

The axial force of this CMNSM is divided into three parts for calculation, and specific expressions are listed below. According to Eqs. (1)–(11), the axial force of the top moving magnet is presented as

$$\begin{aligned}
F_z^u(z) = & \sum_{u_z}^{N_c} \sum_{t_z}^{N_c} 2\pi r_1 I_{1u} \left( I_{4u} P_r^c(r_1, z + z_{2t}, r_6, z_{1u}) + I_{3u} P_r^c(r_1, z + z_{2t}, r_5, z_{1u}) \right) \\
& + \sum_{u_z}^{N_c} \sum_{t_z}^{N_c} 2\pi r_2 I_{2u} \left( I_{4u} P_r^c(r_2, z + z_{2t}, r_6, z_{1u}) + I_{3u} P_r^c(r_2, z + z_{2t}, r_5, z_{1u}) \right) \\
& + \sum_{t_z}^{N_c} \sum_{n_z}^{N_s} 2\pi r_1 I_{1u} \left( Q_{s3} P_r^q(r_1, z + z_{2t}, r_3, z_{3n}) + Q_{s4} P_r^q(r_1, z + z_{2t}, r_4, z_{3n}) \right) \\
& + \sum_{t_z}^{N_c} \sum_{n_z}^{N_s} 2\pi r_2 I_{2u} \left( Q_{s3} P_r^q(r_2, z + z_{2t}, r_3, z_{3n}) + Q_{s4} P_r^q(r_2, z + z_{2t}, r_4, z_{3n}) \right) \tag{S1} \\
& + \sum_{t_z}^{N_c} \sum_{k_r}^{N_v} \sum_{j_z}^{N_v} 2\pi r_1 I_{1u} Q_{v2} P_r^q(r_1, z + z_{2t}, r_{vk}, z_{vj}) + \sum_{t_z}^{N_c} \sum_{k_r}^{N_v} \sum_{j_z}^{N_v} 2\pi r_2 I_{2u} Q_{v2} P_r^q(r_2, z + z_{2t}, r_{vk}, z_{vj}) \\
& + \sum_{w_z}^{N_c} \sum_{t_z}^{N_c} 2\pi r_1 I_{1u} \left( I_{4b} P_r^c(r_1, z + z_{2t}, r_6, z_{6w}) + I_{3b} P_r^c(r_1, z + z_{2t}, r_5, z_{6w}) \right) \\
& + \sum_{w_z}^{N_c} \sum_{t_z}^{N_c} 2\pi r_2 I_{2u} \left( I_{4b} P_r^c(r_2, z + z_{2t}, r_6, z_{6w}) + I_{3b} P_r^c(r_2, z + z_{2t}, r_5, z_{6w}) \right).
\end{aligned}$$

By the same principle, the axial force of lower moving magnet can be calculated as

$$\begin{aligned}
F_z^l(z) = & \sum_{u_z}^{N_c} \sum_{q_z}^{N_c} 2\pi r_1 I_{1b} \left( I_{4u} P_r^c(r_1, z + z_{5q}, r_6, z_{1u}) + I_{3u} P_r^c(r_1, z + z_{5q}, r_5, z_{1u}) \right) \\
& + \sum_{u_z}^{N_c} \sum_{q_z}^{N_c} 2\pi r_2 I_{2b} \left( I_{4u} P_r^c(r_2, z + z_{5q}, r_6, z_{1u}) + I_{3u} P_r^c(r_2, z + z_{5q}, r_5, z_{1u}) \right) \\
& + \sum_{q_z}^{N_c} \sum_{n_z}^{N_s} 2\pi r_1 I_{1b} \left( Q_{s3} P_r^q(r_1, z + z_{5q}, r_3, z_{3n}) + Q_{s4} P_r^q(r_1, z + z_{5q}, r_4, z_{3n}) \right) \\
& + \sum_{q_z}^{N_c} \sum_{n_z}^{N_s} 2\pi r_2 I_{2b} \left( Q_{s3} P_r^q(r_2, z + z_{5q}, r_3, z_{3n}) + Q_{s4} P_r^q(r_2, z + z_{5q}, r_4, z_{3n}) \right) \tag{S2} \\
& + \sum_{q_z}^{N_c} \sum_{k_r}^{N_v} \sum_{j_z}^{N_v} 2\pi r_1 I_{1b} Q_{v2} P_r^q(r_1, z + z_{5q}, r_{vk}, z_{vj}) + \sum_{q_z}^{N_c} \sum_{k_r}^{N_v} \sum_{j_z}^{N_v} 2\pi r_2 I_{2b} Q_{v2} P_r^q(r_2, z + z_{5q}, r_{vk}, z_{vj}) \\
& + \sum_{w_z}^{N_c} \sum_{q_z}^{N_c} 2\pi r_1 I_{1b} \left( I_{4b} P_r^c(r_1, z + z_{5q}, r_6, z_{6w}) + I_{3b} P_r^c(r_1, z + z_{5q}, r_5, z_{6w}) \right) \\
& + \sum_{w_z}^{N_c} \sum_{q_z}^{N_c} 2\pi r_2 I_{2b} \left( I_{4b} P_r^c(r_2, z + z_{5q}, r_6, z_{6w}) + I_{3b} P_r^c(r_2, z + z_{5q}, r_5, z_{6w}) \right).
\end{aligned}$$

Similarly, the axial force of the middle moving magnet is

$$\begin{aligned}
F_z^m(z) = & \sum_{i_z}^{N_s} \sum_{u_z}^{N_c} Q_{s1} \left( I_{4u} P_z^c(r_1, z + z_{4i}, r_6, z_{1u}) + I_{3u} P_z^c(r_1, z + z_{4i}, r_5, z_{1u}) \right) \\
& + \sum_{i_z}^{N_s} \sum_{u_z}^{N_c} Q_{s2} \left( I_{4u} P_z^c(r_2, z + z_{4i}, r_6, z_{1u}) + I_{3u} P_z^c(r_2, z + z_{4i}, r_5, z_{1u}) \right) \\
& + \sum_{p_r}^{N_v} \sum_{m_z}^{N_v} \sum_{u_z}^{N_c} Q_{v1} \left( I_{4u} P_z^c(r_{vp}, z + z_{vm}, r_6, z_{1u}) + I_{3u} P_z^c(r_{vp}, z + z_{vm}, r_5, z_{1u}) \right) \\
& + \sum_{i_z}^{N_s} \sum_{n_z}^{N_s} Q_{s1} \left( Q_{s3} P_z^q(r_1, z + z_{4i}, r_3, z_{3n}) + Q_{s4} P_z^q(r_1, z + z_{4i}, r_4, z_{3n}) \right) \\
& + \sum_{i_z}^{N_s} \sum_{n_z}^{N_s} Q_{s2} \left( Q_{s3} P_z^q(r_2, z + z_{4i}, r_3, z_{3n}) + Q_{s4} P_z^q(r_2, z + z_{4i}, r_4, z_{3n}) \right) \\
& + \sum_{i_z}^{N_s} \sum_{j_z}^{N_v} \sum_{k_r}^{N_v} (Q_{s1} + Q_{s2}) Q_{v2} P_z^q(r_1, z + z_{4i}, r_{vk}, z_{vj}) \tag{S3} \\
& + \sum_{p_r}^{N_v} \sum_{m_z}^{N_v} \sum_{n_z}^{N_s} Q_{v1} \left( Q_{s3} P_z^q(r_{vp}, z + z_{vm}, r_3, z_{3n}) + Q_{s4} P_z^q(r_{vp}, z + z_{vm}, r_4, z_{3n}) \right) \\
& + \sum_{j_z}^{N_v} \sum_{k_r}^{N_v} \sum_{m_z}^{N_v} \sum_{p_r}^{N_v} Q_{v1} Q_{v2} P_z^q(r_{vp}, z + z_{vm}, r_{vk}, z_{vj}) \\
& + \sum_{i_z}^{N_s} \sum_{w_z}^{N_c} Q_{s1} \left( I_{4b} P_z^c(r_1, z + z_{4i}, r_6, z_{6w}) + I_{3b} P_z^c(r_1, z + z_{4i}, r_5, z_{6w}) \right) \\
& + \sum_{i_z}^{N_s} \sum_{w_z}^{N_c} Q_{s2} \left( I_{4b} P_z^c(r_2, z + z_{4i}, r_6, z_{6w}) + I_{3b} P_z^c(r_2, z + z_{4i}, r_5, z_{6w}) \right) \\
& + \sum_{p_r}^{N_v} \sum_{m_z}^{N_v} \sum_{w_z}^{N_c} Q_{v1} \left( I_{4b} P_z^c(r_{vp}, z + z_{vm}, r_6, z_{6w}) + I_{3b} P_z^c(r_{vp}, z + z_{vm}, r_5, z_{6w}) \right)
\end{aligned}$$

In Eqs. (S1)–(S3),  $z$  represents the axial displacement of the moving magnet, which is equal to 0 when the CMNSM is at the equilibrium position.