

Optical synthetic aperture circle-array optimization based on genetic algorithm

Yuntao HE (✉), Yuesong JIANG, Guangda LIU

School of Electronic Information Engineering, Beihang University, Beijing 100191, China

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Abstract An optimization model of circle array was set up from the basic optical synthetic aperture imaging principle. The circle array was optimized by adopting a genetic algorithm with an improved real coding method coding the location of sub-apertures. The measure function was designed based on maximizing the distances between u - v coverage dots and minimizing the redundant array. The point spread function, optical transfer function and diffractive imaging were analyzed with the circle array synthetic aperture imaging system. The optimized result of 8 to 16 sub-apertures on a circle array was obtained, and they were compared to the results achieved through simulated annealing algorithm. Using the emulator program, the point spread function was analyzed and contrasted to that of a uniform circle array. Results show that the real coding genetic algorithm can resolve the array optimization well, cost less time and get a better optimization compared with the simulated annealing algorithm.

Keywords synthetic aperture, array optimization, genetic algorithm, u - v coverage, real coding

1 Introduction

Optical aperture synthesis offers an effective route to achieve high resolution by interferometrically combining the signals from a number of smaller distributed apertures substituting using a single large aperture [1]. Each pair of sub-apertures (or baseline) in the array defines a spatial frequency at which the mutual coherence function of the source is measured, thus an image of the source can be obtained by Fourier transforming these data. Therefore, higher image quality depends on suf-

ficient sampling in the spatial frequency domain which we called “the coverage in u - v plane”, besides using CLEAN and maximum entropy methods (MEM) processing [2]. In order to obtain a u - v coverage measured by the maximum uniformity and the minimum redundancy of spatial frequency, a key method is to introduce advanced random algorithms optimizing the sub-aperture array [3–5]. The circle array, different from other two-dimensional (2D) arrays, could achieve non-redundancy of u - v coverage [6]. At present, array optimization is realized generally by simulated annealing algorithm and synthetic aperture array optimization using genetic algorithm (GA) is not so common. In this paper, the genetic algorithm will be applied to circle array optimization.

The genetic algorithm is probably the most widely known and used type of evolution algorithms for its powerful and comprehensive random searching and optimization. It has been widely used in areas such as adaptive control, combination optimization, pattern recognition, machine learning and artificial biology. genetic algorithm adopts population searching techniques, and generates a new population by applying elements selection, crossovers and mutations to the current population. Therefore, the population could be evolved step-by-step into a status of containing or close to the optimized solution. Encoding is a primary problem and a key step when using GA [7,8]. As the positions of sub-apertures are constrained to an angle range of 0 – 2π in optimizing a circle array of optical synthetic aperture, real-encoding approach could apply solution space to present the search space and improve the calculation efficiency. In this paper, we compared the achieved optimized array with uniform array to study the applicability of GA by analyzing the points spread function [5]. The measure functions were also compared to those using simulated annealing algorithm (SAA) in Ref. [5]. The optimized results are of great reference value to the design of optical and other frequency synthetic aperture array.

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E-mail: taoyunhe@ee.buaa.edu.cn

2 Optimization model for synthetic aperture array

Optical synthetic aperture imaging is based on interferometric imaging. According to Van Cittert Zernike theorem, a mutual intensity (or coherent factor) μ_{12} is equal to the normalized Fourier transform of optical source intensity distribution. The amplitude of interferometric fringes is the amplitude of complex mutual factor and the target's Fourier phase is the phase of complex mutual factor. Each complex mutual factor corresponds to a u - v sampling point (u, v) . There are two approaches to achieve enough u - v points to form a well-covered u - v plane: one approach applies two sub-apertures to sample each u - v point asynchronously and the other approach uses enough sub-apertures to realize full u - v coverage synchronously. Then the target brightness temperature could be recovered by a Fourier transform to the whole u - v sampling plane.

Because the vectors formed by sub-apertures in the entrance pupil plane u - v plane is one-to-one corresponded to the space frequency points of the image, these sub-aperture pairs that have the same vectors $(\Delta x, \Delta y)$ would sample the same space frequency points (u, v) , which is called redundancy, and the array is a redundant array. The redundancy could increase the signal-to-noise ratio (SNR) when the imaging system does not contain optical aberration, but these redundancies do not provide any new information for the imaging system. When the redundancy exists in an aberration system or a nonuniform medium, the redundancy is harmful to the imaging quality. That is because the fringe contrast decays, and the measure precision of signal amplitudes is decreased as the same space frequency points would have a different phase. In order to reduce the redundancies or even to implement a non-redundant array, it is necessary to apply a penalty factor to those redundancies. Based on Cornwell's optimization of correlation array [4–6], the measure function was as

$$E(r_1, r_2, \dots, r_N) = \sum_{i=1}^N \sum_{j=1}^{i-1} \log d_{ij}, \quad (1)$$

$$d_{ij} = \begin{cases} \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2}, & \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2} > \varepsilon, \\ C, & \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2} \leq \varepsilon, \end{cases} \quad (2)$$

where r_i is the i th aperture coordinate; (u_i, v_i) and (u_j, v_j) is the i th and j th u - v point corresponded, respectively; ε is a constant determined by minimum sampling interval; C is a positive number far less than ε and it is assigned a minimum number of the calculation software so that the measure function would be 'weight' to avoid the redundancies. The sub-apertures have the same weight for an array formed by all N identical sub-apertures. The non-redundant points can be $N(N+1)+1$ at most and be distributed

symmetrically in u - v coverage plane since the complex correlation of sub-apertures is also symmetrical. Besides, constraint should be imposed on the position of sub-apertures when Eq. (1) is used as the measure function to obtain homogeneous u - v coverage. For example, the distance of two sub-apertures should be larger than the physical dimension size of the two sub-apertures in an actual project.

3 Genetic algorithm applied in array optimization

GA is based on natural selection and genetic mechanism. Reproductions $P(t)$ (t represents the generation number of evolution) are formed by many individuals which are potential solutions for the optimization problem. Each individual should be measured and yield a fitness value. Some individuals need to suffer random evolution to generate new individuals, which is called genetic operation. These new individuals called offspring $C(t)$ would be measured and calculated to get fitness values again. Then a new reproduction is formed by selecting better fitness individuals from the father reproduction and sub reproduction. After a given generation, the algorithm would become the most fit convergence, and the convergence individual probably represents an optimized solution.

A general maximum optimization problem could be expressed as

$$\max f(x), \text{ s.t. } g(\mathbf{X}) \in D^r, \quad (3)$$

where $f(x)$ is the measure function, $\mathbf{X} = [x_1, x_2, \dots, x_N]^T$ is the decision variable, and D^r is the collection of feasible solutions. As the coordinate values of sub-apertures are real numbers in a circle synthetic aperture array, a real-coding method is adopted to improve the calculation precision and efficiency. In this paper, the sub-aperture positions were denoted by their angles in a circle directly, and were constrained in a range of $0-2\pi$. The distance of any two sub-apertures should be no less than the diameter of a sub-aperture. The real-coding method in GA simplifies the calculation as the actual coding process was bypassed. The specific steps of using GA to solve problems were as follows:

Step 1 The initialization of reproductions: set the evolution generation counter $t=0$; set the maximum evolution generation T ; generate M individuals $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_M$ as the initial reproduction $P(0)$ goes randomly.

Step 2 Fitness calculation: Eq. (1) was used as the fitness function to calculate every fitness value $f_i = E(\mathbf{X}_i)$ of every individual \mathbf{X}_i . Calculate the probability after finishing all the fitness value calculation:

$$p_i = f_i / \sum_{j=1}^M f_j, \quad i = 1, 2, \dots, M. \quad (4)$$

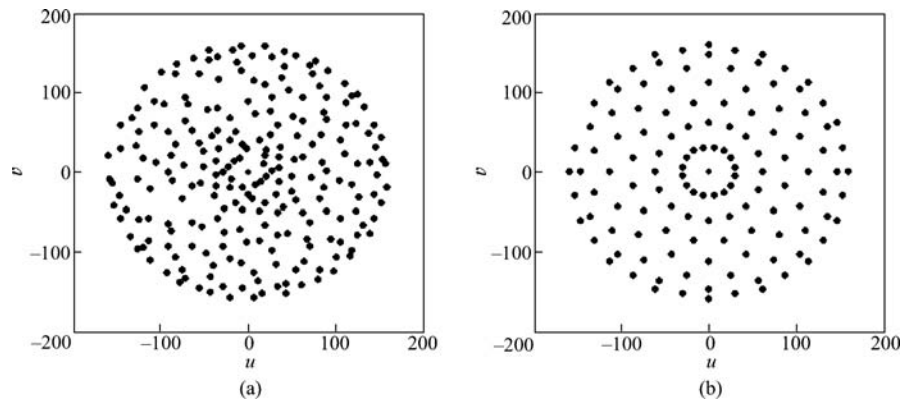


Fig. 1 (u, v) coverage of 16-element arrays. (a) With optimized arrangement; (b) with uniform arrangement

$$P(x,y) = \sum_{n=1}^N \beta_{\text{circ}} \left(\sqrt{(x-r_n \cos \theta_n)^2 + (y-r_n \sin \theta_n)^2} / a \right). \quad (10)$$

As the amplitude spread function is defined to be the convolution of 2D Fourier transform to the pupil function, we could get

$$h(u,v) = \frac{2\pi a J_1(a\sqrt{u^2+v^2})}{\sqrt{u^2+v^2}} \sum_{i=1}^N \exp[-j(ur_i \cos \theta_i, y - vr_i \sin \theta_i)]. \quad (11)$$

In the expression, u and v are defined as

$$u = \frac{2\pi x_i}{\lambda f}, \quad v = \frac{2\pi y_i}{\lambda f}. \quad (12)$$

Here, f is the imaging lens' focus. x_i and y_i are the coordinates of the imaging plane, and λ is the optical wavelength. As the optical frequency is very large, the actual detected signal is the optical intensity which is the square of the absolute value of the amplitude spread function $I(u,v) = |h(u,v)|^2$. The resulting diffraction-limited intensity distribution (PSF) in the image plane is the squared modulus of amplitude spread function (Fourier transformation of the pupil function), which is given by

$$S(u,v) = \left| \frac{2J_1(a\sqrt{u^2+v^2})}{a\sqrt{u^2+v^2}} \right|^2 \left\{ \frac{1}{N} + \frac{2}{N^2} \sum_{l=1}^{N-1} \sum_{i=l+1}^N \cos \left[2r_i \times \sqrt{u^2+v^2} \sin \frac{\theta_i - \theta_l}{2} \cos \left(\varphi + \frac{\theta_i + \theta_l}{2} \right) \right] \right\}. \quad (13)$$

In Eq. (13),

$$\varphi = \arcsin \frac{u}{\sqrt{u^2+v^2}} = \arccos \frac{v}{\sqrt{u^2+v^2}}. \quad (14)$$

The optical transfer function (OTF) is a Fourier transform to point spread function. Applying a Fourier transform to Eq. (13), we could get

$$T_{\text{OTF}}(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(u,v) \times \exp[-j2\pi(f_x u + f_y v)] du dv \Big|_{\substack{f_x = x/(\lambda f) \\ f_y = y/(\lambda f)}}. \quad (15)$$

From Eq. (13) and Eq. (15), we obtain that the optical intensity and OTF were intimately related to the number of sub-aperture N , diameters of sub-aperture and the circle array, and the location of each sub-apertures for an optical synthetic aperture array with definite optical wavelength and definite imaging focus. If an incoherent radiative spread source $O(u,v)$ was imaged to get a diffracted image $H(u,v)$ by an optical synthetic aperture circle array, we could get the relationship of $O(u,v)$ and $H(u,v)$ as follows according to the imaging theory:

$$H(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(u-u', v-v') O(u', v') du' dv'. \quad (16)$$

Eq. (16) indicated that the imaging quality was determined by PSF of the synthetic aperture imaging system, which contained N , a , r , and θ_i .

4.3 PSF comparison of optimized array and non-optimized array

Computer simulations were used to analyze the point spread function to measure the optimized results with GA. In the simulation, $N=16$, $a=1$ mm, $R=80$ mm, $\lambda=808$ nm, $f=300$ mm. By using the optimized circle array parameter in Table 1, the imaging characteristics were compared to that with a uniform circle array in Fig. 2.

From Figs. 2(a) and 2(b), we can conclude that the side-lobe of PSF with optimized array was distinctly reduced and imaging qualities with optimized array would be improved as the imaging process was the convolution of

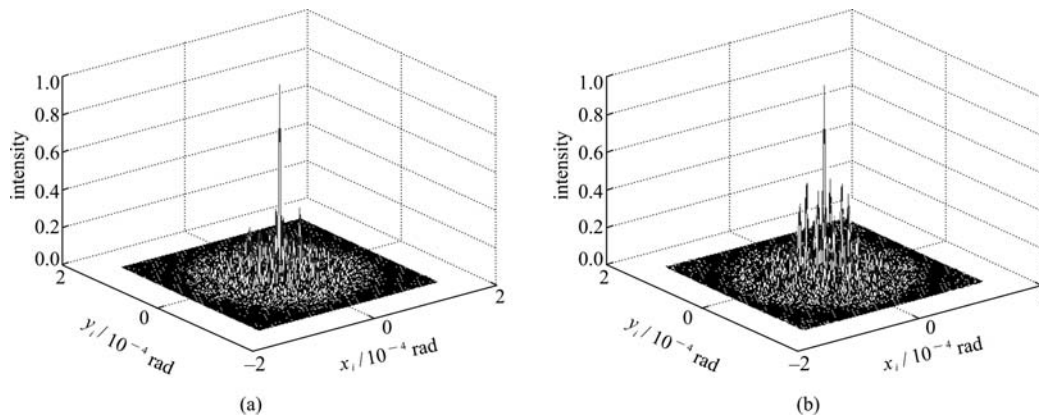


Fig. 2 PSF of 16-element arrays. (a) With array optimized by GA; (b) with uniform distribution array

Table 2 Contrast of measure function between SAA and GA optimization for 8 to 16 sub-apertures

N	8	9	10	11	12	13	14	15	16
SAA	3398	5584	8691	12904	18593	25914	35195	46803	61054
GA	3413	5604	8713	12966	18617	25938	35234	46833	61088

system PSF and targets image in an optical synthetic aperture system.

4.4 Optimization results comparison using GA and SAA

To make sure that GA is efficient in the optimization of synthetic aperture imaging array, we compared the optimization results of sub-apertures using GA to that from Ref. [5] using SAA. According to Eq. (1), the measure functions of 8–16 sub-apertures optimized are listed in Table 2.

It can be concluded from the data in Table 2 that the measure function using GA is larger than that using SAA. Thus, the optimization results with GA are better according to the former optimization model. On one hand, the simulated annealing algorithm was less efficient in synthetic aperture array optimization, because that the status of the whole searched space in SAA was not explicit, which was a disadvantage to the search process to reach the best search spaces. On the other hand, the GA could use the search results information from more than one small search area, which means that the GA could optimize all the sub-apertures synchronously. Thus the GA has a higher efficiency and costs less time in array optimization.

5 Conclusion

This paper analyzed the optimization model for synthetic aperture array, and optimized the optical synthetic aperture array by introducing the real-coded genetic algorithm. The measure function was chosen based on minimum redundant and most uniform u - v coverage. Arrays with 8–16 sub-apertures were optimized and the

results were compared with the previous ones using the simulated annealing algorithm. The results showed that the genetic algorithm was very practical and more efficient. By comparing the imaging PSF of the optimized 2D optical synthetic aperture circle array to the imaging PSF of un-optimized array, we can conclude that the imaging quality of the optimized array with genetic algorithm is remarkably improved. The genetic algorithm used in this paper is a universal optimization method for the synthetic aperture arrays, and can be also applied to other kind of plane arrays like “Y” shape, “T” shape and shapes that are even more complicated. In those situations, the constraint of parameters should be changed according to the array shapes. The results of this paper could be valuable for interferometric array design and the other related real projects.

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