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RESEARCH ARTICLE

Freeform optical element for uniform illumination

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Abstract The energy from a source was rearranged through reflection or refraction by a freeform optical element in order to get the desired uniform illumination. The numerical results of first-order partial differential equation sets had been investigated to obtain the freeform optical element, and the equations could be used to get the characteristics of the light source and the desired illumination. For example, a light emitting diode (LED) with a Lambertian light emitting surface of 1 mm × 1 mm was applied as the light source. Two kinds of freeform reflectors and one freeform lens were designed for different illuminations, and the simulated uniformity was near to 90%. Considering the size of these freeform optical elements, the illumination system can be very compact and efficient if the freeform optical element is applied in the illumination system of projectors with LED as source.

Keywords freeform optical element, uniform illumination, partial differential equation sets, numerical solutions

1 Introduction

Freeform optical elements are always used to get the desired illumination. Approximately, there are two ways to design them. One is by trial and error method, which always takes a lot of time [1, 2]. The other is to construct the freeform optical element by solving a set of differential equations [3–8]. In most cases, only the circle illumination is considered. The freeform reflector educed from the first-order partial differential equation sets proposed in this paper could create a uniform rectangular illumination by a single reflection. If this kind of freeform optical element is applied in projectors for illuminating, the system can be simplified and the efficiency can be high without multiplying the reflectance and transmittance on optical sets such as light tunnel or fly eye lens [9].

2 Partial differential equations

In order to design the freeform optical element, three equations are proposed. Figure 1 indicates the vectors which would appear in these equations. Assuming that the center of source S is located at the origin of an orthogonal coordinate system and T is the target plane for the illumination, points on T can be expressed as $t(x, y, z)$. The freeform optical element P is located in a spherical coordinates system, which shares the same origin with the orthogonal system mentioned before. So points on P can be expressed as $(\theta, \varphi, \rho(\theta, \varphi))$, and the normal vector at the point p of the reflector is $\boldsymbol{N}$. $\boldsymbol{I}$ is the vector of the light from source to point p, while $\boldsymbol{O}$ is the vector from point p to t after reflection or refraction by P.

Here, $\boldsymbol{N}$ stands for the slope of the reflector, while $\boldsymbol{I}$ and $\boldsymbol{O}$ have connection with points p and t respectively, so these three vectors can be expressed as

$$\boldsymbol{N}=\left(N_x\boldsymbol{i},N_y\boldsymbol{j},N_z\boldsymbol{k}\right), \tag{1}$$

$$\boldsymbol{I}=\left(I_x\boldsymbol{i},I_y\boldsymbol{j},I_z\boldsymbol{k}\right), \tag{2}$$

$$\boldsymbol{O}=(\boldsymbol{t}-\boldsymbol{p})/|\boldsymbol{t}-\boldsymbol{p}|. \tag{3}$$

These three vectors are all unit vectors, and connect with each other by the reflective or refractive law, which means the position of point t can be expressed by the coordinates of point p and the slope of the freeform surface as

$$x=\frac{N_x[n_{\boldsymbol{O}}(z-p_z)-n_{\boldsymbol{I}}I_z|\boldsymbol{t}-\boldsymbol{p}|]}{n_{\boldsymbol{O}}N_z}+p_x + \frac{n_{\boldsymbol{I}}}{n_{\boldsymbol{O}}}I_x|\boldsymbol{t}-\boldsymbol{p}|, \tag{4}$$

$$y=\frac{N_y[n_{\boldsymbol{O}}(z-p_z)-n_{\boldsymbol{I}}I_z|\boldsymbol{t}-\boldsymbol{p}|]}{n_{\boldsymbol{O}}N_z}+p_y + \frac{n_{\boldsymbol{I}}}{n_{\boldsymbol{O}}}I_y|\boldsymbol{t}-\boldsymbol{p}|, \tag{5}$$

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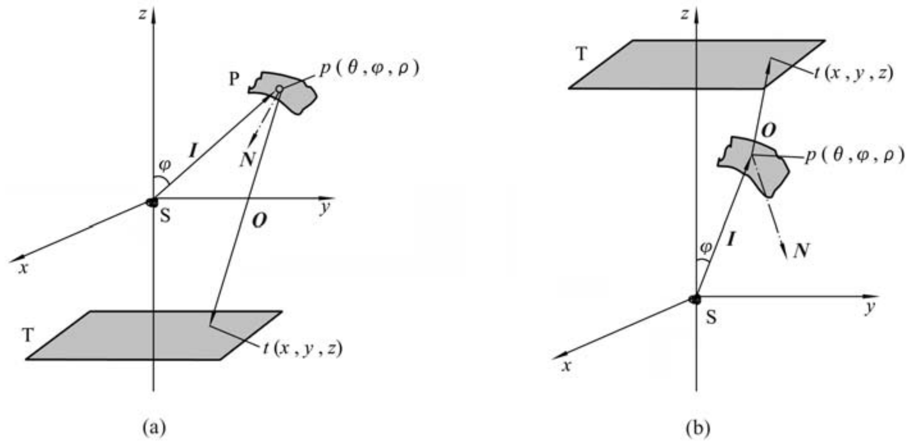

Fig. 1 Vectors in reflection and refraction. (a) Reflection; (b) refraction

where n_I and n_O are the refractive index of the incidence and emergent medium respectively. N_x, N_y and N_z including the first-order partial difference on the directions of θ and φ of $\rho(\theta, \varphi)$ [10] stand for the slope on point p of the freeform surface.

Also, the process of reflection should observe energy conservation, which means the output of the source should be equal to the flux inside the target plane. Assuming the viewing angle of the source is ϕ, then as the following equation shows:

$$\int_0^{2\pi} \mathrm{d}\theta \int_0^{\phi} I(\boldsymbol{I}(\varphi)) \sin\varphi \mathrm{d}\varphi = \int E(\boldsymbol{t}) \mathrm{d}A, \tag{6}$$

where $E(\boldsymbol{t})$ means the luminance at point t, $I(\boldsymbol{I}(\varphi))$ is the intensity at the direction of $\boldsymbol{I}(\varphi)$, A is the illuminated area. Equation (6) indicates the relationship between θ, φ and x, y, z. Its exact form depends on the topological mapping from the source to the target plane. When plane T is known, which means x, y, z are known, by substituting Eq. (6) into Eqs. (4) and (5), the first-order partial differential equation sets on the direction of θ and φ of $\rho(\theta, \varphi)$ can be educed. However, it is difficult to get the analytic results, so the numerical methods are employed by applying a pure surface-emitting light emitting diode (LED) as the source [11].

3 Freeform reflector

When the freeform surface is a reflective one, the index in Eqs. (4) and (5) follows the relationship $n_I = -n_O$. To simplify the calculation, assume that the light-emitting surface of LED is on the x-y plane with its center located at the origin. The LED has a Lambertian luminous surface of 1 mm $\times$ 1 mm and a viewing angle of 120°. The target plane T is 4:3 rectangular with a diagonal length of 0.5″. It is perpendicular to the z axis, and the coordinate of its center is (15, 0, 0), in order to avoid the probability that the reflected light might be blocked by the source [12]. The vertex of the freeform reflector is 10 mm above the LED, just as Fig. 2 shows.

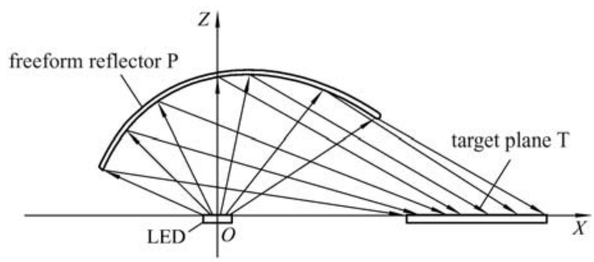

Fig. 2 LED reflective illumination system

As the emitting area of the LED is small enough compared with the size of the freeform reflector, the source can be treated as a point source, which means

$$\boldsymbol{I} = (\sin\varphi\cos\theta\boldsymbol{i}, \sin\varphi\sin\theta\boldsymbol{j}, \cos\varphi\boldsymbol{k}), \tag{7}$$

$$\boldsymbol{p} = (\theta,\varphi,\rho(\theta,\varphi)) = \rho(\theta,\varphi) \times \boldsymbol{I}\,. \tag{8}$$

When a uniform illumination is desired, $E(\boldsymbol{t})$ in Eq. (6) becomes a constant E. Considering the characteristics of LED, then

$$I(\boldsymbol{I}(\varphi)) = I \times \cos\varphi. \tag{9}$$

Using the topological mapping as Fig. 3 shows, and substituting Eq. (9) into Eq. (6), Eq. (6) can be changed into

$$\varphi = 0.5 \times \arccos\left(1 - \frac{8 \times E \times (X - 15) \times Y}{I \times \pi}\right), \tag{10}$$

where X and Y are the coordinates of the corner of the rectangle, and both of them are positive. Once φ has been

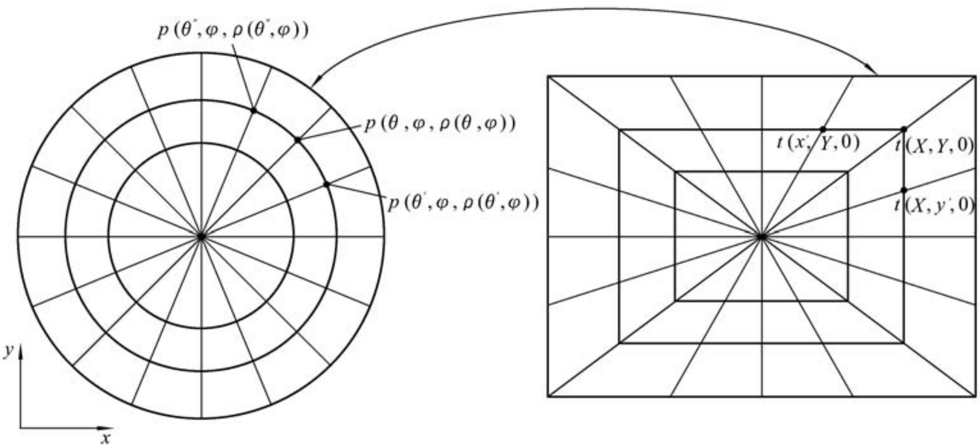

Fig. 3 Topological mapping from source to target plane

confirmed, θ in the first quadrant can be expressed as

$$\begin{cases} \theta = \dfrac{y'}{Y} \times \dfrac{\pi}{4}, & 0 \leqslant \theta \leqslant \dfrac{\pi}{4}, \\ \theta = \dfrac{X - x'}{X} \times \dfrac{\pi}{4} + \dfrac{\pi}{4}, & \dfrac{\pi}{4} \leqslant \theta \leqslant \dfrac{\pi}{2}, \end{cases} \tag{11}$$

where x' and y' are the coordinates on the target plane, and the values of θ in other quadrants can be easily calculated in the same way. Since the target plane is fixed, the values of x, y and z on the target plane are known.

After replacing the corresponding items in Eqs. (4), (5) by Eqs. (10) and (11), the unknown of the equation sets are $\rho(\theta,\varphi)$ and the first-order derivates of $\rho(\theta,\varphi)$. Using the numerical methods [11], by discretizing the equations with the grids depicted in Fig. 3, the values of $\rho(\theta,\varphi)$ can be calculated. The numerical results stand for the contour of the freeform reflector.

Figure 4 shows the freeform reflector in solid form. Its projective length on x axis is about 23 mm, and about 21 mm on y axis.

Fig. 4 Sketch map of the freeform reflector

Figure 5 shows the simulated results. Figure 5(a) shows the rectangular illumination on the target plane, which is 4:3 rectangular with a diagonal length of about 0.5″. Figure 5(b) shows the energy distributing across the center of the rectangle, and the vertical axis stands for the normalized illumination while the horizontal axis stands for the spatial distribution of the illumination on its center line.

From Fig. 5(b), it is seen that the uniformity is quite well, near to 90%. The energy inside the desired rectangle is about 95% of the output of the source if there's no energy loss when reflecting.

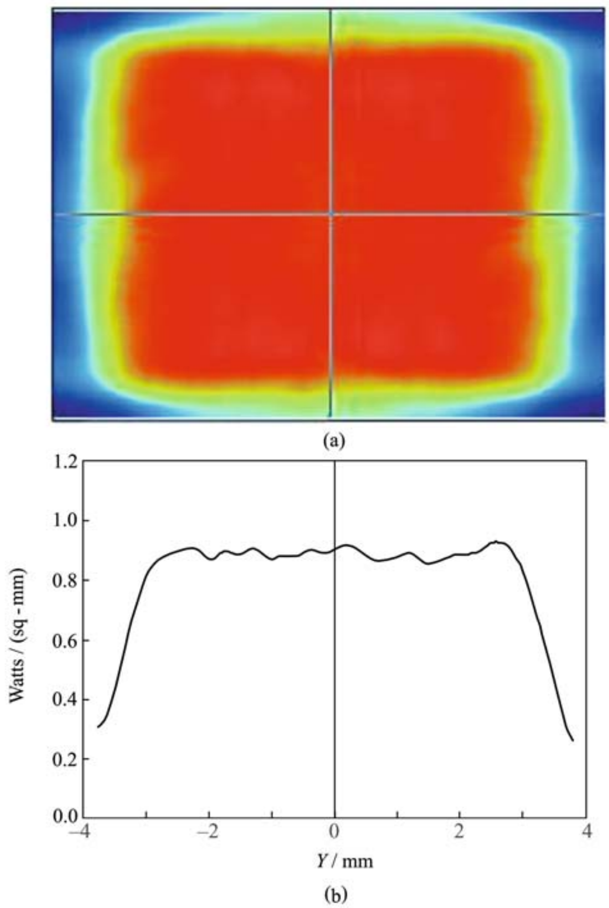

Fig. 5 Results of the illumination simulation

4 Freeform lens

The calculation of refractive surface and the reflective one are much alike. Still assume that the light-emitting surface of the LED is on the x-y plane with its center located at the origin. The LED has a Lambertian luminous surface of 1 mm × 1 mm and an emitting angle of 180°. The target plane T is a circle with a radius of 5 mm and it is perpendicular to the z axis, and the coordinate of its center is (20, 0, 0). The vertex of the freeform reflector is 10 mm above the LED, just as Fig. 6 shows.

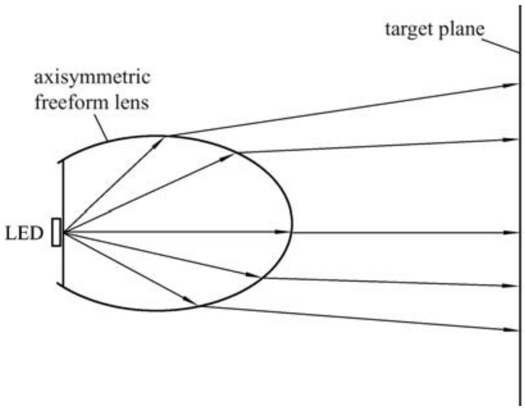

Fig. 6 LED refractive illumination system

Since the illumination is a circle, the lens can be axisymmetric. Then ρ_θ equals to 0, which means the partial differential equations become ordinary differential ones, and Eq. (10) is changed into

$$\varphi = 0.5 \times \arccos\left(1 - \frac{2 \times E \times r^2}{I}\right). \quad (12)$$

Here, r is the radius of the illumination circle. By substituting Eq. (12) into Eqs. (4) and (5), ordinary differential equation sets can be educed. The solution of the equation sets stands for the feature of the generatrix of the freeform lens. By revolving this generatirx round z axis, the axisymmetirc lens can be derived.

Figure 7 shows the axisymmetric freeform lens in solid form. Its projective length on the x axis is about 4.6 mm with the height of 10 mm.

Figure 8 shows the simulated results. Figure 8(a) shows the circle illumination on the target plane with a radius of about 0.5 mm. Figure 8(b) shows the energy distributing across the center, and the vertical axis stands for the normalized illumination while the horizontal axis stands for the spatial distribution of the illumination on its center line. Obviously, the results are also quite good.

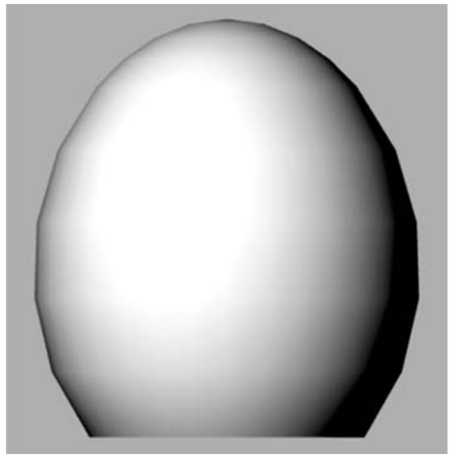

Fig. 7 Sketch map of the freeform lens

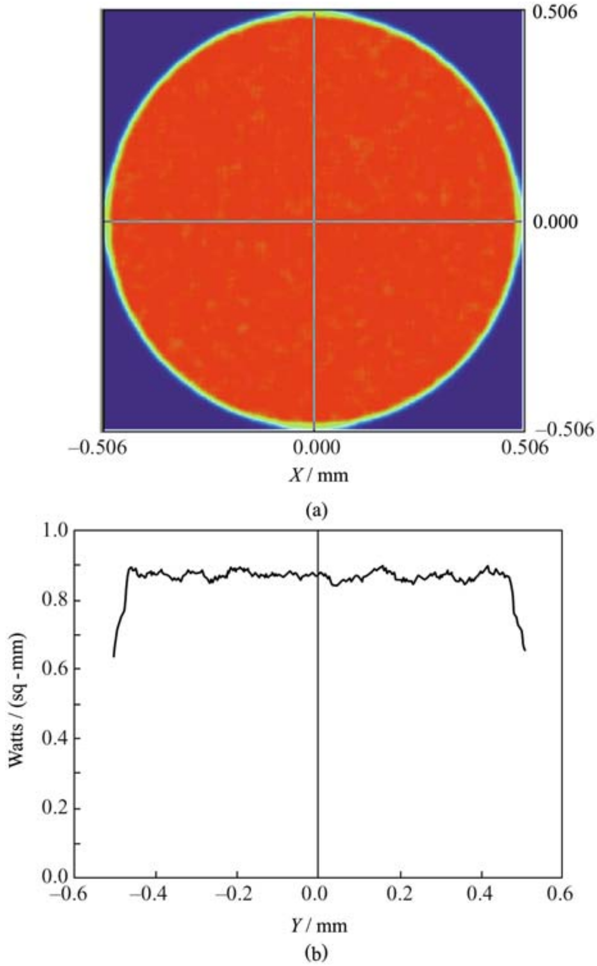

Fig. 8 Results of the illumination simulation

5 Conclusions

The first-order partial differential equation sets proposed in this paper can be applied to construct the freeform optical element for the desired uniform illumination, the given characteristics of the source and desired illumination. It only takes less than two minutes to get the answer and the uniformity is quite good. Supposing that the target plane is a digital micro-mirror device (DMD) in a digital light processor (DLP) projector, the illumination system could be quite compact and have high efficiency with the single freeform optical element illuminating DMD.

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