

Bei CHEN, Quan XU, Mo CHEN, Huagan WU, Bocheng BAO, 2021. Initial-condition-switched boosting extreme multistability and mechanism analysis in a memcapacitive oscillator. *Frontiers of Information Technology & Electronic Engineering*, 22(11):1517-1531. <https://doi.org/10.1631/FITEE.2000622>

Initial-condition-switched boosting extreme multistability and mechanism analysis in a memcapacitive oscillator

Key words: Extreme multistability; Initial-condition-switched boosting; Memcapacitive oscillator; Mechanism analysis

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Motivation

- Due to the memcapacitive characteristics, more and more memcapacitive oscillating circuits have been constructed to explore the nonlinear phenomenon.
- Extreme multistability has potential advantages for some engineering applications, e.g., secure communications.
- The initial conditions of the state variables may induce initial offset boosting coexisting attractors with extreme multistability, while the forming mechanism of the coexisting attractors has rarely been discussed.

Main idea

- The cosine function is treated as the inverse memcapacitance nonlinearity in the proposed memcapacitive oscillator, which could be an effective way to implement the initial-condition-switched boosting extreme multistability.
- The incremental integral reconstruction method is an effective method to explore the mechanism of initial-condition-switched boosting behaviors and is employed in this paper. The mechanism of the initial-condition-switched boosting behaviors can be reflected in the bifurcation analysis of the initial-condition-related parameters of the reconstructed system.

Method

1. The mathematical model of the memcapacitor is as follows:

$$C_M^{-1}(\sigma) = a + b \cos \sigma,$$

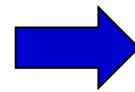
$$\dot{\sigma} = q.$$

Here q is the charge at both ends of the memcapacitor, σ is the time integration of q , and $C_M^{-1}(\sigma)$ stands for the inverse memcapacitor related to σ . The presented inverse memcapacitance possesses triangular nonlinearity.

Method (Cont'd)

2. The memcapacitive oscillator is composed of four linear circuit elements and one memcapacitor. The mathematical model of the memcapacitor is as follows:

$$\begin{aligned}
 L \frac{di_L}{dt} &= v_C, \\
 C \frac{dv_C}{dt} &= \frac{1}{R} [(a + b \cos \sigma) q_M - v_C] - i_L, \\
 \frac{dq_M}{dt} &= \frac{1}{R} v_C + \left(G - \frac{1}{R} \right) (a + b \cos \sigma) q_M, \\
 \frac{d\sigma}{dt} &= q_M.
 \end{aligned}$$



$$\begin{aligned}
 \dot{x} &= \mu y, \\
 \dot{y} &= -x - y + [\alpha + \beta \cos(\eta u)] z, \\
 \dot{z} &= y - \gamma [\alpha + \beta \cos(\eta u)] z, \\
 \dot{u} &= z
 \end{aligned}$$

Dimensionless form

Method (Cont'd)

3. The three-dimensional dimensionality reduction model is written as

$$\begin{aligned}\dot{X} &= \mu Y + \eta_1, \\ \dot{Y} &= -X - Y + \alpha Z + \frac{\beta}{\eta} \sin(\eta Z + \eta \eta_0) - \frac{\beta}{\eta} \sin(\eta \eta_0) + \eta_2, \\ \dot{Z} &= Y - \gamma \alpha Z - \frac{\gamma \beta}{\eta} \sin(\eta Z + \eta \eta_0) + \frac{\gamma \beta}{\eta} \sin(\eta \eta_0) + \eta_3.\end{aligned}$$

Here, denote $X = \int_0^t x d\tau$, $Y = \int_0^t y d\tau$, $Z = \int_0^t z d\tau$, and

$$\eta_0 = u(0), \eta_1 = x(0), \eta_2 = y(0), \eta_3 = z(0).$$

These initial-condition-related parameters are explicitly expressed in the dimensionless model.

Method (Cont'd)

4. Equivalent realization circuit

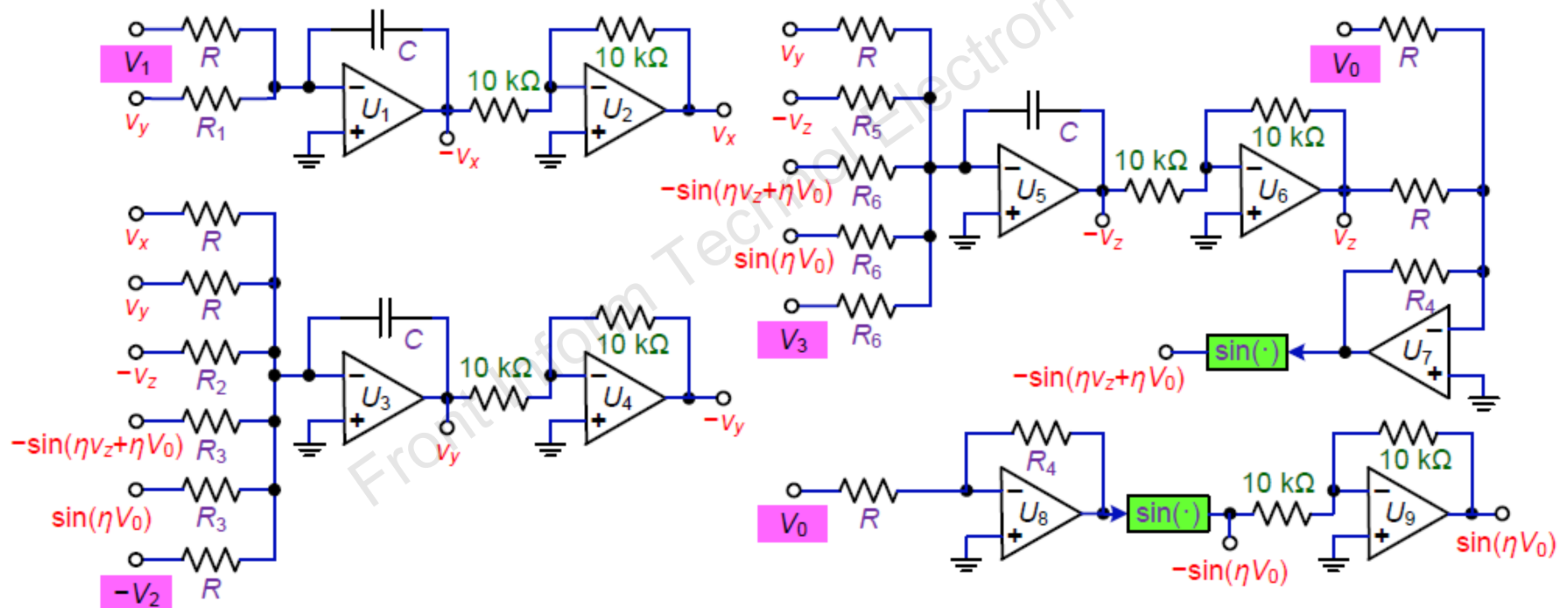


Fig. 14 Analog implementation circuit for the dimensionality reduction system

Major results

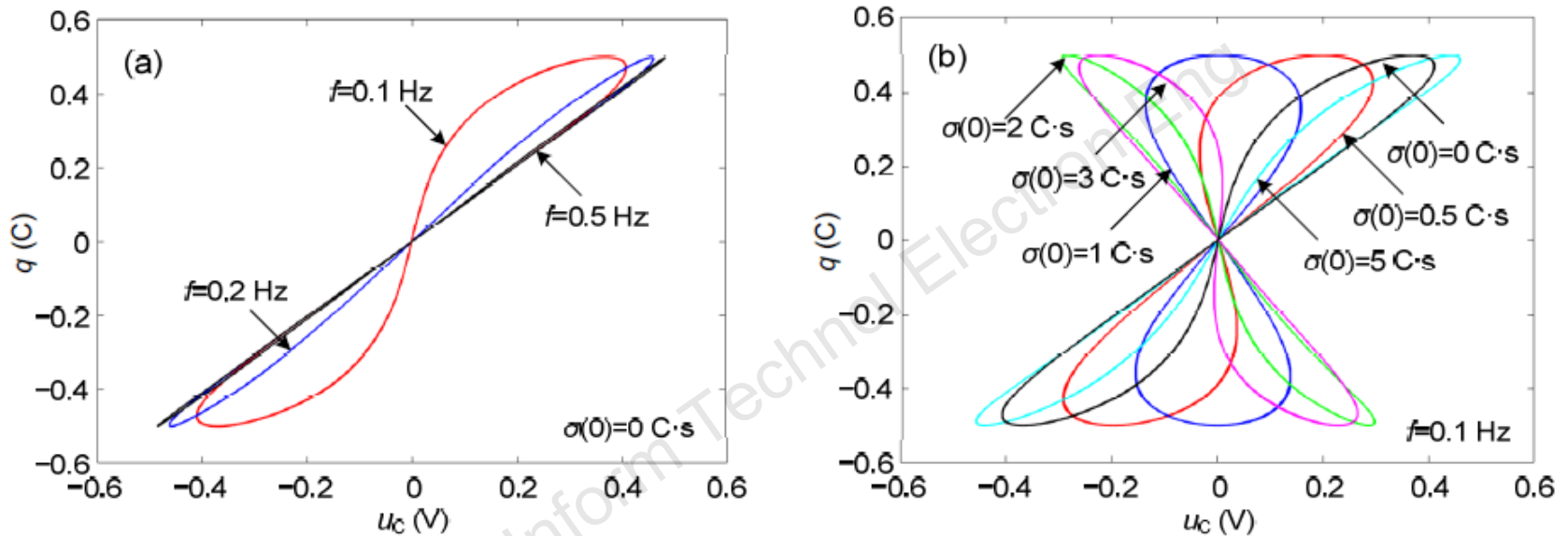


Fig. 1 Pinched hysteresis loops of the memcapacitor model: (a) pinched hysteresis loops with $\sigma(0) = 0 \text{ C}\cdot\text{s}$; (b) pinched hysteresis loops with $f = 0.1 \text{ Hz}$

$u_C(t) = C_M^{-1}(\sigma) q(t)$ is the terminal voltage of the memcapacitor

Major results (Cont'd)

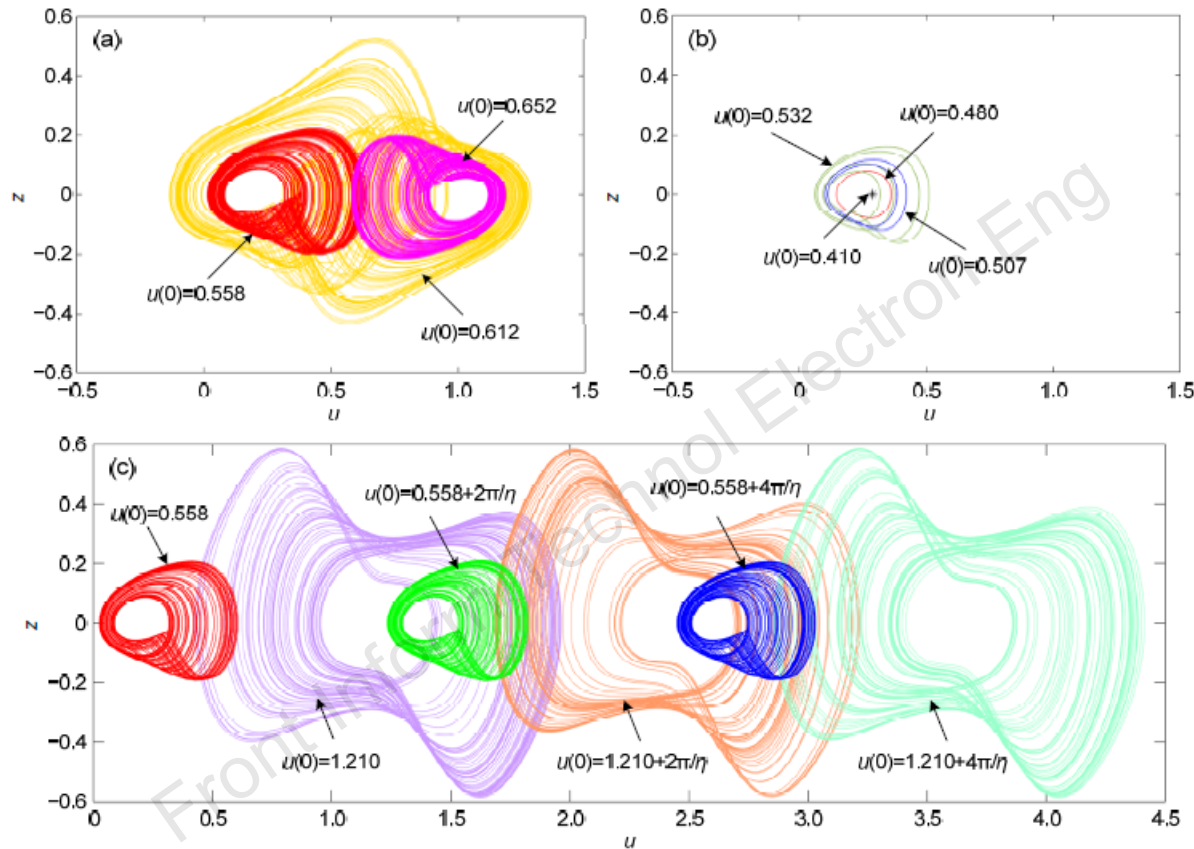
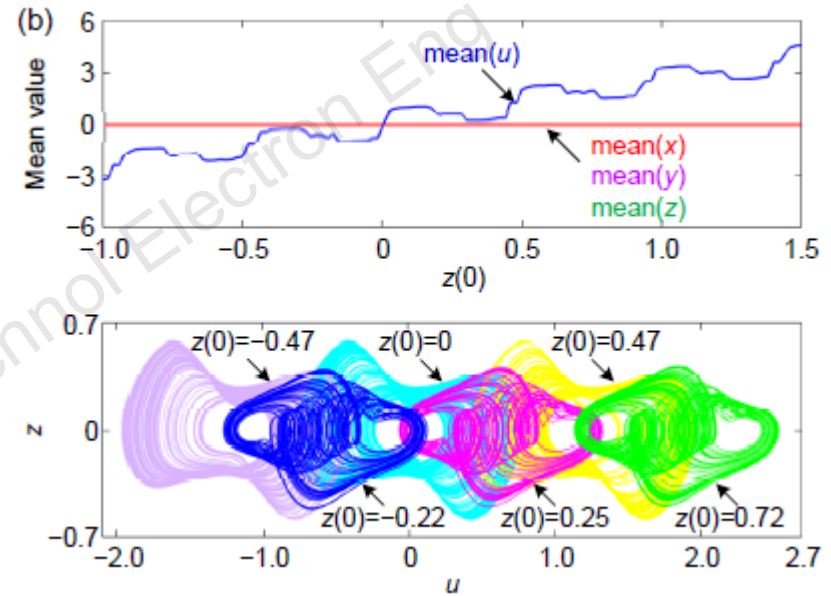
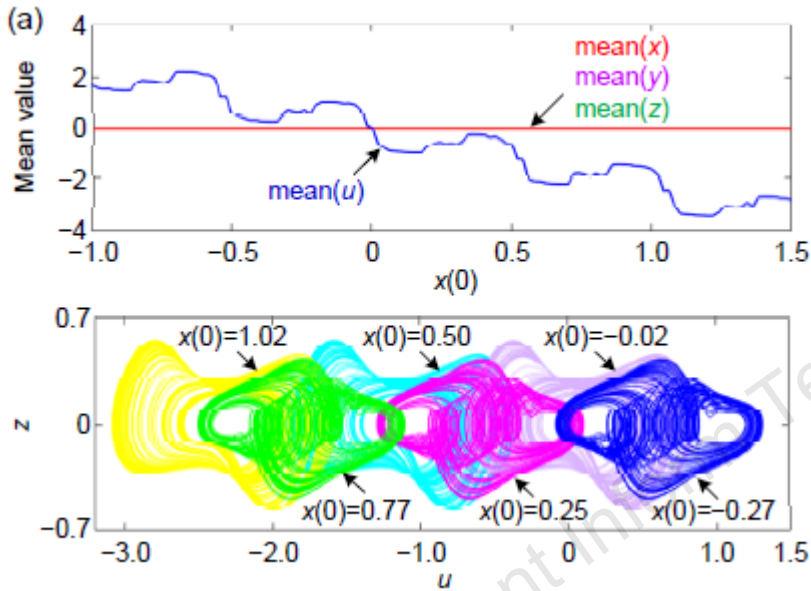


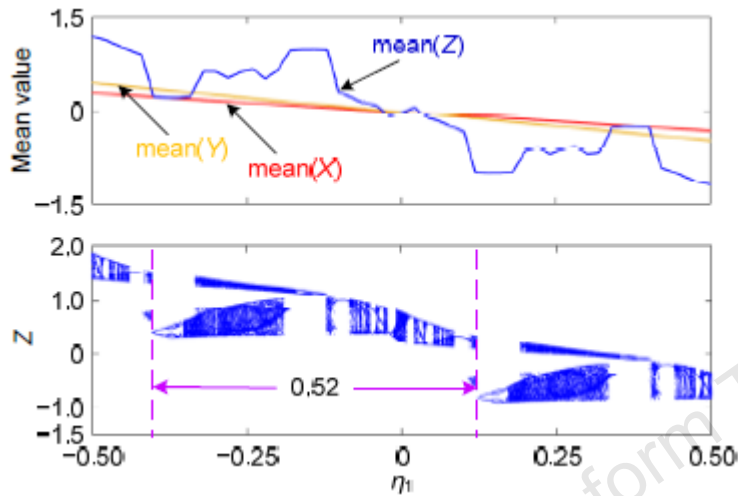
Fig. 7 Initial $u(0)$ dependent coexisting behaviors and initial-condition-switched boosting behaviors with fixed $x(0)=10^{-9}$, $y(0)=z(0)=0$: (a) coexisting chaotic attractors in an individual period, where the left spiral attractor (red), right spiral attractor (magenta), and double-scroll attractor (yellow) correspond to $u(0)=0.558$, 0.652 , and 0.612 , respectively; (b) coexisting point and periodic attractors in an individual period, where the stable point (black), period-1 limit cycle (red), period-2 limit cycle (blue), and period-4 limit cycle (dark green) correspond to $u(0)=0.410$, 0.480 , 0.507 , and 0.532 , respectively; (c) initial offset-boosted coexisting attractors in three adjacent periods, where the red, green, and blue spiral attractors correspond to $u(0)=0.558$, $0.558+2\pi/\eta$, and $0.558+4\pi/\eta$, respectively, and the lavender, dark orange, and light green chaotic attractors correspond to $u(0)=1.210$, $1.210+2\pi/\eta$, and $1.210+4\pi/\eta$, respectively. References to color refer to the online version of this figure

Major results (Cont'd)

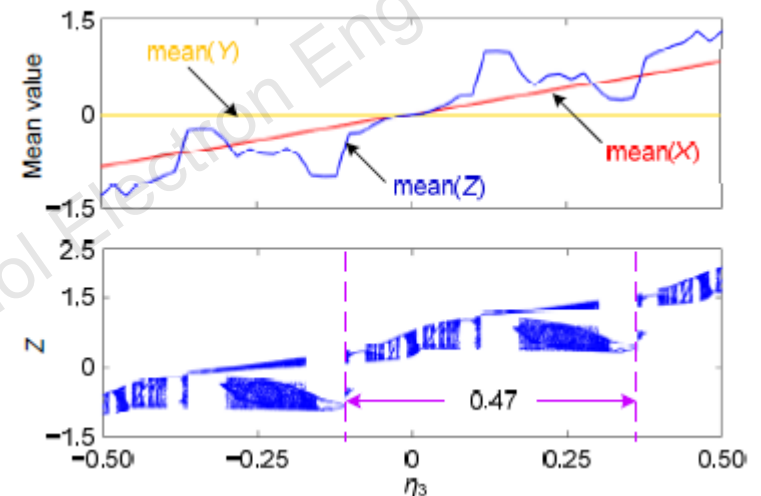


Initial-condition-switched boosting behaviors illustrated by the mean values of the four state variables and corresponding phase portraits: (a) $y(0)=10^{-9}$ and $z(0)=u(0)=0$; (b) $x(0)=10^{-9}$ and $y(0)=u(0)=0$

Major results (Cont'd)



(a)



(b)

Initial parameter-switched boosting behaviors of the dimensionality reduction system illustrated by the state mean values and bifurcation diagrams: (a) η_1 -switched boosting behavior with $\eta_2=10^{-9}$ and $\eta_3=\eta_0=0$; (b) η_3 -switched boosting behavior with $\eta_1=10^{-9}$ and $\eta_2=\eta_0=0$

Major results (Cont'd)

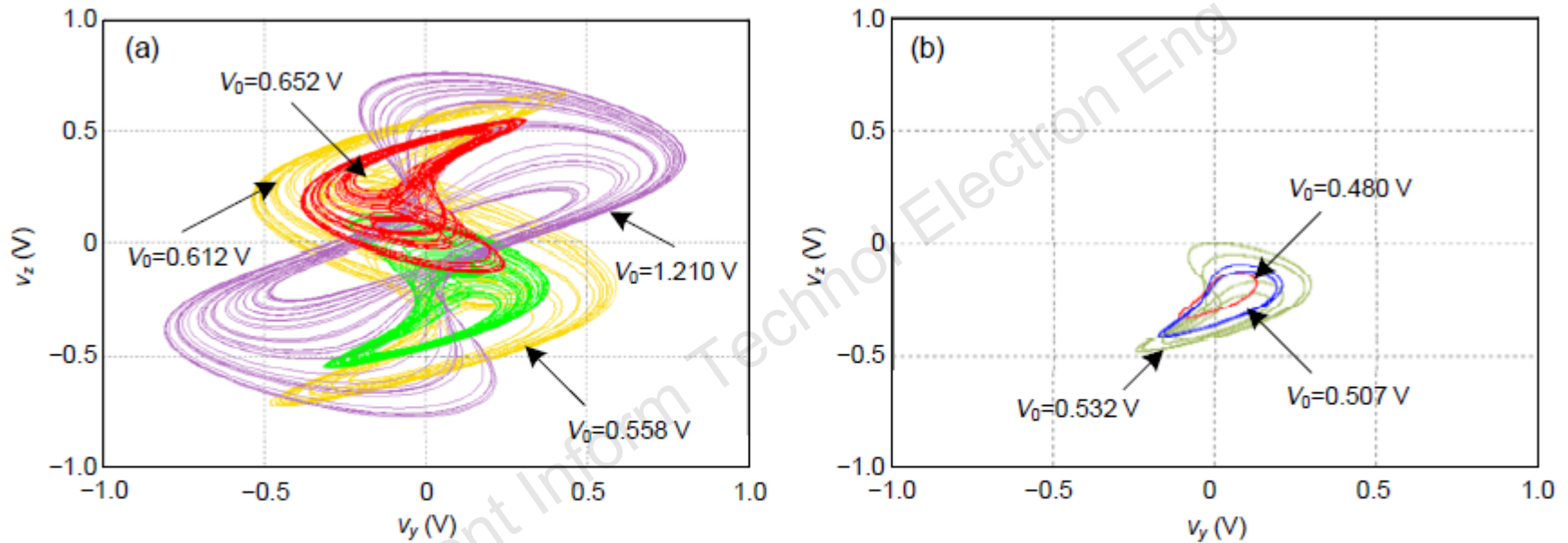


Fig. 15 Power SIMulation (PSIM) circuit simulations of the dimensionality reduction system for different initial conditions: (a) chaotic behaviors with $V_0=0.558/0.612/0.652/1.210$ V, $V_1=10^{-9}$ V, and $V_2=V_3=0$ V; (b) periodic behaviors with $V_0=0.480/0.507/0.532$ V, $V_1=10^{-9}$ V, and $V_2=V_3=0$ V

Conclusions

- The 4D memcapacitive oscillator was constructed by introducing a cosine memcapacitor into an oscillator.
- The 4D memcapacitive oscillator possessed a line equilibrium set and its stability periodically evolved with the initial condition of the memcapacitor.
- The initial-condition-switched boosting extreme multistability was revealed.
- Based on a reduced-order system constructed by the integral transformation, the mechanism of the initial-condition-switched boosting behavior was uncovered.
- The PSIM circuit simulations further confirmed the initial-condition-switched boosting extreme multistability.