omega_{SM} = 3.3576$ μm, $\omega_{DO} = 3.2878$ μm and $\omega_M = 3.3457$ μm. They are all different from each other; furthermore, referring to the matching efficiency between the field of the LP_{01} mode and its Gaussian field, they are not optimal either.

Substituting Eqs. (10)–(12) into Eq. (7), we can obtain a characteristic curve for the matching efficiency η between the field of the LP_{01} mode and its Gaussian field. It is displayed in Fig. 4.

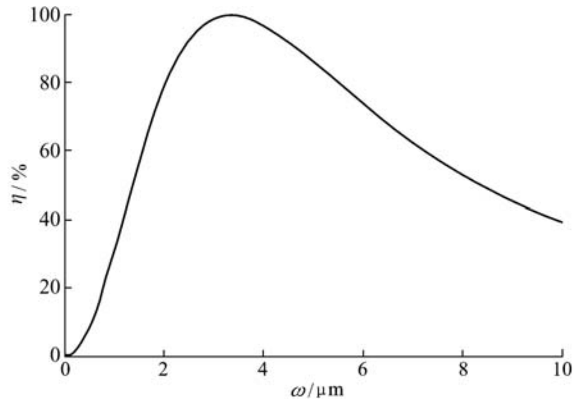

Fig. 4 Characteristic curve of matching efficiency

It is obvious that the matching efficiency η varies with increasing ω and η to the maximum $\eta_{\max} = 99.63\%$ while $\omega_G = 3.3193$ μm. Compared with the three kinds of mode-field radii $\omega_{SM} = 3.3576$ μm, $\omega_{DO} = 3.2878$ μm and $\omega_M = 3.3457$ μm, we can draw the conclusion that they are not optimal. Only if the matching efficiency is maximal will the mode-field radius be optimal.

We can also obtain all the numerical data on mode-field radius ω_G along with the increased V based on the maximal matching efficiency method. The solid line in Fig. 5 shows that the mode-field radius ω_G decreases with increasing normalized frequency V, but there is no direct equation between them.

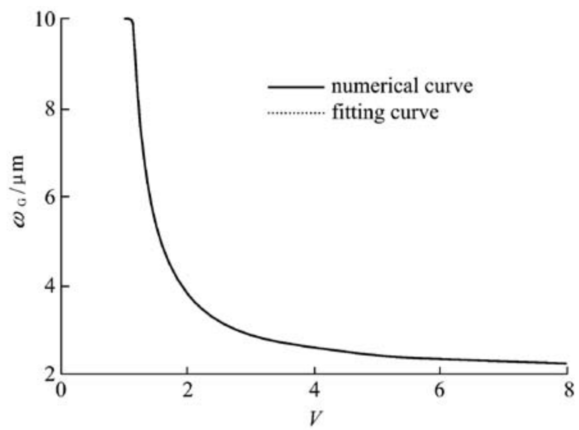

Fig. 5 Relational curve of mode-field radius

Through a similar procedure above, an equation between the mode-field radius and the normalized frequency is introduced by mathematical modeling and combined with the curve fitting method:

$$\omega_G = 0.6818\exp(2.909V^{-1.2}) + 1.367. \qquad (16)$$

A dotted line is displayed in Fig. 5 from Eq. (16), and it is clear that it is in excellent agreement with the solid line. The numerical data has indicated that the relative error α varies with the normalized frequency V, when the normalized frequency satisfies the relation $1.2 < V < 10$ and the maximal relative error is smaller than $\pm 2\%$.

Furthermore, the matching efficiency $\eta(\Psi_{\text{eigen}},\Psi_G)$ as well as $\eta(\Psi_{\text{eigen}},\Psi_M)$ are given to explain the accuracy of Eq. (16), where $\Psi_G(r) = \exp(-r^2/\omega_G^2)$ and $\Psi_M(r) = \exp(-r^2/\omega_M^2)$. The results are displayed in Fig. 6.

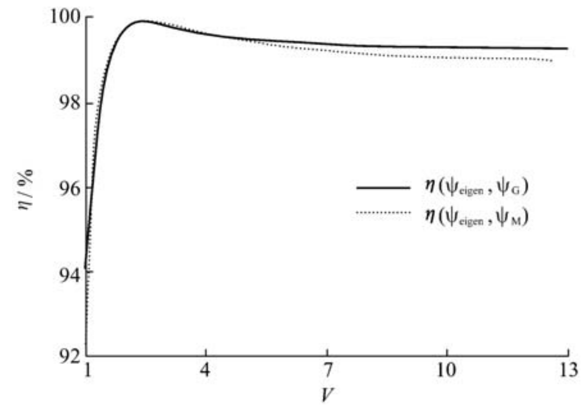

Fig. 6 Comparative curves of two kinds of matching efficiencies

Figure 6 shows that they have some difference, and the matching efficiency $\eta(\Psi_{\text{eigen}},\Psi_G)$ is higher than $\eta(\Psi_{\text{eigen}},\Psi_M)$ as a whole. The numerical calculation indicates that only if $1.58 < V < 15.53$, $\eta(\Psi_{\text{eigen}},\Psi_M) \geqslant 99\%$, however, within the parameter range $V > 1.61$, the matching efficiency $\eta(\Psi_{\text{eigen}},\Psi_G)$ always satisfies $\eta(\Psi_{\text{eigen}},\Psi_G) \geqslant 99\%$. Consequently, ω_G is more apt to be used for the mode-field radius than ω_M is in the Gaussian approximation.

4 Conclusions

For a symmetric step index single-mode optical waveguide, a characteristic curve of the matching efficiency between the eigen mode-field and its Gaussian approximate field is given by the numerical analysis method. Based on that, the numerical relationship between the mode-field half width and the normalized frequency is introduced by using the maximal matching efficiency method. Furthermore, the modeling $\omega = a\exp(bV^c) + d$ is presented and two equations of mode-field half widths as a function of normalized frequency are given according to the curve fitting method. The numerical calculation indicates that they are accurate within a reasonable parameter range. These conclusions may provide theoretical support for quick beam parameters analysis.

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