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Trajectory optimization with constraints for alpine skiers based on multi-phase nonlinear optimal control

Key words: Trajectory optimization; Optimal control; Pseudospectral method; Optimal trajectory; Numerical solution

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Contributions

1. A simplified and effective trajectory optimization model is established for a Super-G course with multiple gates, whose course data is taken from the official website of FIS.

2. Gliding inflection points are marked on the optimal skier trajectory. These points are the transition points for concavity and convexity in the ski trajectory; however, they have not been presented in the optimal ski trajectory in other works in terms of concept and coordinate labeling.

3. Several practical proposals are put forward after analysis. It is hoped that these reasonable suggestions can help in the daily training and provide efforts for performance improvements for alpine skiers.

Main idea

1. A time-saving skier trajectory is tremendously important for alpine skiers to improve training and skiing performance.

2. A trajectory optimization model is designed, where the objective is to minimize the runtime between the starting point and the finish line.

3. The original trajectory optimization problem is converted into a multi-phase nonlinear optimal control problem solved with a pseudospectral method, and the parameters are optimized to discover the optimal trajectory.

4. The proposed model and our code are proved by simulations.

Method

1. Considering the complexity and convergence of the trajectory optimization model, a simplified trajectory optimization model is established based on the optimal control theory.

2. Radau pseudospectral method is adopted for the solution to the trajectory optimization problem, the interpolation polynomial of which is the Lagrange polynomial and the discrete point is the Legendre-Gauss-Radau (LGR) point.

3. Using a numerical solution carried out by MATLAB optimization toolbox, the optimal trajectory is obtained with constraints and the code is proven to work properly.

Major results

When $-0.046 \le u(t) \le 0.046$

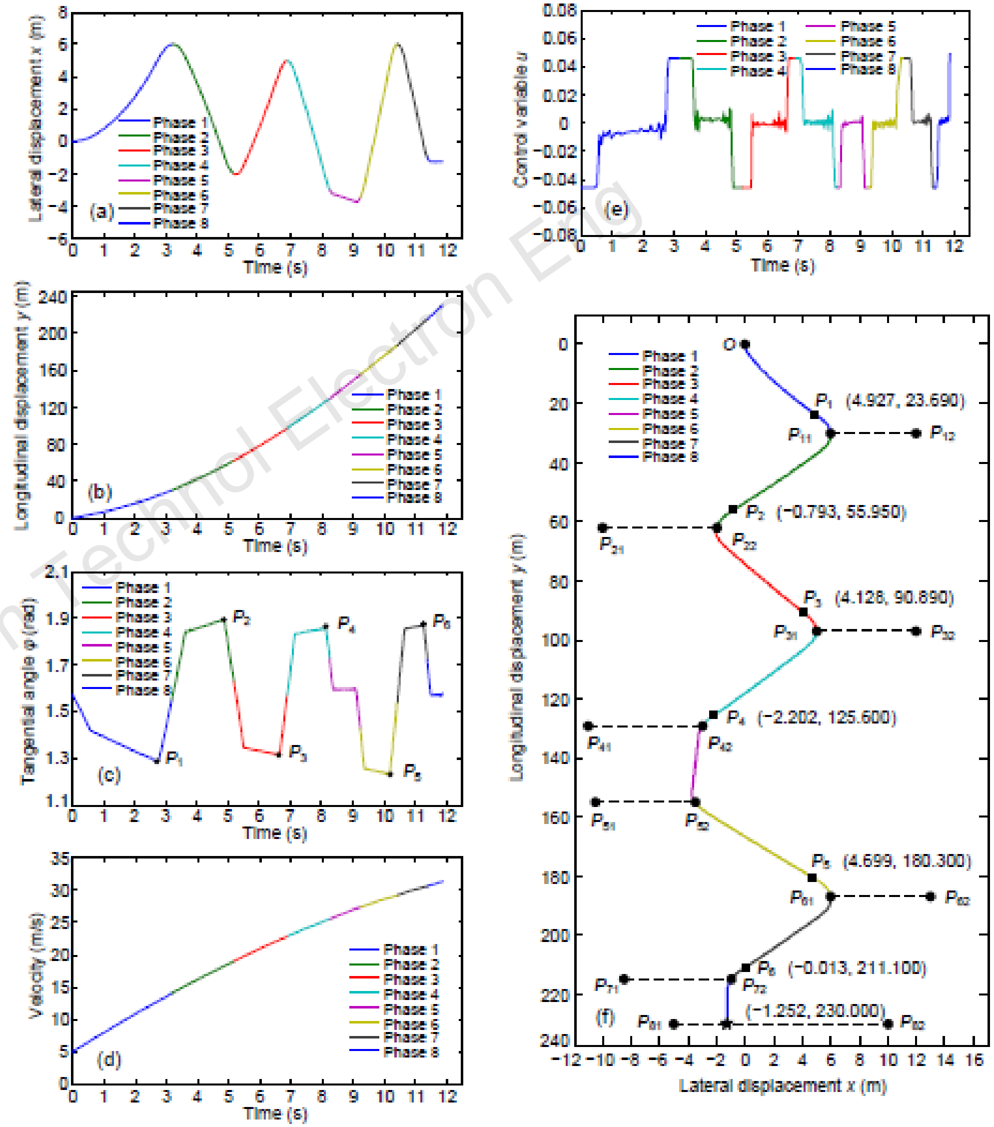

Fig. 5 Changes in the lateral (a) and longitudinal (b) displacements over time, tangential angle curve (c), velocity curve (d), control variable changing curve (e), and the optimal trajectory curve where the coordinates of the gliding inflection points are shown (f)

Major results

When $-0.025 \le u(t) \le 0.025$

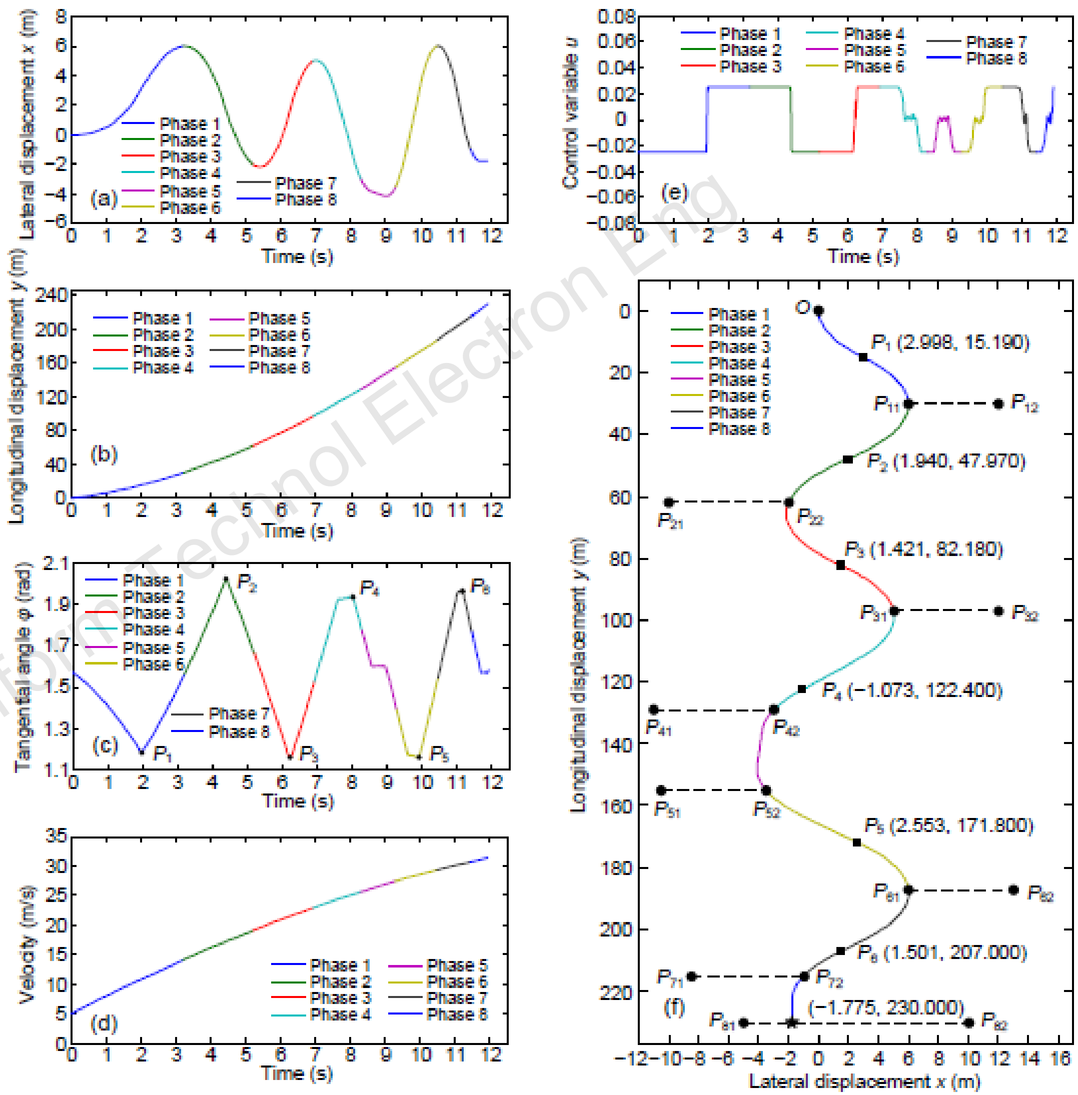

Fig. 6 Simulation results when the control variable is constrained by $-0.025 \le u(t) \le 0.025$ under the constraint conditions of Eqs. (18)–(22): (a) lateral displacements; (b) longitudinal displacements; (c) tangential angle curve; (d) velocity curve; (e) control variable changing curve; (f) the optimal trajectory curve

Major results

When $-0.058 \le u(t) \le 0.058$

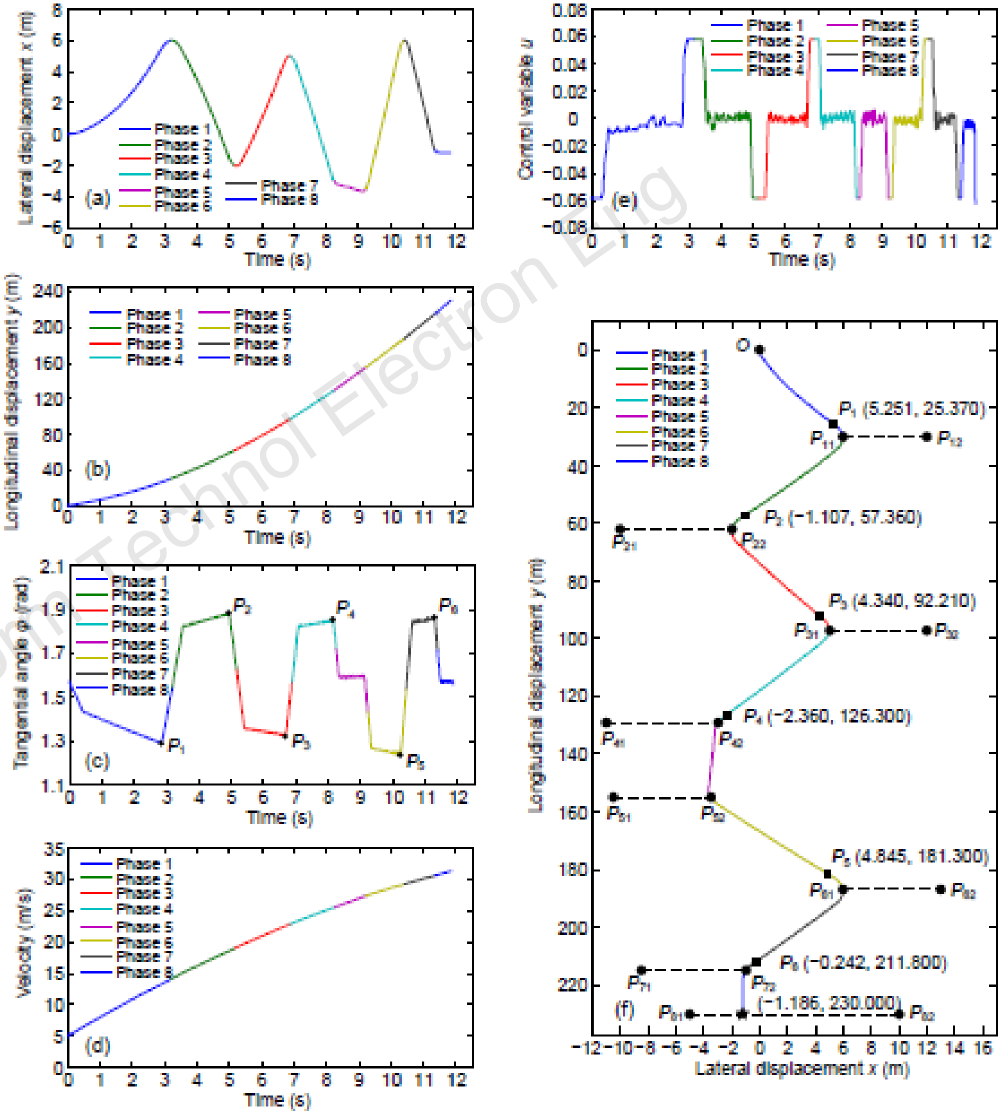

Fig. 7 Simulation results when the control variable is constrained by $-0.058 \le u(t) \le 0.058$ under the constraint conditions of Eqs. (18)–(22): (a) lateral displacements; (b) longitudinal displacements; (c) tangential angle curve; (d) velocity curve; (e) control variable changing curve; (f) the optimal trajectory curve

Major results

Numerical results of our trajectory optimization model

Table 1 Minimum time for the optimal trajectories with different upper and lower limits of control variables

Upper and lower limits of $u(t)$	Convergence/ Divergence	Final time (s)	Upper and lower limits of $u(t)$	Convergence/ Divergence	Final time (s)
±0.019	d	–	±0.040	c	11.8712
±0.021	d	–	±0.042	c	11.8687
±0.023	d	–	±0.044	c	11.8666
±0.025	c	11.9195	±0.046	c	11.8647
±0.030	c	11.8919	±0.048	c	11.8631
±0.031	c	11.8887	±0.050	c	11.8616
±0.033	c	11.8834	±0.052	c	11.8602
±0.035	c	11.8791	±0.054	c	11.8590
±0.037	c	11.8755	±0.056	c	11.8579
±0.039	c	11.8725	±0.058	c	11.8569

Skiing time listed is accurate to four decimal places for comparison. "–" means that the numerical results cannot be worked out

Major results

Code testing results for the one-phase trajectory optimization problem when $-0.046 \le u(t) \le 0.046$

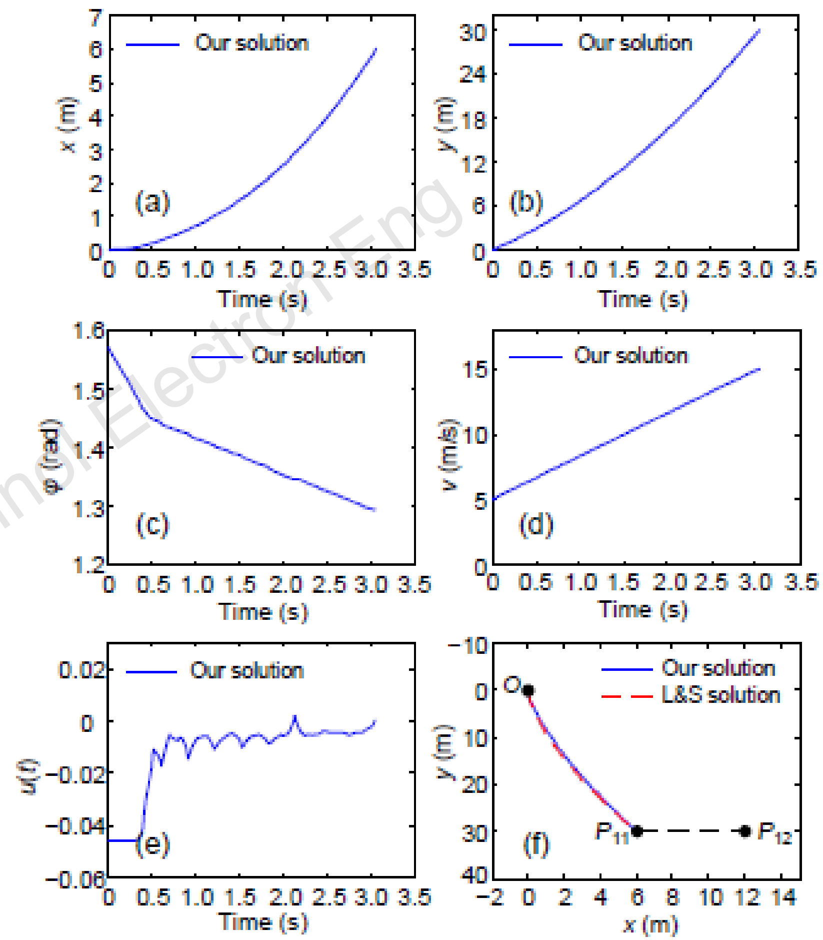

Fig. 8 Code testing results for the one-phase trajectory optimization problem (start-to-first gate) under the constraint conditions of Eqs. (37)–(41): (a) lateral displacements; (b) longitudinal displacements; (c) tangential angle curve; (d) velocity curve; (e) control variable changing curve; (f) the optimal trajectory curve

Conclusions

1. A trajectory optimization model for alpine skiers in the Super-G has been established, which is proved to be reasonable and effective by numerical simulations.

2. After comprehensive consideration of safety and time, the optimal skier trajectory is obtained within a reasonable range of control variables, on which the inflection points are marked.

3. Several skiing suggestions for alpine skiers have been presented through simulation analysis to provide alpine skiers with help and reference.

Prof. Xiao-lan YAO received her BS, MS, and PhD from Beijing Institute of Technology, China. Her research interests focus on modeling, optimization, and control of complex industrial control systems.