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Stochastic extra-gradient based alternating direction methods for graph-guided regularized minimization

Key words: Stochastic optimization; Graph-guided minimization; Extra-gradient method; Fused logistic regression; Graph-guided regularized logistic regression

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Motivations

1. There are many problems arising from statistics, machine learning, and genetic engineering. They can be formulated as minimization problems. One example is the logistic regression model.

2. The optimization problem can be solved by the alternating direction method of multipliers (ADMM) through introducing an auxiliary variable and reformulating it as the following linearly constrained minimization problem.

3. We propose two methods, named stochastic extra-gradient alternating direction method with Lagrangian function (SEGL) and augmented Lagrangian function (SEGAL), to solve linear constrained optimization problems in large scale.

Main ideas

1. Stochastic extra-gradient alternating direction method with Lagrangian function(SEGL).

2. Stochastic extra-gradient alternating direction method with augmented Lagrangianfunction (SEGAL).

Stochastic extra-gradient alternating direction method with Lagrangian function (SEGL)

Algorithm 1 Stochastic extra-gradient alternating direction method with Lagrangian function (SEGL)

Input: $\boldsymbol{x}^0$, $\boldsymbol{y}^0$, and $\boldsymbol{\lambda}^0$.

Output: $\tilde{\boldsymbol{y}}^t = \sum_{k=0}^{t} \alpha^{k+1} \boldsymbol{y}^{k+1}$, $\tilde{\boldsymbol{x}}^t = \sum_{k=0}^{t} \alpha^{k+1} \bar{\boldsymbol{x}}^{k+1}$, and $\tilde{\boldsymbol{\lambda}}^t = \sum_{k=0}^{t} \alpha^{k+1} \bar{\boldsymbol{\lambda}}^{k+1}$.

1: **for** $k = 0, 1, 2, \ldots$ **do**
2: Choose two data samples ξ_1^{k+1} and ξ_2^{k+1} randomly
3: Update $\boldsymbol{y}^{k+1}$ according to Eq. (5)
4: Update $\bar{\boldsymbol{x}}^{k+1}$ according to Eq. (7)
5: Update $\bar{\boldsymbol{\lambda}}^{k+1}$ according to Eq. (8)
6: Update $\boldsymbol{x}^{k+1}$ according to Eq. (9)
7: Update $\boldsymbol{\lambda}^{k+1}$ according to Eq. (10)
8: **end for**

Stochastic extra-gradient alternating direction method with augmented Lagrangian function (SEGAL)

Algorithm 2 Stochastic extra-gradient alternating direction method with augmented Lagrangian function (SEGAL)

Input: $\boldsymbol{x}^0$, $\boldsymbol{y}^0$, and $\boldsymbol{\lambda}^0$.

Output: $\tilde{\boldsymbol{y}}^t = \sum_{k=0}^{t} \alpha^{k+1} \boldsymbol{y}^{k+1}$, $\tilde{\boldsymbol{x}}^t = \sum_{k=0}^{t} \alpha^{k+1} \bar{\boldsymbol{x}}^{k+1}$, and $\tilde{\boldsymbol{\lambda}}^t = \sum_{k=0}^{t} \alpha^{k+1} \bar{\boldsymbol{\lambda}}^{k+1}$.

1: **for** $k = 0, 1, 2, \ldots$ **do**
2: Choose two data samples ξ_1^{k+1} and ξ_2^{k+1} randomly
3: Update $\boldsymbol{y}^{k+1}$ according to Eq. (5)
4: Update $\bar{\boldsymbol{x}}^{k+1}$ according to Eq. (11)
5: Update $\bar{\boldsymbol{\lambda}}^{k+1}$ according to Eq. (8)
6: Update $\boldsymbol{x}^{k+1}$ according to Eq. (12)
7: Update $\boldsymbol{\lambda}^{k+1}$ according to Eq. (10)
8: **end for**

Conclusions

1. The proposed algorithm inherits the stability and efficiency of the extra-gradient alternating method and the ability of stochastic optimization algorithms to handle large-scale problems.

2. We have compared our SEGL and SEGAL algorithms with 6 existing stochastic ADMM-type algorithms and 2 adaptive stochastic ADMM-type algorithms.

3. The experiments results demonstrated the efficacy of the proposed SEGL and SEGAL beyond other competing algorithms.