

Electronic Supplementary Material

Ground mobile mechanism with a multi-loop reconfigurable trunk

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Table S1 List of D-H parameters

Name	Links ($i \rightarrow j$)	α_i	a_i	d_j	θ_j
KC ₁	$O_{xyz} \rightarrow L_{123}$	0	0	0	θ_{123}
	$L_{123} \rightarrow L_{124}$	0	1	0	θ_{124}
	$L_{124} \rightarrow P_{2a}$	$-\pi/2$	0	0	θ_{125}
	$P_{2a} \rightarrow P_{2b}$	0	$3r/2$	0	$\pi/2$
	$P_{2b} \rightarrow L_{231}$	$-2\pi/3$	0	$\sqrt{3}r$	θ_{231}
	$L_{231} \rightarrow L_{232}$	$-\pi/2$	0	0	θ_{232}
	$L_{232} \rightarrow L_{233}$	0	1	0	θ_{233}
	$L_{233} \rightarrow L_{234}$	$\pi/2$	0	1	θ_{234}
	$L_{234} \rightarrow L_{235}$	$-\pi/2$	0	0	θ_{235}
	$L_{235} \rightarrow P_3$	$-\pi/2$	0	0	θ_{236}
	$P_3 \rightarrow O_{P_3}$	$-\pi/2$	r	0	0
KC ₂	$O_{xyz} \rightarrow L_{121}$	0	1	0	θ_{122}
	$L_{121} \rightarrow P_{1a}$	$-\pi/2$	0	0	θ_{121}
	$P_{1a} \rightarrow P_{1b}$	0	$3r/2$	0	$\pi/2$
	$P_{1b} \rightarrow L_{141}$	$-2\pi/3$	0	$\sqrt{3}r$	θ_{141}
	$L_{141} \rightarrow L_{142}$	$-\pi/2$	0	0	θ_{142}
	$L_{142} \rightarrow L_{143}$	0	1	0	θ_{143}
	$L_{143} \rightarrow L_{144}$	$\pi/2$	0	1	θ_{144}
	$L_{144} \rightarrow L_{145}$	$-\pi/2$	0	0	θ_{145}
	$L_{144} \rightarrow P_4$	$-\pi/2$	0	0	θ_{146}
	$P_4 \rightarrow O_{P_4}$	$-\pi/2$	r	0	0
KC ₃	$P_{2a} \rightarrow P_{2b}$	0	$3r/2$	0	$\pi/2$
	$P_{2b} \rightarrow L_{241}$	$2\pi/3$	0	$-\sqrt{3}r$	θ_{241}
	$L_{241} \rightarrow L_{242}$	$-\pi/2$	0	0	θ_{242}
	$L_{242} \rightarrow L_{243}$	0	1	0	θ_{243}
	$L_{243} \rightarrow L_{244}$	$\pi/2$	0	1	θ_{244}
	$L_{244} \rightarrow L_{245}$	$-\pi/2$	0	0	θ_{245}
	$L_{245} \rightarrow P_4$	$-\pi/2$	0	0	θ_{246}
	$P_4 \rightarrow O_{P_4}$	$-\pi/2$	r	0	0
KC ₄	$P_{1a} \rightarrow P_{1b}$	0	$3r/2$	0	$\pi/2$
	$P_{1b} \rightarrow L_{131}$	$2\pi/3$	0	$-\sqrt{3}r$	θ_{131}
	$L_{131} \rightarrow L_{132}$	$-\pi/2$	0	0	θ_{132}
	$L_{132} \rightarrow L_{133}$	0	1	0	θ_{133}
	$L_{133} \rightarrow L_{134}$	$\pi/2$	0	1	θ_{134}
	$L_{134} \rightarrow L_{135}$	$-\pi/2$	0	0	θ_{135}

	$L_{135} \rightarrow P_{3a}$	$-\pi / 2$	0	0	θ_{136}
	$P_{3a} \rightarrow P_{3b}$	0	$3r/2$	0	$\pi / 2$
	$P_{3b} \rightarrow L_{341}$	$-2\pi / 3$	0	$\sqrt{3}r$	θ_{341}
	$L_{341} \rightarrow L_{342}$	$-\pi / 2$	0	0	θ_{342}
	$L_{342} \rightarrow L_{343}$	0	1	0	θ_{343}
	$L_{343} \rightarrow L_{344}$	0	1	0	θ_{344}
	$L_{344} \rightarrow P_4$	$-\pi / 2$	0	0	θ_{345}
	$P_4 \rightarrow O_{P_4}$	$-\pi / 2$	r	0	0
KC ₅	$L_{123} \rightarrow R_m \rightarrow L_{343}$	0	0	$2r$	θ_m
	$O_{121} \rightarrow O_{P_1}$	$-\pi / 2$	r	0	0
	$O_{125} \rightarrow O_{P_2}$	$-\pi / 2$	r	0	0
Others	$O_L \rightarrow L_{141}$	$\pi / 6$	0	$-r \cdot \tan \frac{\pi}{3}$	θ_{141}
	$O_L \rightarrow L_{131}$	$-\pi / 6$	0	$-r \cdot \tan \frac{\pi}{3}$	θ_{131}

Equations (S1) and (S2) are listed as follows:

$$\begin{aligned}
{}_{146}^{O_L} \mathbf{R}^{(1,1)} &= c_6 (c_5 (s_1 s_4 - c_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) - s_5 (c_1 c_2 s_3 + c_1 c_3 s_2)) - s_6 (c_4 s_1 + s_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) \\
{}_{146}^{O_L} \mathbf{R}^{(1,2)} &= -c_6 (c_4 s_1 + s_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) - s_6 (c_5 (s_1 s_4 - c_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) - s_5 (c_1 c_2 s_3 + c_1 c_3 s_2)) \\
{}_{146}^{O_L} \mathbf{R}^{(1,3)} &= s_5 (s_1 s_4 - c_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) + c_5 (c_1 c_2 s_3 + c_1 c_3 s_2) \\
{}_{146}^{O_L} \mathbf{R}^{(2,1)} &= c_6 (A + B) + s_6 \left(s_4 \left(c_3 \left(s_2 / 2 + \sqrt{3} c_2 s_1 / 2 \right) + s_3 \left(c_2 / 2 - \sqrt{3} s_1 s_2 / 2 \right) \right) + \sqrt{3} c_1 c_4 / 2 \right) \\
A &= s_5 \left(c_3 \left(c_2 / 2 - \sqrt{3} s_1 s_2 / 2 \right) - s_3 \left(s_2 / 2 + \sqrt{3} c_2 s_1 / 2 \right) \right) \\
B &= c_5 \left(c_4 \left(c_3 \left(s_2 / 2 + \sqrt{3} c_2 s_1 / 2 \right) + s_3 \left(c_2 / 2 - \sqrt{3} s_1 s_2 / 2 \right) \right) - \sqrt{3} c_1 s_4 / 2 \right) \\
{}_{146}^{O_L} \mathbf{R}^{(2,2)} &= C + D + s_3 (c_2 / 2 - \sqrt{3} s_1 s_2 / 2) - \sqrt{3} c_1 s_4 / 2) \\
C &= c_6 (s_4 (c_3 (s_2 / 2 + \sqrt{3} c_2 s_1 / 2) + s_3 (c_2 / 2 - \sqrt{3} s_1 s_2 / 2)) + \sqrt{3} c_1 c_4 / 2) \\
D &= -s_6 (s_5 (c_3 (c_2 / 2 - \sqrt{3} s_1 s_2 / 2) - s_3 (s_2 / 2 + \sqrt{3} c_2 s_1 / 2)) + c_5 (c_4 (c_3 (s_2 / 2 + \sqrt{3} c_2 s_1 / 2) \\
{}_{146}^{O_L} \mathbf{R}^{(2,3)} &= E - c_5 (c_3 (c_2 / 2 - \sqrt{3} s_1 s_2 / 2) - s_3 (s_2 / 2 + \sqrt{3} c_2 s_1 / 2)) \\
E &= s_5 (c_4 (c_3 (s_2 / 2 + \sqrt{3} c_2 s_1 / 2) + s_3 (c_2 / 2 - \sqrt{3} s_1 s_2 / 2)) - \sqrt{3} c_1 s_4 / 2) \\
{}_{146}^{O_L} \mathbf{R}^{(3,1)} &= F - c_6 \left(G + s_5 \left(c_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2) + s_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2) \right) \right) \\
F &= s_6 \left((c_1 c_4) / 2 + s_4 \left(c_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2) - s_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2) \right) \right) \\
G &= c_5 \left((c_1 s_4) / 2 - c_4 \left(c_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2) - s_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2) \right) \right) \\
{}_{146}^{O_L} \mathbf{R}^{(3,2)} &= H + I + s_5 (c_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2) + s_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2)) \\
H &= c_6 ((c_1 c_4) / 2 + s_4 (c_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2) - s_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2))) \\
I &= s_6 (c_5 ((c_1 s_4) / 2 - c_4 (c_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2) - s_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2))) \\
{}_{146}^{O_L} \mathbf{R}^{(3,3)} &= J - s_5 ((c_1 s_4) / 2 - c_4 (c_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2) - s_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2))) \\
J &= c_5 (c_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2) + s_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2)) \\
{}_{146}^{O_L} \mathbf{d}^{(1,1)} &= 238 c_1 c_2 - 238 c_1 c_2 s_3 - 238 c_1 c_3 s_2 \\
{}_{146}^{O_L} \mathbf{d}^{(1,2)} &= 119 s_2 + 238 c_3 (c_2 / 2 - \sqrt{3} s_1 s_2 / 2) - 238 s_3 (s_2 / 2 + \sqrt{3} c_2 s_1 / 2) + K \\
K &= 119 \sqrt{3} c_2 s_1 + 6924130064655945 / 140737488355328 \\
{}_{146}^{O_L} \mathbf{d}^{(1,3)} &= L - 238 s_3 (c_2 s_1 / 2 - \sqrt{3} s_2 / 2) - 119 \sqrt{3} s_2 \\
L &= 119 c_2 s_1 - 238 c_3 (s_1 s_2 / 2 + \sqrt{3} c_2 / 2) - (6924130064655945 \times \sqrt{3}) / 140737488355328
\end{aligned} \tag{S1}$$

$$\begin{aligned}
{}_{136}^{O_L} \mathbf{R}^{(1,1)} &= c_6 \left(c_5 (s_1 s_4 - c_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) - s_5 (c_1 c_2 s_3 + c_1 c_3 s_2) \right) - s_6 \left(c_4 s_1 + s_4 (c_1 s_2 s_3 - c_1 c_2 c_3) \right) \\
{}_{136}^{O_L} \mathbf{R}^{(1,2)} &= -c_6 \left(c_4 s_1 + s_4 (c_1 s_2 s_3 - c_1 c_2 c_3) \right) - s_6 \left(c_5 (s_1 s_4 - c_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) - s_5 (c_1 c_2 s_3 + c_1 c_3 s_2) \right) \\
{}_{136}^{O_L} \mathbf{R}^{(1,3)} &= s_5 (s_1 s_4 - c_4 (c_1 s_2 s_3 - c_1 c_2 c_3)) + c_5 (c_1 c_2 s_3 + c_1 c_3 s_2) \\
{}_{136}^{O_L} \mathbf{R}^{(2,1)} &= -c_6 (A + B) - s_6 \left(s_4 \left(c_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) + s_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) \right) - \sqrt{3} c_1 c_4 / 2 \right) \\
A &= s_5 \left(c_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) - s_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) \right) \\
B &= c_5 \left(c_4 \left(c_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) + s_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) \right) + \sqrt{3} c_1 s_4 / 2 \right) \\
{}_{136}^{O_L} \mathbf{R}^{(2,2)} &= s_6 (C + D) - c_6 \left(s_4 \left(c_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) + s_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) \right) - \sqrt{3} c_1 c_4 / 2 \right) \\
C &= s_5 \left(c_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) - s_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) \right) \\
D &= c_5 \left(c_4 \left(c_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) + s_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) \right) + \sqrt{3} c_1 s_4 / 2 \right) \\
{}_{136}^{O_L} \mathbf{R}^{(2,2)} &= E - s_5 \left(c_4 \left(c_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) + s_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) \right) + \sqrt{3} c_1 s_4 / 2 \right) \\
E &= c_5 \left(c_3 (c_2 / 2 + \sqrt{3} s_1 s_2 / 2) - s_3 (s_2 / 2 - \sqrt{3} c_2 s_1 / 2) \right) \\
{}_{136}^{O_L} \mathbf{R}^{(3,1)} &= c_6 (F + G) - s_6 \left((c_1 c_4) / 2 + s_4 \left(c_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) - s_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) \right) \right) \\
F &= c_5 \left((c_1 s_4) / 2 - c_4 \left(c_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) - s_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) \right) \right) \\
G &= s_5 \left(c_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) + s_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) \right) \\
{}_{136}^{O_L} \mathbf{R}^{(3,2)} &= H - s_6 \left(I + s_5 \left(c_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) + s_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) \right) \right) \\
H &= -c_6 \left((c_1 c_4) / 2 + s_4 \left(c_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) - s_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) \right) \right) \\
I &= c_5 \left((c_1 s_4) / 2 - c_4 \left(c_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) - s_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) \right) \right) \\
{}_{136}^{O_L} \mathbf{R}^{(3,3)} &= J - c_5 \left(c_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) + s_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) \right) \\
J &= s_5 \left((c_1 s_4) / 2 - c_4 \left(c_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) - s_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) \right) \right) \\
{}_{136}^{O_L} \mathbf{d}^{(1,1)} &= 238c_1 c_2 - 238c_1 c_2 s_3 - 238c_1 c_3 s_2 \\
{}_{136}^{O_L} \mathbf{d}^{(1,2)} &= K + 119\sqrt{3}c_2 s_1 - 6924130064655945 / 140737488355328 \\
K &= 238s_3 \left(s_2 / 2 - \sqrt{3} c_2 s_1 / 2 \right) - 238c_3 \left(c_2 / 2 + \sqrt{3} s_1 s_2 / 2 \right) - 119s_2 \\
{}_{136}^{O_L} \mathbf{d}^{(1,3)} &= L + 238s_3 \left((c_2 s_1) / 2 + \sqrt{3} s_2 / 2 \right) - 119\sqrt{3} s_2 \\
L &= 238c_3 \left((s_1 s_2) / 2 - \sqrt{3} c_2 / 2 \right) - 119c_2 s_1 - (6924130064655945 \times \sqrt{3}) / 140737488355328
\end{aligned} \tag{S2}$$