$$\boldsymbol{R}_{01}=\begin{bmatrix}\cos\theta_B & -\sin\theta_B & l_1\\ \sin\theta_B & \cos\theta_B & 0\\ 0 & 0 & 1\end{bmatrix},\quad \boldsymbol{R}_{12}=\begin{bmatrix}\cos\theta_A & -\sin\theta_A & l_2\\ \sin\theta_A & \cos\theta_A & 0\\ 0 & 0 & 1\end{bmatrix},\quad \boldsymbol{R}_T=\begin{bmatrix}\cos\gamma & -\sin\gamma & a\\ \sin\gamma & \cos\gamma & b\\ 0 & 0 & 1\end{bmatrix},$$

$$\frac{d\boldsymbol{R}_{01}}{dt}=\begin{bmatrix}-\frac{d\theta_B}{dt}\sin\theta_B & -\frac{d\theta_B}{dt}\cos\theta_B & 0\\ \frac{d\theta_B}{dt}\cos\theta_B & \frac{d\theta_B}{dt}\sin\theta_B & 0\\ 0 & 0 & 0\end{bmatrix},\quad \frac{d\boldsymbol{R}_{12}}{dt}=\begin{bmatrix}-\frac{d\theta_B}{dt}\sin\theta_B & -\frac{d\theta_B}{dt}\cos\theta_B & 0\\ \frac{d\theta_B}{dt}\cos\theta_B & \frac{d\theta_B}{dt}\sin\theta_B & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\frac{d\boldsymbol{R}_T}{dt}=\begin{bmatrix}-\frac{d\alpha}{dt}\sin\gamma & -\frac{d\alpha}{dt}\cos\gamma & V_x\\ \frac{d\alpha}{dt}\cos\gamma & \frac{d\alpha}{dt}\sin\gamma & V_y\\ 0 & 0 & 0\end{bmatrix}. \quad (S1)$$

1) Step 1: Link 3 bends up/returns to zero

While two pulleys differential-rotate positively, the linkage mechanism of UTMTR bends toward the upper side from the initial position, Link 3 would revolve first, and Link 1 would stay at 0°. The linear model is shown in Fig. S1(a). We use blue lines to indicate the configuration after transformation.

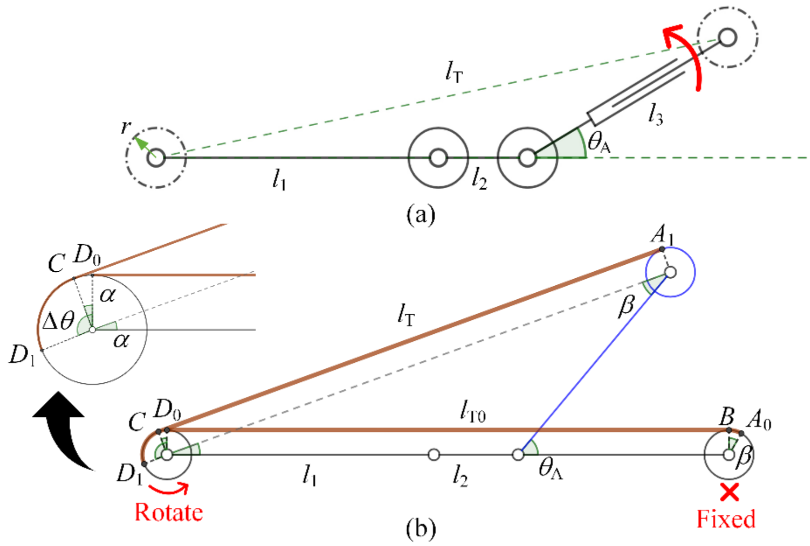

Fig. S1 Linear model of underactuated tension-motivated tracked robot in step 1: (a) motion of the linkage, (b) geometric relationships.

We can obtain the geometric relationship described in Eq. (S2).

$$l_T^2=(l_1+l_2)^2+l_3^2+2(l_1+l_2)l_3\cos\theta_A. \quad (S2)$$

By solving Eqs. (7) and (S2), we can obtain the relationships between the pulleys' differential-rotate angle and θ_{A} as Eq. (S3) shows.

$$\begin{cases} \theta_{\mathrm{A}} = \arccos \dfrac{2 l_Z l_{\mathrm{T}} - l_Z^2 - \left(l_1 + l_2\right)^2}{2\left(l_1 + l_2\right)\left(l_Z - l_{\mathrm{T}}\right)}, \\ l_Z = L - 2\pi r - l_1 - l_2, \end{cases} \quad \text{(S3)}$$

where l_{T} refers to the tightened segment of the track belt, which is related to the two pulleys' differential-rotate angle $\Delta\theta$. To determine the exact relationship, we fix the front pulley of the UTMTR and rotate the rear pulley to obtain the $\Delta\theta$ during this process, as Fig. S1(b) shows. From the initial configuration (marked with black lines), the rear pulley rotates counter-clockwise to reach a configuration with positive $\Delta\theta$ (marked with blue lines). In this process, A_0BD_0 and A_1CD_1 (marked with brown lines) are the same segment of the track belt. Therefore, we have

$$A_0B + BD_0 = r\beta + l_{\mathrm{T0}} = l_{\mathrm{T}} + r\left(\Delta\theta - \alpha\right) = A_1C + CD_1, \quad \text{(S4)}$$

where α is the angle between tightened segment of the track belt and Link 1, and β is the angle between the tightened segment of the track belt and Link 3. Then, we can derive the relationship between l_{T} and $\Delta\theta$ in step 1 as

$$\begin{cases} l_{\mathrm{T}} = r\left(\alpha + \beta\right) + l_{\mathrm{T0}} - r\Delta\theta = l_{\mathrm{T0}} + r\left(\theta_{\mathrm{A}} - \Delta\theta\right), \\ l_{\mathrm{T0}} = 0.5\left(L - 2\pi r\right). \end{cases} \quad \text{(S5)}$$

Before θ_{A} reaches θ_{lim}, these relationships would always be available whether $\Delta\theta$ is increasing or decreasing. Therefore, we can refer to this step as a reversible step, which means the UTMTR can perform the step backward according to the same geometric relationship.

2) Step 2: Link 1 bends up

When Link 3 reaches its limitation while $\Delta\theta$ still increasing, Link 1 would begin to revolve, and Link 3 is fixed in this process. As Fig. S2(a) shows, when observing in the ground coordinate system, we can see that Link 2 would rise while Link 1 remains on the ground.

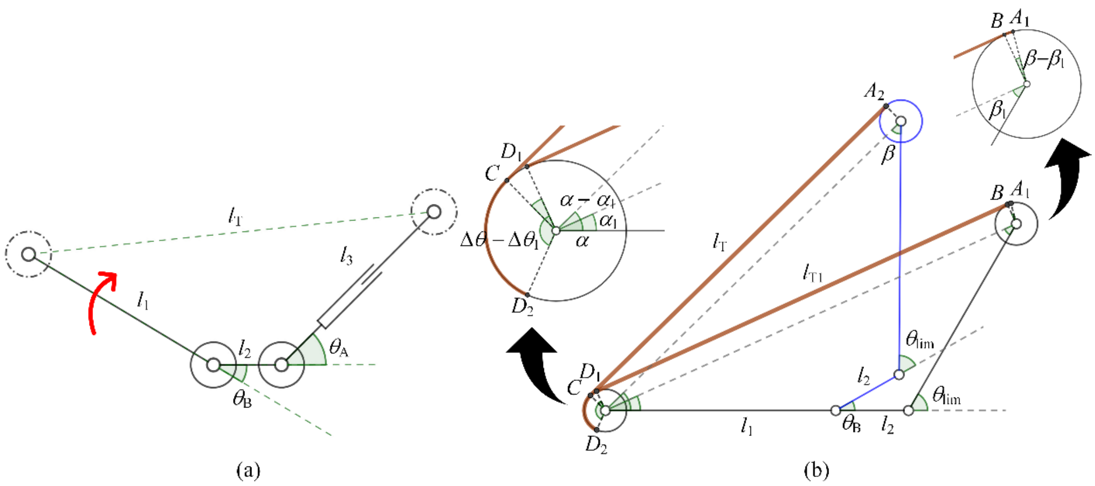

Fig. S2 Linear model of underactuated tension-motivated tracked robot in step 2: (a) motion of the linkage, (b) geometric relationships.

Similar to the method we used in step 1, we fixed the front pulley and trace a tightened segment during the transforming process (Fig. S2(b)). We have

$$A_1B+BD_1=r\left(\beta-\beta_1\right)+l_{T1}=l_T+r\left(\left(\Delta\theta-\Delta\theta_1\right)-\left(\alpha-\alpha_1\right)\right)=A_2C+CD_2, \quad \text{(S6)}$$

where α_1, β_1, $\Delta\theta_1$, and l_{T1} are values of α, β, $\Delta\theta$, and l_T at the end of step 1 (when θ_A reaches θ_{lim}), respectively. The $\Delta\theta_1$ can be obtained according to Eqs. (S3) and (S5) as

$$\Delta\theta_1=\frac{l_{T0}}{r}-\frac{\left(l_1+l_2\right)^2+l_Z^2+2l_Z\left(l_1+l_2\right)\cos\theta_{lim}}{2r\left[l_Z+\left(l_1+l_2\right)\cos\theta_{lim}\right]}+\theta_{lim}. \quad \text{(S7)}$$

Then, we can obtain the relationship between l_T and $\Delta\theta$ in step 2 as

$$\begin{cases} l_T=r\left[\left(\alpha+\beta\right)-\left(\alpha_1+\beta_1\right)\right]+l_{T1}-r\left(\Delta\theta-\Delta\theta_1\right)=l_{T1}+r\left(\theta_B-\Delta\theta+\Delta\theta_1\right), \\ l_{T1}=l_{T0}+r\left(\theta_{lim}-\Delta\theta_1\right)=\dfrac{\left(l_1+l_2\right)^2+l_Z^2+2l_Z\left(l_1+l_2\right)\cos\theta_{lim}}{2\left[l_Z+\left(l_1+l_2\right)\cos\theta_{lim}\right]}, \\ l_T=l_{T0}+r\left(\theta_{lim}+\theta_B-\Delta\theta\right). \end{cases} \quad \text{(S8)}$$

In this step, if $\Delta\theta$ begins to decrease, then Link 1 would be temporarily locked by the angular damper, and Link 3 would revolve (which is the action step 3), so this step is not reversible.

3) Step 3: Link 3 returns to zero/raises up

After step 2 is performed, the UTMTR needs two steps to return to the initial position. Step 3 is the first one.

As Fig. S3(a) shows, while decreasing $\Delta\theta$ from step 2, Link 3 would revolve first while Link 1 would keep its position. The linear model is the same as in step 2. The relationship between l_T and $\Delta\theta$ in step 3 can be calculated as shown in Fig. S3(b), which is,

$$A_3C+CD_3=r\left(\beta_2-\beta\right)+l_T=l_{T2}+r\left(\left(\Delta\theta_2-\Delta\theta\right)+\left(\alpha-\alpha_2\right)\right)=A_2B+BD_2, \quad \text{(S9)}$$

where α_2, β_2, $\Delta\theta_2$, and l_{T2} are values of α, β, $\Delta\theta$, and l_T at the end of step 2, respectively.

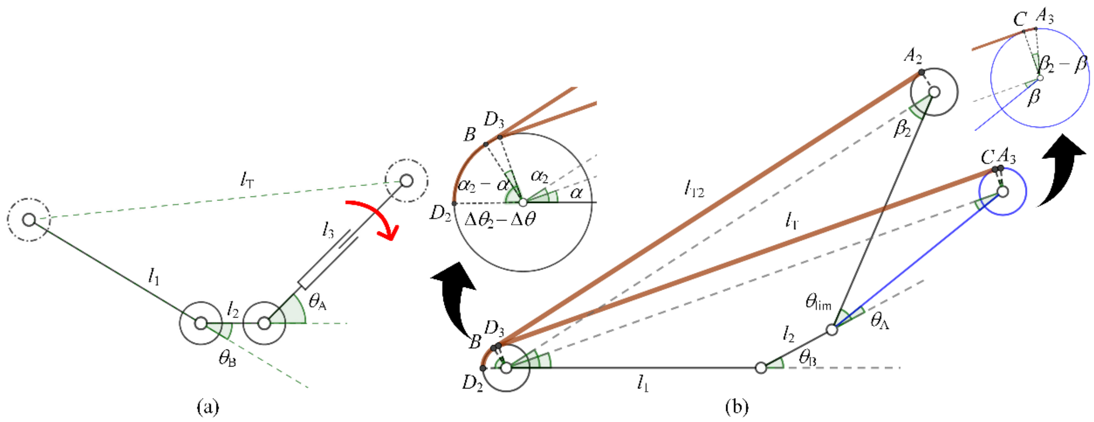

Fig. S3 Linear model of underactuated tension-motivated tracked robot in step 3: (a) motion of the linkage, (b) geometric relationships.

According to Eq. (S9), we can obtain the value of l_T in step 3 as

$$\begin{cases} l_{\mathrm{T2}} = l_{\mathrm{T1}} + r\left(\theta_{\mathrm{B}} - \Delta\theta_2 + \Delta\theta_1\right) = l_{\mathrm{T0}} + r\left(\theta_{\mathrm{lim}} + \theta_{\mathrm{B}} - \Delta\theta_2\right), \\ l_{\mathrm{T}} = l_{\mathrm{T2}} + r\left(-\Delta\theta + \Delta\theta_2 - \theta_{\mathrm{lim}} + \theta_{\mathrm{A}}\right) = l_{\mathrm{T0}} + r\left(\theta_{\mathrm{B}} + \theta_{\mathrm{A}} - \Delta\theta\right). \end{cases} \quad \text{(S10)}$$

The UTMTR would not reverse the action in this step although it seems reversible because it could easily cause unpredicted configurations. Unlike step 1, Link 1 has a positive revolve angle.

4) Step 4: Link 1 returns zero

After Link 3 returns to the initial position and meet its limitation, Link 1 would begin to revolve and finally return to the initial configuration, as Fig. S4(a) shows.

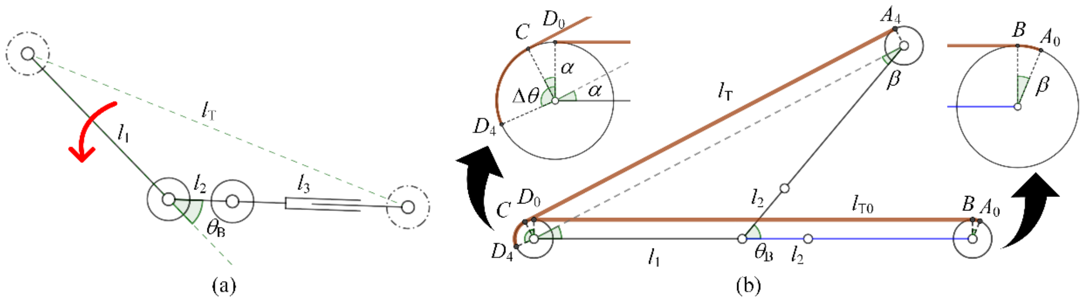

Fig. S4 Linear model of underactuated tension-motivated tracked robot in step 4: (a) motion of the linkage, (b) geometric relationships.

The geometric relationships can be described in Eq. (S11).

$$l_{\mathrm{T}}^2 = l_1^2 + \left(l_2 + l_3\right)^2 + 2l_1\left(l_2 + l_3\right)\cos\theta_{\mathrm{B}}. \quad \text{(S11)}$$

The solution of Eq. (S11) is shown in Eq. (S12):

$$\begin{cases} \theta_{\mathrm{B}} = \arccos\dfrac{2l_Y l_{\mathrm{T}} - l_Y^2 - l_1^2}{2l_1\left(l_Y - l_{\mathrm{T}}\right)}, \\ l_Y = L - 2\pi r - l_1. \end{cases} \quad \text{(S12)}$$

Evidentially, this step cannot be performed backward, because when $\Delta\theta$ begins to increase, Link 3 would revolve positively and leave the zero position. For the convenience of discussing the relationships between l_{T} and $\Delta\theta$ in this step, we pretend that the transformation acts in this step are reversible (Fig. 9(b)), so we can obtain the relationship as

$$A_0B + BD_0 = r\beta + l_{\mathrm{T0}} = l_{\mathrm{T}} + r\left(\Delta\theta - \alpha\right) = A_4C + CD_4. \quad \text{(S13)}$$

Then, by solving Eq. (S13), we can obtain

$$l_{\mathrm{T}} = r(\alpha+\beta)+l_{\mathrm{T0}}-r\Delta\theta = l_{\mathrm{T0}}+r(\theta_{\mathrm{B}}-\Delta\theta). \quad (\mathrm{S14})$$

As for any certain θ_{B}, the value of $\Delta\theta$ in step 4 is equal to the final value at the end of step 3. Thus, we can derive the exact value of $\Delta\theta_3$ according to Eqs. (S12) and (S14), that is,

$$\begin{cases} l_{\mathrm{T}} = \dfrac{2l_1 l_Y \cos\theta_{\mathrm{B}} + l_Y^2 + l_1^2}{2(l_Y + l_1\cos\theta_{\mathrm{B}})} = l_{\mathrm{T0}} + r(\theta_{\mathrm{B}} - \Delta\theta_3), \\ \Delta\theta_3 = \dfrac{l_{\mathrm{T0}}}{r} - \dfrac{2l_1 l_Y \cos\theta_{\mathrm{B}} + l_Y^2 + l_1^2}{2r(l_Y + l_1\cos\theta_{\mathrm{B}})}. \end{cases} \quad (\mathrm{S15})$$

5) Step 5: Link 1 bends down/returns zero

While the $\Delta\theta$ is still decreasing after the UTMTR returns to its initial condition, the linkage mechanism would bend toward the lower side, Link 3 would stay in the zero position, and Link 1 would revolve negatively, as Fig. S5(a) shows.

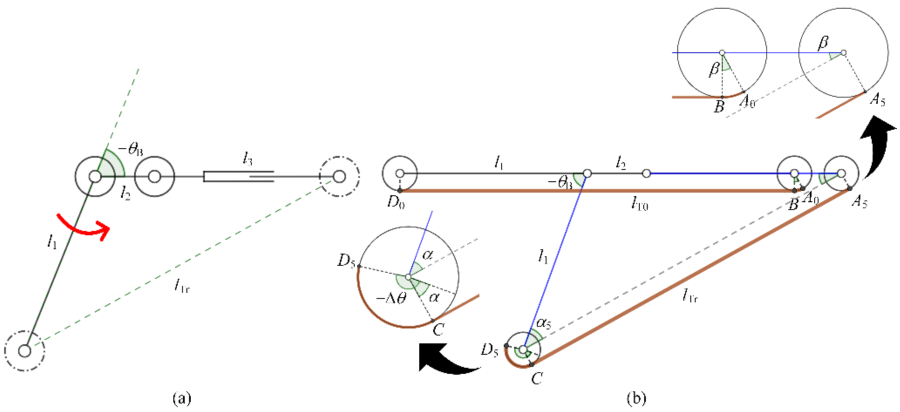

Fig. S5 Linear model of underactuated tension-motivated tracked robot in step 5: (a) motion of the linkage, (b) geometric relationships.

In this step, the tightened segment of the track belt is different from steps 1–4, and we use l_{Tr} to represent the length of this segment. This segment is not an ideal straight line; it would bend when touching obstacles. Thus, to minimize the calculation error caused by this factor, the UTMTR usually chooses to perform step 5 when the contact point is near the front pulley.

As Fig. S5(b) shows, the relationship between l_{Tr} and $\Delta\theta$ in step 5 can be described as:

$$A_0B + BD_0 = r\beta + l_{\mathrm{T0}} = l_{\mathrm{Tr}} + r(-\Delta\theta - \alpha) = A_5C + CD_5. \quad (\mathrm{S16})$$

By solving Eq. (S16) we can obtain

$$l_{\mathrm{Tr}} = r(\Delta\theta + \alpha + \beta) + l_{\mathrm{T0}} = l_{\mathrm{T0}} + r(\Delta\theta - \theta_{\mathrm{B}}). \quad (\mathrm{S17})$$

As Link 3 is fixed by restriction mechanisms in this step, step 5 is also reversible.