Front. Optoelectron. China 2008, 1(1–2): 50–57
DOI 10.1007/s12200-008-0001-9

REVIEW ARTICLE

On the theory of temporal aberrations for dynamic electron optics

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Abstract A new theory for temporal aberrations of dynamic optics by applying the direct integral method is put forward in the present paper. A new definition of temporal aberration is given, in which a certain initial energy of electron emission emitted from a photocathode along the axial direction $\sqrt{\varepsilon_{z1}}$ $(0 \leqslant \sqrt{\varepsilon_{z1}} \leqslant \sqrt{\varepsilon_{0\max}})$ was taken as a criterion. New expressions of the temporary aberration coefficients in integral forms for the electron optical imaging systems have been deduced. An electrostatic concentric spherical system model is used to test and verify expressions of the coefficients given by the "direct integral method" and "τ variation method". The analytical solutions prove that both methods are correct and equivalent. Compared to the "τ variation method", the direct integral method only needs to carry out the integral calculation for the three geometrical temporal aberration coefficients of the second order, which is more convenient and suitable for computation in the practical design. Finally, results of the study for the theory of temporal aberrations of electron optical imaging systems, from the point of view of methodology, have been elaborated.

Keywords cathode lenses, electron optical imaging systems, dynamic electron optics, theory of temporal aberrations

1 Introduction

The theory of temporal aberrations, which holds an important place in the study of electron optics with its wide beam focusing and system design, was first investigated by Monastyrski M A and Schelev M Y in 1980 [1]. This initial study brought forward the "τ variation method" to calculate the temporal aberration coefficients of the first and second orders, in which a certain initial energy of electron emission emitted from the cathode surface along the axial direction $\sqrt{\varepsilon_{z1}} = 0$ as a criterion is considered. However, the derived expressions of geometrical temporal aberration coefficients have been related to solve differential equations. Work on the "τ variation method" raised possibilities for scientists working on electron optics, particularly to consider the influence of temporal aberrations of the second order in designing image intensifiers and high-speed photographic image tubes that had scientific significance and practical applications.

The study of lateral aberrations and temporal aberrations of electron optical imaging systems in the 1950s and 1970s started to investigate the paraxial lateral chromatic aberration of the second order and the paraxial temporal chromatic aberration of the first order. Conclusions were as follows:

The spatial-trajectory-spread of electron optical imaging systems is mainly determined by the second-order paraxial lateral aberration, which can be expressed by the Recknagel-Artimovich formula [2,3],

$$\Delta r_2^* = \frac{2M}{E_c} \sqrt{\varepsilon_r} \left(\sqrt{\varepsilon_z} - \sqrt{\varepsilon_{z1}} \right). \tag{1}$$

The transit-time-spread of electron optical imaging systems is mainly determined by paraxial temporal chromatic aberration of the first order as

$$\Delta T_1 = \sqrt{\frac{2m_0}{e}} \frac{1}{E_c} \sqrt{\varepsilon_z}, \tag{2}$$

which has been given by Savoisky-Fanchenko [4] and proven by Csorba [5].

In Eqs. (1) and (2), $\sqrt{\varepsilon_z}$ and $\sqrt{\varepsilon_r}$ are the initial axial energy and initial radial energy of electrons emitted from the photocathode, respectively, M is the magnification of the system, E_c is the strength of the electric-field at the photocathode, E_c takes a negative value, and e/m_0 is the ratio of electron charge to mass. In Eq. (1), $\sqrt{\varepsilon_{z1}}$

Translated from *Transactions of Beijing Institute of Technology*, 2006, 26(5): 377–382 [译自: 北京理工大学学报]

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$(0 \leqslant \sqrt{\varepsilon_{z1}} \leqslant \sqrt{\varepsilon_{0\max}})$ is a given initial axial energy of electron corresponding to paraxial image position, and lateral aberrations are inspected only at this ideal image plane.

The difference between Eq. (1) expressing paraxial lateral aberration of the second order and Eq. (2) expressing first-order paraxial temporal aberration is obvious. However, we are able to find out the non-harmonious and non-symmetry between the two equations. It may be seen that neither the lateral aberration (in second order approximation) nor the temporal aberration (in first order approximation) depends on the concrete electrode structure and axial potential distribution. It is strange to find that the former depends on $\sqrt{\varepsilon_{z1}}$, which corresponds to the position of the ideal image plane, but the latter does not.

In fact, in terms of temporal and spatial aberrations, the initial energy spread of electrons emitted from the photocathode expressed as the spatial-trajectory-spread characteristics at the image plane are actually the same as the transit-time spread characteristics at a certain z plane. To define the temporal aberration, there should be a criterion to evaluate the time difference between two moving electrons. The non-harmonious relationship between Eqs. (1) and (2) can only be explained by Eq. (1) taking $\sqrt{\varepsilon_{z1}}=0$ as a criterion to explore the temporal aberrations formed by initial axial energies $\sqrt{\varepsilon_z}\neq 0$ of electrons emitted from the photocathode.

If we take $\sqrt{\varepsilon_{z1}}$ $(0\leqslant\sqrt{\varepsilon_{z1}}\leqslant\sqrt{\varepsilon_{0\max}})$, a certain initial energy of electron emission emitted from the photocathode along the axial direction, as a criterion to explore ΔT_1, i.e., the paraxial temporal chromatic aberration of the first order, then Eq. (2) should be written as follows:

$$\Delta T_1=\sqrt{\frac{2m_0}{e}}\frac{1}{E_c}(\sqrt{\varepsilon_z}-\sqrt{\varepsilon_{z1}}). \tag{3}$$

Eq. (3) has been strictly verified by an electrostatic concentric spherical system model [6,7]. It is evident from the equation that temporal aberrations and its theory should include the relation with $\sqrt{\varepsilon_{z1}}$.

2 Definition of temporal aberrations

According to our study of aberration theory of electron optics with wide beam focusing, as well as the aberration theory for a concentric spherical system of electrostatic focus, the lateral aberrations $\Delta\boldsymbol{r}$ of electron optical imaging systems can be defined as [8,9]:

$$\Delta\boldsymbol{r}=\Delta\boldsymbol{r}(z_i)=\boldsymbol{r}_{\text{practical}}\left(z_i,\varepsilon_r^{1/2},\varepsilon_z^{1/2},\boldsymbol{r}_0\right)-\boldsymbol{r}_{\text{paraxial}}\left(z_i,\varepsilon_{r1}^{1/2},\varepsilon_{z1}^{1/2},\boldsymbol{r}_0\right), \tag{4}$$

where, $r_{\text{practical}}$, r_{paraxial} express the lateral height at an image plane z_i, formed by the practical ray and paraxial ray of electrons emitted from the photocathode going through the system, respectively; and $\boldsymbol{r}_0$ is the initial height of electron emission. Taking the cylindrical coordinate system $(z,\boldsymbol{r})$, the axial coordinate z is chosen from the photocathode $z_0=0$.

In literatures [8,10] we have proven that Eq. (4) can be expressed by

$$\Delta\boldsymbol{r}=\Delta\boldsymbol{r}^*+\delta\boldsymbol{r}=\Delta\boldsymbol{r}_2^*+\Delta\boldsymbol{r}_3^*+\delta\boldsymbol{r}_3, \tag{5}$$

i.e., lateral aberrations $\Delta\boldsymbol{r}$ are defined as the sum of paraxial lateral aberration $\Delta\boldsymbol{r}^*$ and geometrical lateral aberration $\delta\boldsymbol{r}$, and the paraxial lateral aberration $\Delta\boldsymbol{r}^*$ is composed of paraxial lateral aberrations of second-order $\Delta\boldsymbol{r}_2{}^*$ and third-order $\Delta\boldsymbol{r}_3{}^*$. The paraxial lateral aberration of third-order $\Delta\boldsymbol{r}_3{}^*$ and the geometrical lateral aberration of third-order $\delta\boldsymbol{r}_3$ are in the same order of magnitude. $\delta\boldsymbol{r}_3$ has been investigated in a lot of literatures [11–13].

Since lateral aberrations and temporal aberrations of electron optical imaging systems actually are the same, with the initial energy spread and the initial angular spread of electrons emitted from the photocathode expressed as spatial-trajectory-spread characteristics at the ideal image plane or as transit-time spread characteristics at a certain z plane, including the ideal image plane, temporal aberrations can be similarly defined as Eq. (4):

$$\Delta t=\Delta t(z)=t_{\text{practical}}\left(z,\varepsilon_r^{1/2},\varepsilon_z^{1/2},\boldsymbol{r}_0\right)-t_{\text{paraxial}}\left(z,\varepsilon_r^{1/2},\varepsilon_{z1}^{1/2},\boldsymbol{r}_0\right), \tag{6}$$

where, $t_{\text{practical}}$, t_{paraxial} express the time when the practical ray and paraxial ray of electrons emitted from the photocathode go through the system, respectively.

The difference between Eqs. (4) and (6) is that the lateral aberration of electron optical imaging systems is measured at an ideal image plane z_i (real or virtual) and corresponds to $\sqrt{\varepsilon_{z1}}$, while the temporal aberration of such systems is measured at a certain z plane, including the real image plane. The lateral aberrations are inspected only at this ideal image plane.

We have proven that t_{paraxial} does not depend on $\varepsilon_r^{1/2}$ and $\boldsymbol{r}_0$ [14]. Substituting t, t^* for $t_{\text{practical}}$, t_{paraxial}, respectively; then Eq. (6) can be expressed as

$$\Delta t=t\left(z,\varepsilon_r^{1/2},\varepsilon_z^{1/2},\boldsymbol{r}_0\right)-t^*\left(z,\varepsilon_{z1}^{1/2}\right). \tag{7}$$

Similar to Eq. (5), we may express Eq. (7) as

$$\Delta t=\Delta T\left(z,\varepsilon_z^{1/2},\varepsilon_{z1}^{1/2}\right)+\Delta\tau\left(z,\varepsilon_r^{1/2},\varepsilon_z^{1/2},\boldsymbol{r}_0\right)=\Delta T+\Delta\tau, \tag{8}$$

where,

$$\Delta T = t^*\left(z,\varepsilon_z^{1/2}\right) - t^*\left(z,\varepsilon_{z1}^{1/2}\right) = \Delta T_1 + \Delta T_2, \tag{9}$$

$$\Delta\tau = t\left(z,\varepsilon_r^{1/2},\varepsilon_z^{1/2},\boldsymbol{r}_0\right) - t^*\left(z,\varepsilon_z^{1/2}\right) = \Delta\tau_2, \tag{10}$$

in which ΔT is called the paraxial temporal aberration, which expresses time difference between two paraxial rays with different initial axial energies. ΔT can be divided into two parts: the paraxial temporal aberration of first order ΔT_1 and the paraxial temporal aberration of second order ΔT_2. $\Delta\tau$ ($\Delta\tau_2$) is called the geometrical temporal aberration of the second order, which expresses the time difference between practical ray and paraxial ray with the same initial axial energies. Here, ΔT_2 and $\Delta\tau_2$ are in the same order of magnitude. The definition of temporal aberrations expressed by Eqs. (9) and (10) will be the starting point of our investigation.

3 Expressions for temporal aberration coefficients solved by "direct integral method" [14–16]

The most direct and simplest approach for solving temporal aberration coefficients is undoubtedly to integrate the electron motion equation in the electrostatic field with axial symmetry,

$$\ddot{z} = \frac{e}{m_0}\frac{\partial\varphi}{\partial z}. \tag{11}$$

After the axial velocity $\dot{z}$ is obtained, we may integrate $1/\dot{z} = \mathrm{d}t/\mathrm{d}z$, then we can derive the expression of time t.

Using the Scherzer series expansions of spatial potential distribution $\varphi = \varphi(z,\boldsymbol{r})$,

$$\varphi(z,\boldsymbol{r}) = \phi(z) - \frac{\boldsymbol{r}^2}{4}\phi''(z) + \frac{\boldsymbol{r}^4}{64}\phi^{IV}(z) - \cdots \tag{12}$$

and substituting Eq. (12) to (11), we get

$$\dot{z}^2 = \frac{2e}{m_0}\int\phi'(z)\mathrm{d}z - \frac{2e}{m_0}\int\frac{\boldsymbol{r}^2}{4}\phi'''(z)\mathrm{d}z + \frac{2e}{m_0}\varepsilon_z.$$

To solve the integral $\int\frac{\boldsymbol{r}^2}{4}\phi'''(z)\mathrm{d}z$ in the expression above, we will meet a problem for solving double integrals — the most difficult part of the investigation. The problem was settled by using the paraxial electron motion equation and its two special solutions.

Through a series of complicated transformations, we may obtain the time expression of an electron which reaches a certain position z in the system under the following initial conditions: the initial position vector $\boldsymbol{r}_0$, the initial direction angle θ_0, the initial emission angle α_0, the initial position angle β_0, and the initial emission velocity v_0, i.e., initial energy of electron ε_0 ($v_0 = (2e/m_0)^{1/2} \times \varepsilon_0^{1/2}$, $\varepsilon_z^{1/2} = \varepsilon_0^{1/2}\cos\alpha_0$, $\varepsilon_r^{1/2} = \varepsilon_0^{1/2}\sin\alpha_0$),

$$\begin{aligned} & t\left(z,\varepsilon_r^{1/2},\varepsilon_z^{1/2},\boldsymbol{r}_0\right) \\ &= \int_0^z \frac{\mathrm{d}z}{\left(\frac{2e}{m_0}\right)^{1/2}[\phi(z)+\varepsilon_z]^{1/2}} \\ &\times\left\{1 + \frac{1}{2}\frac{1}{[\phi(z)+\varepsilon_z]}\left[\varepsilon_r\left(v'^2\phi_* + \frac{1}{4}\phi''v^2 - 1\right)\right.\right. \\ &+ r_0^2\left(w'^2\phi_* + \frac{1}{4}\phi''w^2 - \frac{1}{4}\phi_0''\right) \\ &\left.\left. + 2r_0\sqrt{\varepsilon_r}\cos(\theta_0-\beta_0)\left(v'w'\phi_* + \frac{1}{4}\phi''vw\right)\right]\right\}, \end{aligned} \tag{13}$$

where $\phi(z)$ is the axial potential distribution, $\phi'(z)$, $\phi''(z)$ are the first order and second order derivatives of $\phi(z)$ with respect to z. $v = v(z)$, $w = w(z)$ are two linear independent special solutions of the paraxial electron motion equation satisfying the following initial conditions:

$$\begin{aligned} & v(z_0=0)=0, \quad v'(z_0=0)=\frac{1}{\sqrt{\varepsilon_z}}; \\ & w(z_0=0)=1, \quad w'(z_0=0)=0. \end{aligned} \tag{14}$$

Considering the paraxial condition of electron trajectories and expanding the first order term in expression (13), from the definition of paraxial temporal aberration (9), we can get the expressions of paraxial temporal aberrations ΔT and its aberration coefficients:

$$\Delta T = a_2\left(\varepsilon_z^{1/2} - \varepsilon_{z1}^{1/2}\right) + A_{22}(\varepsilon_z - \varepsilon_{z1}), \tag{15}$$

where a_2 is the paraxial temporal chromatic aberration coefficient of the first order,

$$a_2 = -\sqrt{\frac{2m_0}{e}}\frac{1}{\phi'(0)} = \sqrt{\frac{2m_0}{e}}\frac{1}{E_c}. \tag{16}$$

A_{22} is the paraxial temporal chromatic aberration coefficient of the second order,

$$A_{22} = \frac{1}{2}\sqrt{\frac{2m_0}{e}}\left\{\frac{1}{\sqrt{\phi(z)}}\frac{1}{\phi'(z)} + \int_0^z\frac{\phi''(z)}{\sqrt{\phi(z)}[\phi'(z)]^2}\mathrm{d}z\right\}. \tag{17}$$

Similarly, expanding the second order term in expression (13), from the definition of geometrical temporal aberrations (10), we can get the expressions of geometrical temporal aberrations $\Delta\tau$ and its aberration coefficients as follows:

$$\Delta\tau = A_{11}\varepsilon_r + 2A_{13}\varepsilon_r^{1/2}r_0 + A_{33}r_0^2, \tag{18}$$

in which,

$$A_{11}=\int_0^z \frac{1}{2(2e/m_0)^{1/2}[\phi(z)+\varepsilon_z]^{3/2}}\left(v'^2\phi_*+\frac{1}{4}\phi''v^2-1\right)\mathrm{d}z, \quad (19)$$

$$A_{13}=\int_0^z \frac{1}{2(2e/m_0)^{1/2}[\phi(z)+\varepsilon_z]^{3/2}}\cos(\theta_0-\beta_0) \times\left(v'w'\phi_*+\frac{1}{4}\phi''vw\right)\mathrm{d}z, \quad (20)$$

$$A_{33}=\int_0^z \frac{1}{2(2e/m_0)^{1/2}[\phi(z)+\varepsilon_z]^{3/2}} \times\left(w'^2\phi_*+\frac{1}{4}\phi''w^2-\frac{1}{4}\phi''_0\right)\mathrm{d}z, \quad (21)$$

where A_{11} is the temporal spherical aberration coefficient, A_{13} is the temporal aberration coefficient of field of curvature, and A_{33} is the temporal distortion aberration coefficient of the second order.

4 Verification of temporal aberration coefficients deduced from "direct integral method" by an electrostatic concentric spherical system model [7]

The correctness of expressions for temporal aberration coefficients deduced from the "direct integral method" should be tested and verified. We take a two-electrode electrostatic concentric spherical system as an ideal model, in which the electrostatic potential distribution and the moving electron rays can be expressed by distinct analytical expressions. Although the system has been investigated by many authors [17–19], the time of moving electrons expressed by analytical solution has not been given. If we can get the analytical expression of time for moving electron rays in this system, it may be used as a criterion to test and verify the correctness of the "direct integral method".

Suppose that in the two-electrode electrostatic concentric spherical system the curvature radii of spherical cathode C and of the mesh-spherical anode A are R_c and R_a, respectively, the common curvature center is O. Because of spherical symmetry, we can use the polar coordinate (ρ,φ) to describe the electron ray. Locating the origin of the polar coordinate at the curvature center O, the potential of the photocathode is $\phi_c=0$, the potential of mesh-anode A with respect to the photocathode C is ϕ_{ac}, and the potential at polar coordinate ρ with respect to photocathode is $\phi_{\rho c}$.

The motion of electron path follows the law of energy conservation and the law of angular momentum conservation:

$$\frac{1}{2}m_0v^2=\frac{1}{2}m_0v_0^2+e\phi_{\rho c}, \quad (22)$$

$$\rho^2\dot{\varphi}=R_c^2\dot{\varphi}_0=R_cv_0\sin\alpha_0. \quad (23)$$

Starting from Eqs. (22) and (23), we may obtain the analytical solution of time t, when the electron ray emitted from the photocathode goes through:

$$t=\sqrt{\frac{m_0}{2e}}\frac{1}{\phi_{ac}/(n-1)-\varepsilon_0}\times\Bigg\{\sqrt{-\rho^2\left[\phi_{ac}/(n-1)-\varepsilon_0\right]+\phi_{ac}/(n-1)R_c\rho-R_c^2\varepsilon_0\sin^2\alpha_0}-\frac{1}{2}\frac{\phi_{ac}/(n-1)}{\sqrt{\phi_{ac}/(n-1)-\varepsilon_0}}$$
$$\times R_c\arctan\frac{-\phi_{ac}/(n-1)R_c+2\rho\left[\phi_{ac}/(n-1)-\varepsilon_0\right]}{2\sqrt{\phi_{ac}/(n-1)-\varepsilon_0}\sqrt{-\rho^2\left[\phi_{ac}/(n-1)-\varepsilon_0\right]+\phi_{ac}/(n-1)R_c\rho-R_c^2\varepsilon_0\sin^2\alpha_0}}+R_c\sqrt{\varepsilon_0\cos^2\alpha_0}$$
$$+\frac{1}{2}\frac{\phi_{ac}/(n-1)}{\sqrt{\phi_{ac}/(n-1)-\varepsilon_0}}R_c\arctan\frac{2\varepsilon_0-\phi_{ac}/(n-1)}{2\sqrt{\phi_{ac}/(n-1)-\varepsilon_0}\sqrt{\varepsilon_0\cos^2\alpha_0}}\Bigg\}, \quad (24)$$

where $n=R_c/R_a$.

Expanding Eq. (24) to the forms of expressions Eqs. (15) and (18) according to definitions Eqs. (9) and (10), we can get the expressions for paraxial temporal chromatic aberration coefficients of first order a_2 and of second order A_{22}:

$$a_2=\sqrt{\frac{2m_0}{e}}\frac{1}{E_c}, \quad (25)$$

$$A_{22}=-\sqrt{\frac{2m_0}{e}}\frac{1}{E_c}\left(\frac{n-1}{\phi_{ac}}\right)^{1/2}\left[\frac{-(R_c+z)(2R_c-z)}{4R_c\sqrt{-z(R_c+z)}}+\frac{3}{4}\left(\arcsin\sqrt{\frac{-z}{R_c}}\right)\right]. \quad (26)$$

Now we shall test and verify the correctness of expressions for temporal aberration coefficients deduced from "direct integral method". Substituting the expression of axial potential distribution $\phi(z)$ in the electrostatic concentric spherical electrostatic system model

$$\phi(z) = \frac{\phi_{ac}}{n-1} \frac{-z}{(z+R_c)}, \tag{27}$$

and the two special solutions of paraxial ray equation [5,9]

$$v(z) = \frac{2z}{\phi(z)} \left[\sqrt{\phi(z)+\varepsilon_z} - \sqrt{\varepsilon_z} \right], \tag{28}$$

$$w(z) = 1 + \frac{1}{R_c} z - \frac{\sqrt{\varepsilon_z}}{R_c} \frac{2z}{\phi(z)} \left[\sqrt{\phi(z)+\varepsilon_z} - \sqrt{\varepsilon_z} \right] \tag{29}$$

into Eqs. (16) and (17), we can get the same results as formulae Eqs. (25) and (26). Thus we have proven that the direct integral method is correct.

Similarly, from Eqs. (19)–(21), by using the analytical solutions of electrostatic concentric spherical electrostatic system models Eqs. (27)–(29), we can also obtain the solutions of coefficients of the geometrical temporal aberrations,

$$A_{11} = \frac{3}{4} \sqrt{\frac{2m_0}{e}} \left(\frac{n-1}{\phi_{ac}} \right)^{1/2} \frac{1}{E_c R_c} \times \left[-\sqrt{z}\sqrt{-z-R_c} - R_c \arctan \frac{\sqrt{z}}{\sqrt{-z-R_c}} \right], \tag{30}$$

$$A_{13} = -\cos(\theta_0 - \beta_0) \frac{1}{2} \sqrt{\frac{2m_0}{e}} \frac{1}{R_c^2 E_c} z, \tag{31}$$

$$A_{33} = \sqrt{\frac{2m_0}{e}} \sqrt{\frac{n-1}{\phi_{ac}}} \frac{3}{8R_c^2} \times \left(\sqrt{z}\sqrt{-z-R_c} - R_c \arctan \frac{\sqrt{z}}{\sqrt{-z-R_c}} \right). \tag{32}$$

5 Comparison for studying temporal aberration theory between "direct integral method" and "τ variation method" [1, 19]

Our study has shown that the expression of coefficients for paraxial temporal chromatic aberration of first order a_2 deduced from τ variation method is completely identical with Eq. (16). The expressions of coefficients for paraxial temporal chromatic aberration of second order A_{22} and coefficients for geometrical temporal aberrations of second order A_{11}, A_{13}, A_{33} can be written as follows:

$$\begin{cases} A_{22} = -\dfrac{\ddot{z}}{2\dot{z}^3} z_{\alpha_2}^2 + \dfrac{\dot{z}_{\alpha_2} z_{\alpha_2}}{\dot{z}^2} - \dfrac{z_{\alpha_2\alpha_2}}{2\dot{z}}, \\ T^{(z)}(z_{\alpha_2\alpha_2}) = \phi\phi''' \dfrac{2}{\phi_0'^2}, \end{cases} \tag{33}$$

$$\begin{cases} A_{11} = -\dfrac{z_{\alpha_1\alpha_1}}{2\dot{z}}, \\ T^{(z)}(z_{\alpha_1\alpha_1}) = -\dfrac{1}{4}\phi''' v^2, \end{cases} \tag{34}$$

$$\begin{cases} A_{13} = -\dfrac{z_{\alpha_1\alpha_3}}{2\dot{z}}, \\ T^{(z)}(z_{\alpha_1\alpha_3}) = -\dfrac{1}{4}\phi''' vw\cos(\theta_0 - \beta_0), \end{cases} \tag{35}$$

$$\begin{cases} A_{33} = -\dfrac{z_{\alpha_3\alpha_3}}{2\dot{z}}, \\ T^{(z)}(z_{\alpha_3\alpha_3}) = -\dfrac{1}{4}\phi''' w^2, \end{cases} \tag{36}$$

where in expressions Eqs. (33)–(36), $z_{\alpha_i\alpha_j}$ is the solution of the following differential equations:

$$T^{(z)}\left(z_{\alpha_i\alpha_j}\right) = \frac{d}{dz}\left(\phi \frac{d}{dz} z_{\alpha_i\alpha_j} - \frac{1}{2}\phi' z_{\alpha_i\alpha_j} \right) \quad (i=1,2,3; j=1,2,3), \tag{37}$$

and each expression in Eq. (33) can be written as

$$\begin{gathered} z_{\alpha_2} = \frac{2}{\phi_0'}\sqrt{\phi}, \ \dot{z} = \sqrt{\frac{2e\phi}{m_0}}, \\ \dot{z}_{\alpha_2} = \sqrt{\frac{2e}{m_0}} \frac{\phi'}{\phi_0'} = \sqrt{\frac{2e}{m_0}} \frac{R_c^2}{(-R_c - z)^2}, \ \ddot{z} = \frac{e}{m_0}\phi'. \end{gathered} \tag{38}$$

Substituting the expression Eq. (27) of axial potential distribution $\phi(z)$ of electrostatic concentric spherical electrostatic system model and the two special solutions Eqs. (28) and (29) of its paraxial ray equation into Eqs. (33)–(36), we can prove that the expressions of coefficients for paraxial temporal chromatic aberration of second order A_{22} and coefficients for geometrical temporal aberrations of second order A_{11}, A_{13}, A_{33} are completely identical with Eqs. (26) and (30)–(32). Thus, we fully proved the correctness and equivalence of the "τ variation method" and "direct integral method" for studying temporal aberration theory.

6 Discussion of scientific method in this research [20]

A British philosopher of science Popper K R has put forward a deductive and inspection method as a scientific method (also called as trial and error method) for research, which can be summarized by the following route:

P_1 (Problem 1) → TT (Tentative Theory) → EE (Elimination of Error) → P_2 (Problem 2)……

The route shows that we can start from a theoretical or historical problem P_1; then we make a tentative answer (a guess or a hypothetical answer), and give a tentative theory TT; after that we submit it to a critical discussion according to evidence collected as the step to eliminate error EE. If it can be confirmed, a new problem P_2 appears.

Now I would like to talk about what we have learned from Popper's scientific method in this research.

Twenty-eight years ago, two Russian scientists put forward a temporal aberration theory for studying electron optical imaging systems, which was named "τ variation method", which no one has been skeptical of since then. When I began to work on the theory, two problems arose in my mind. The first was about the correctness of the "τ variation method" and if I will be able to find a simple and direct approach to study the temporal aberration theory. The second was about the accuracy of coefficients of temporal aberrations given by the "τ variation method", and if I will be able to find an ideal model to test and verify the coefficients of temporal aberrations. These two problems, the correctness of aberration theory and the accuracy of expressions of coefficients given by "τ variation method", have not been thoroughly studied. The first problem that I raised is actually to make an attempt to test the "τ variation method", confirming or negating it, or find a better theory. The second problem that I raised is actually to verify the degree to trust the theory and its feasibility. Such kind of thought is actually Problem 1(P_1) in the deductive and inspection method. This thought gives out a so-called "direct integral method" for studying temporal aberration theory, and a new definition of temporal aberrations has been produced, i.e., that temporal aberrations should be composed of paraxial temporal aberration and geometrical temporal aberration, in which the criterion of comparison is the paraxial electron ray with axial electron energy $\sqrt{\varepsilon_{z1}}$ ($0 \leqslant \sqrt{\varepsilon_{z1}} \leqslant \sqrt{\varepsilon_{0\max}}$). This process is just the tentative theory (TT) in the deductive and inspection method. This research shows that the solution of coefficients of temporal aberrations can be directly expressed by integral forms, and that there is no need to solve the aberration coefficients via differential equations deduced from "τ variation method".

Up to this step, the problem "which one is right and which one is wrong" of these two methods has not been solved. We must figure out an approach to make a critical testing for these two methods. This step is the elimination of error (EE) in the deductive and inspection method. In this step, we have discovered an ideal model, i.e., a two-electrode spherical concentric system with electrostatic focusing, and found the analytical solution of moving time of electrons going through the system. Result of this test and verification has shown that these two methods are not only correct, but also accurate. The correctness comes from the completely identical results obtained from two different approaches, and we have also proven that the result of "τ variation method" can be transformed to the form given by the "direct integral method". The accuracy comes from the result that two approaches are in complete agreement with the analytical solutions given by the ideal model. The study shows that the new "direct integral method" for temporal aberration theory contains the former "τ variation method". In view of this, we can say that the "direct integral method" for temporal aberration theory is a better one. Thus we reach Problem 2 (P_2), and progress of science was promoted.

It seems that the "direct integral method" for temporal aberration theory given by the present paper satisfies a condition of good theoretical pattern in science, which has the following characteristics:

1) The new method (direct integral method) starts from the motion equation of electron optics under the fundamental laws of physics. The premises are clear, the concept is distinct, the mathematical deduction is correct, and the mathematical forms obtained by the new method are simpler and clearer than the former method ("τ variation method").

2) The new method not only can explain all phenomena which can be explained by former one, but also extend the former method in which the criterion of comparison is based on axial energy $\sqrt{\varepsilon_{z1}} = 0$ to the new one in which the criterion of comparison is based on axial energy $0 \leqslant \sqrt{\varepsilon_{z1}} \leqslant \sqrt{\varepsilon_{0\max}}$.

3) The two methods have been verified and tested by using analytical solutions of a two-electrode spherical concentric model, and it has been proven that both methods are correct and accurate. This means that the two approaches for solving temporal aberration theory reached the same goal by taking different routes.

4) A new definition of temporal aberrations has been put forward in dynamic electron optics study with the starting point that temporal and lateral aberrations are actually the space-time expression of same phenomenon. Since lateral aberrations are composed of paraxial lateral aberration and geometrical lateral aberration, the temporal aberrations should also be composed of paraxial temporal aberration and geometrical temporal aberration. The hypothesis of introducing the new definition advances electron optics theory, which connects an internal relationship between static electron optics and dynamic electron optics.

5) The new method has predictive property. It can make a prediction or deduction for dynamic electron optics with electro-magnetic focusing, in which the

temporal aberration has had limited investigation. We shall prove that for either the electron optical imaging systems with electrostatic focusing or electron optical imaging systems with electro-magnetic focusing, the paraxial temporal aberration of the first order is related only to electrostatic field strength at the photocathode, which will be still expressed by Eq. (16).

From the study of this scientific problem, we can find the directive function of methodology.

1) Starting from a logical point of view, we found the contradiction between the spatial aberration theory and the temporal aberration theory, i.e., their non-harmonious relationship and non-symmetry, and we put forward a point of breakthrough for studying temporal aberration theory.

2) We have connected the spatial aberration problem with the temporal aberration problem, and found that the initial energy spread of electrons emitted from the photocathode expressed as spatial-trajectory-spread characteristics at the image plane can also be expressed as transit-time spread characteristics at a certain z plane. The two kinds of aberrations can be unified from the point of view of definition.

3) We have applied a method of analogy in this research, transplanted the method for studying lateral aberration theory to the study of temporal aberration theory, and put forward a new definition for temporal aberrations.

4) We have applied a testing method by using an ideal model, and verified that the new method and the former method are correct and equivalent. Thus we reached a scientific conclusion through a series of comparisons.

Our study shows that the direct integral method to study electron optical imaging systems is only related to solving integral expressions, which are more convenient for computation and could be recommended for the practical design.

7 Conclusions

In this paper, we described what we have obtained and what we have learned in studying the temporal aberration theory in terms of methodology. A new approach to the theory of temporal aberration for electron optical imaging systems called as "direct integral method" is put forward. A new definition of temporal aberration is given, where a certain initial energy of electron emission along the axial direction $\sqrt{\varepsilon_{z1}}$ $(0 \leqslant \sqrt{\varepsilon_{z1}} \leqslant \sqrt{\varepsilon_{0\max}})$ as criterion is considered. New expressions of the temporal geometrical aberration coefficients of second order in integral forms have been obtained by the "direct integral method". All of the formulae for temporal aberration coefficients deduced from the direct integral method and "τ variation method" have been verified by an electrostatic concentric spherical system model. Contrasting their results with analytical solutions shows that these two methods have complete identical solutions. It can be concluded that both methods are equivalent and correct. It should be pointed out that the expressions for solving temporal aberration coefficients of second-order A_{11}, A_{13}, A_{33} and A_{22} given by the direct integral method are related to solve integral expressions, which are more convenient for computation and could be suggested for use in the practical design of electron optical imaging systems. What we have learned from the study for the theory of temporal aberrations from the point of view of methodology has been summarized.

Acknowledgements This project was supported by National Natural Science Foundation of China and International Bureau of Cooperation, NSFC, as well as the Special Science Fund of Doctoral Discipline for Institutions of Higher Learning.

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