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H_∞ reference tracking control design for a class of nonlinear systems with time-varying delays

Key words: H_∞ reference tracking, nonlinear systems, Nonlinear systems, State feedback control, Time-varying delays, Unified model.

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Introduction

- Tracking control has been widely applied to practical engineering processes, but most of them concentrate on stabilization rather than tracking issues.
- It is not easy to achieve the tracking performances for highly nonlinear systems with time-varying delays and external disturbances. Some methods do exist but no unified approach is proposed in terms of different nonlinear systems.
- A unified method is established based on a unified model to deal with the tracking control issue of different nonlinear systems with time-varying delays.
- H_∞ reference tracking control design for a class of nonlinear systems with time-varying delays has been presented in this work.

Method

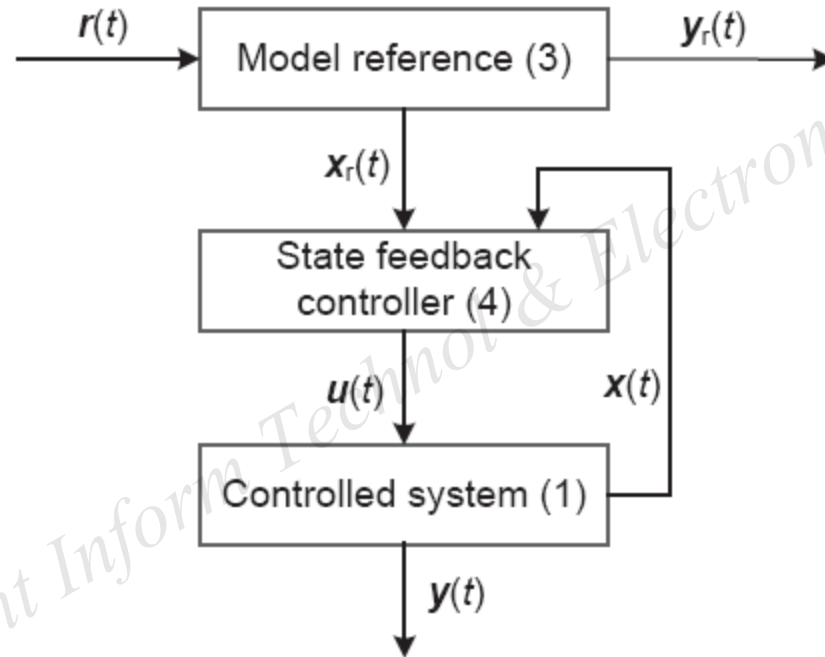


Fig. 1 Diagram of the closed-loop system

Main results: stability analysis

Theorem 1 Given $d \geq 0$, $\mu \geq 0$, and $\gamma > 0$, if there exist symmetric positive definite matrices P , Q_1 , Q_2 , R , a diagonal positive definite matrix Σ , and a symmetric matrix S that satisfy the following linear matrix inequalities (LMIs):

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & M_{23} & M_{24} & M_{25} \\ * & * & M_{33} & 0 & 0 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix} < 0,$$
$$\begin{bmatrix} R & S \\ * & R \end{bmatrix} > 0,$$

Main results: stability analysis

where

$$\begin{aligned}M_{11} &= P\tilde{A} + \tilde{A}^T P + Q_1 + Q_2 - R \\&\quad + \tilde{C}_y^T T \tilde{C}_y + d^2 \tilde{A}^T R \tilde{A}, \\M_{12} &= P\tilde{A}_d + R - S + d^2 \tilde{A}^T R \tilde{A}_d, \\M_{13} &= S, \quad M_{14} = P\tilde{B}_p + \tilde{C}_q^T \Sigma H + d^2 \tilde{A}^T R \tilde{B}_p, \\M_{15} &= \tilde{C}_y^T T \tilde{D}_{yw} + P\tilde{B}_w + d^2 \tilde{A}^T R \tilde{B}_w, \\M_{22} &= -(1 - \mu)Q_2 - 2R + 2S + d^2 \tilde{A}_d^T R \tilde{A}_d,\end{aligned}$$

Main results: stability analysis

$$\begin{aligned}M_{23} &= R - S, \quad M_{24} = \tilde{C}_{qd}^T \Sigma H + d^2 \tilde{A}_d^T R \tilde{B}_p, \\M_{25} &= d^2 \tilde{A}_d^T R \tilde{B}_w, \quad M_{33} = -Q_1 - R, \\M_{44} &= D_p^T \Sigma H + H \Sigma D_p - 2\Sigma + d^2 \tilde{B}_p^T R \tilde{B}_p, \\M_{45} &= d^2 \tilde{B}_p^T R \tilde{B}_w + H \Sigma \tilde{D}_w, \\M_{55} &= -\gamma^2 I + \tilde{D}_{yw}^T T \tilde{D}_{yw} + d^2 \tilde{B}_w^T R \tilde{B}_w, \\H &= \text{diag}(h_1, h_2, \dots, h_L),\end{aligned}$$

then the controlled system is asymptotically stable without external disturbances , and the prescribed L_2 gain is derived.

Main results: controller design

Theorem 2 Given $d \geq 0$, $\mu \geq 0$, and $\gamma > 0$, if there exist symmetric positive definite matrices X , Y_1 , Y_2 , Y_3 , Y_4 , a diagonal positive definite matrix V , and a matrix W that satisfy Eq. (17) (see the next page) and the following LMIs:

$$\begin{bmatrix} Y_3 & Y_4 \\ * & Y_3 \end{bmatrix} > 0,$$

Main results: controller design

$$N_1 = \begin{bmatrix} \Xi_3 & \tilde{A}_d X + Y_3 - Y_4 & Y_4 & \Xi_4 & \tilde{B}_w & \Xi_6 & \Xi_7 \\ * & -(1-\mu)Y_2 - 2Y_3 + 2Y_4 & Y_3 - Y_4 & X\tilde{C}_{qd}^T H & 0 & dX\tilde{A}_d^T & 0 \\ * & * & -Y_1 - Y_3 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_5 & H\tilde{D}_w & dV\tilde{B}_p^T & 0 \\ * & * & * & * & -\gamma^2 I & d\tilde{B}_w^T & \tilde{D}_{yw}^T T \\ * & * & * & * & * & -X - X^T + Y_3 & 0 \\ * & * & * & * & * & * & -T \end{bmatrix} < 0, \quad (17)$$

$$\Xi_3 = (\bar{A}X + \bar{B}_u W)^T + \bar{A}X + \bar{B}_u W + Y_1 + Y_2 - Y_3, \quad \Xi_4 = \tilde{B}_p V + X\bar{C}_q^T H + W^T D_u^T H,$$

$$\Xi_5 = -2V + V D_p^T H + H D_p V, \quad \Xi_6 = dX\bar{A}^T + dW^T \bar{B}_u^T, \quad \Xi_7 = X\bar{C}_y^T T - W^T D_{yu}^T T.$$

then the controlled system is asymptotically stable without external disturbances, and the prescribed L_2 gain is derived. Furthermore, the controller gain is as follows:

$$K = W X^{-1}.$$

Simulation results: sinusoidal signal

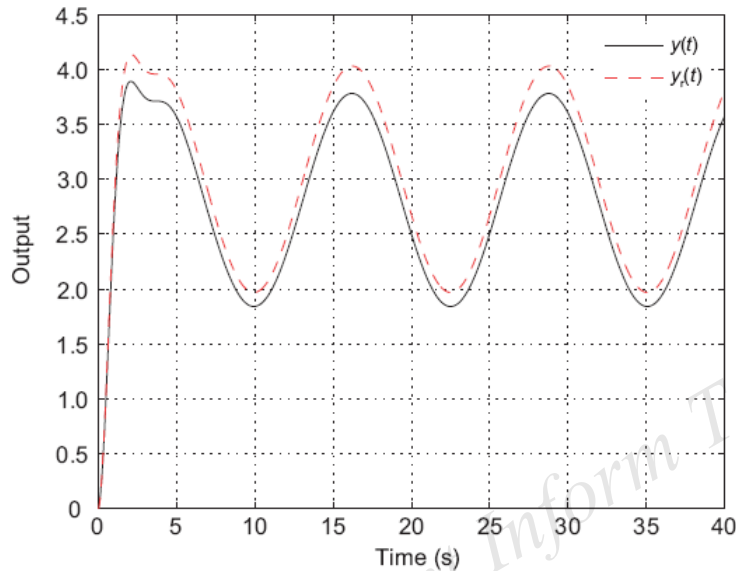


Fig. 2 The output of system (31) and the reference input (33) when $k = 8$

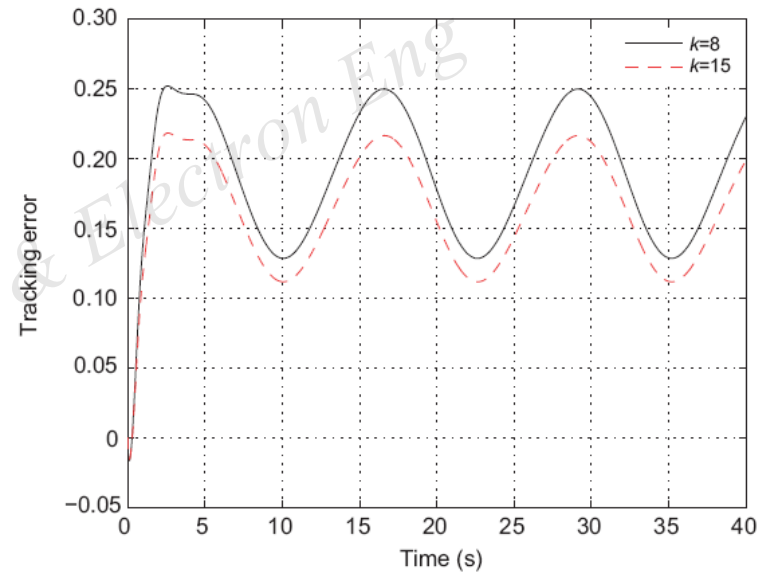


Fig. 3 Tracking errors in the cases of $k = 8$ and $k = 15$, respectively, with reference input (33)

Simulation results: square signal

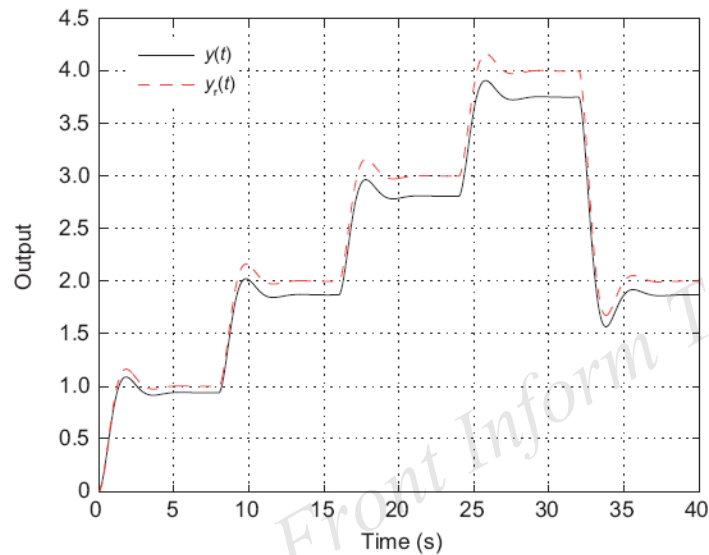


Fig. 4 The output of system (31) and the reference input (34) when $k = 8$

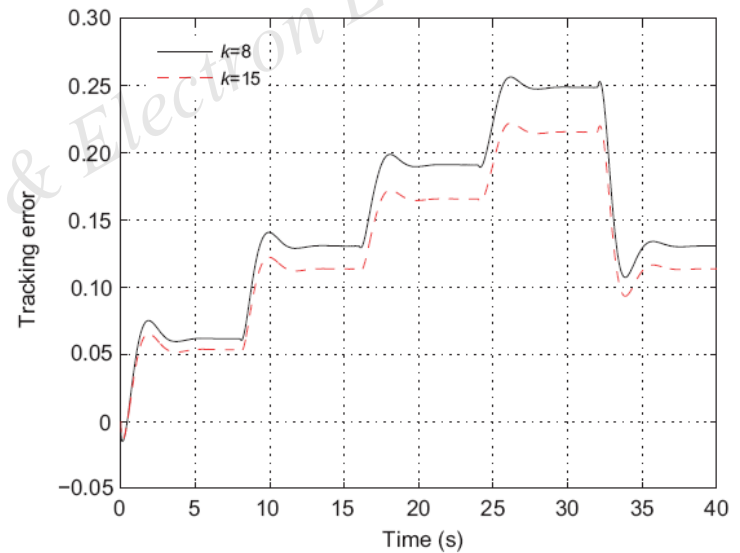


Fig. 5 Tracking errors in the cases of $k = 8$ and $k = 15$, respectively, with reference input (34)

Conclusions

- A state feedback controller is designed which guarantees that the closed-loop system is stable and the output of the system tracks the model reference trajectory in the H_∞ sense even in case of unknown time-varying delays.
- The reference model used in this work is more flexible than previous ones and the scaling factor introduced as a positive integer reduces the tracking error effectively .