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Unusual phenomenon of optimizing the Griewank function with the increase of dimension

Key words: Griewank; Two-scale structure; Multi-scale quantum harmonic oscillator algorithm; Quantum tunnel effect

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Motivation

1. The Griewank function is a typical multimodal benchmark function composed of a quadratic convex function and an oscillatory nonconvex function.
2. Different from most test functions, an unusual phenomenon appears when optimizing the Griewank function.
3. The Griewank function first becomes more difficult and then easier to optimize with the increase of dimension.

Main idea

1. The Griewank function is composed of a quadratic convex function with a unique global minimum, and an oscillatory nonconvex function with numerous local minima.
2. With the increase of dimension, the comparative importance of these two parts alters.
3. The two-scale structure is the core difficulty in the optimization process.
4. Structural, mathematical, and quantum analyses can be used to reveal the reason behind this phenomenon.

Method

1. Three analytical methods are proposed to interpret this phenomenon, including structural, mathematical, and quantum analyses.
2. Frequency transformation and amplitude transformation are implemented on the Griewank function to make a generalization.

Major results

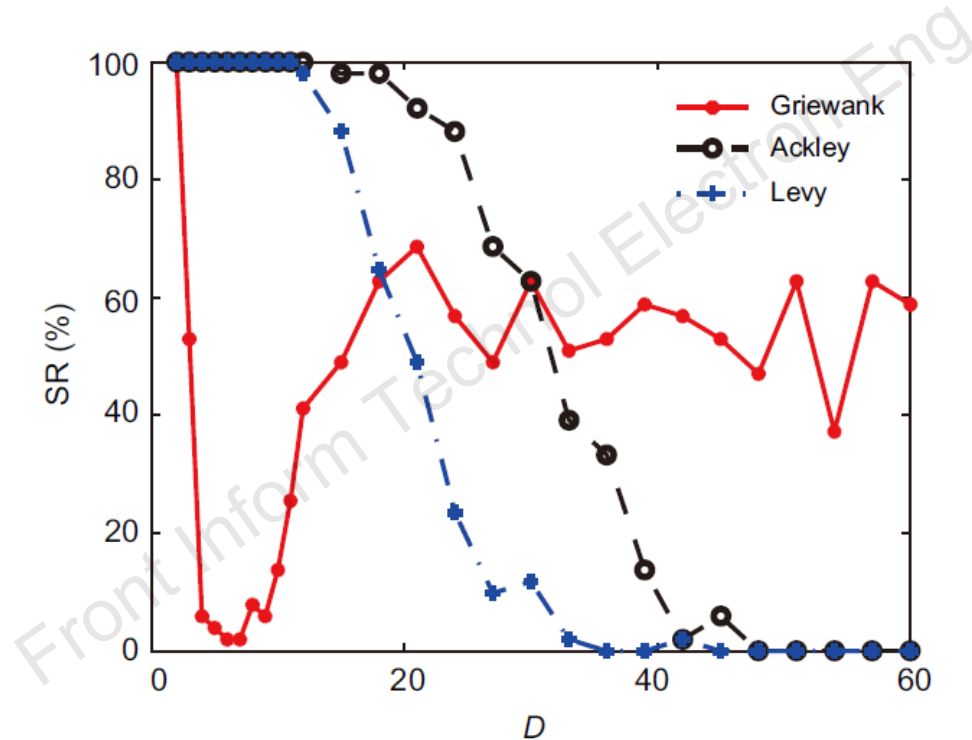


Fig. 3 Success rates (SRs) of Griewank, Ackley, and Levy, optimized with SPSO2011

Major results (Cont'd)

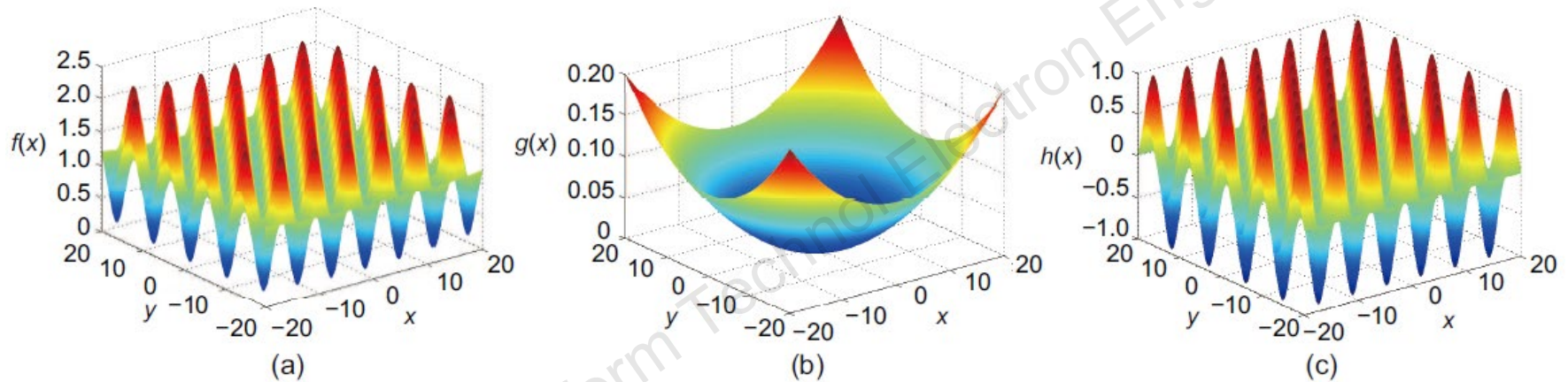


Fig. 2 Three-dimensional images of the Griewank function and its components: (a) Griewank ($f(x)$); (b) $g(x)$; (c) $h(x)$

Major results (Cont'd)

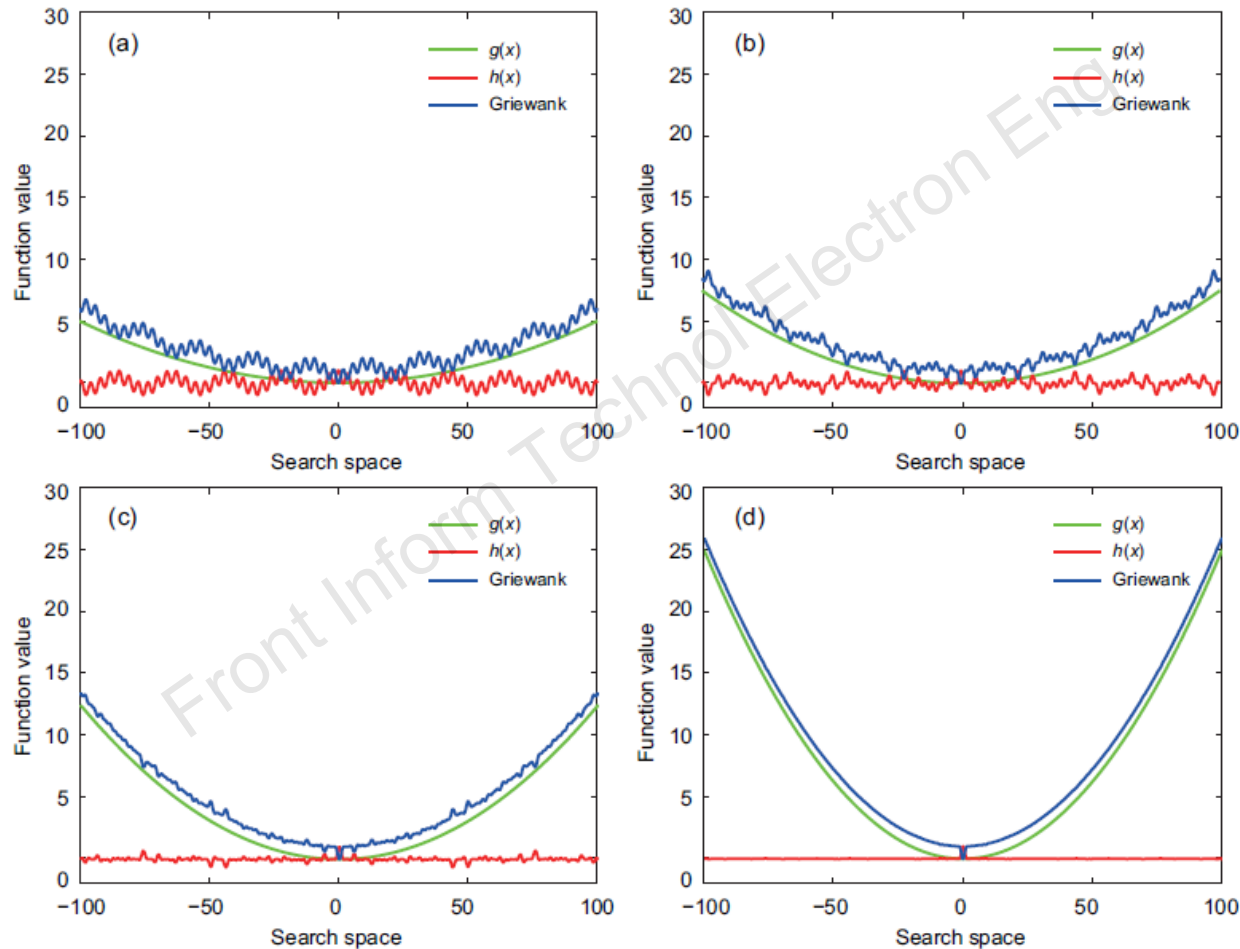


Fig. 4 Projecting an n -dimensional Griewank function to a plane: (a) $n = 2$; (b) $n = 3$; (c) $n = 5$; (d) $n = 10$.
References to color refer to the online version of this figure

Major results (Cont'd)

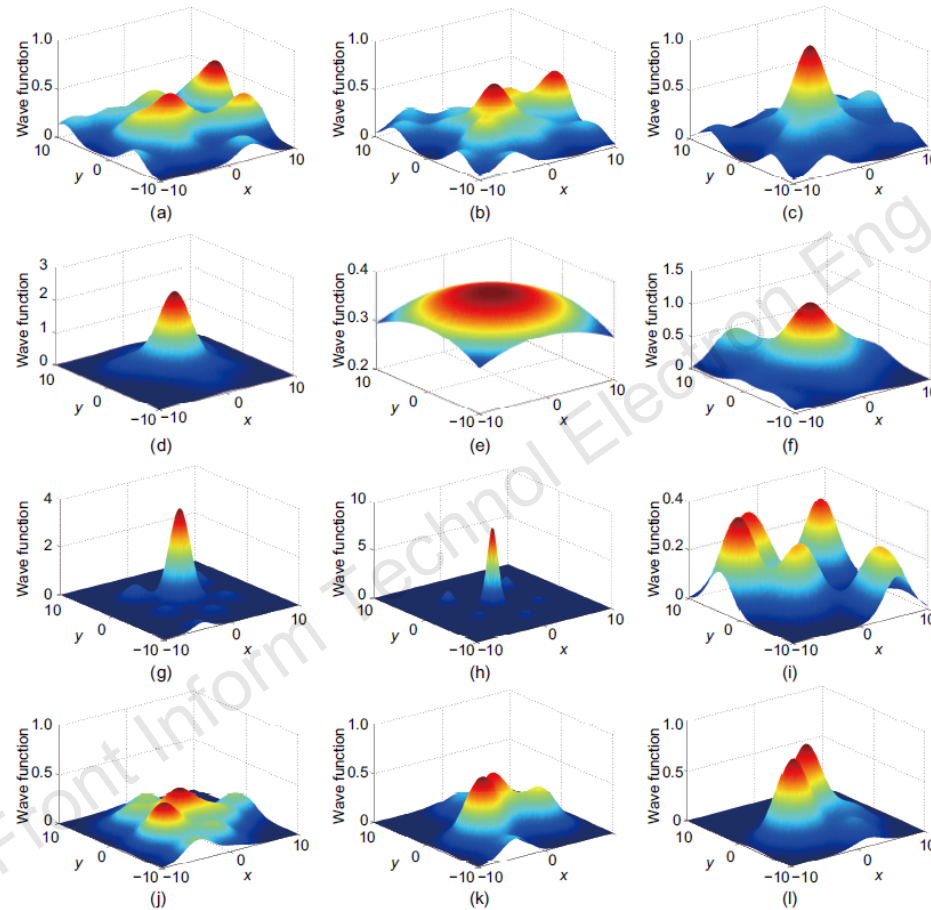


Fig. 9 Wave functions of the Griewank function optimized by MQHOA: (a) initial status ($k=20$, $\sigma_0=2$, $i=1$, $\sigma=2$); (b) competitive sampling areas appear ($k=20$, $\sigma_0=2$, $i=400$, $\sigma=2$); (c) aggregate and approach to the optimal solution ($k=20$, $\sigma_0=2$, $i=450$, $\sigma=2$); (d) the global optimal solution is obtained accurately ($k=20$, $\sigma_0=2$, $i=20\,000$, $\sigma=2$); (e) initial status, very flat in shape ($k=20$, $\sigma_0=20$, $i=1$, $\sigma=20$); (f) competitive sampling areas appear ($k=20$, $\sigma_0=20$, $i=505$, $\sigma=2.5$); (g) the optimal area becomes obvious ($k=20$, $\sigma_0=20$, $i=7611$, $\sigma=1.25$); (h) the global optimal solution is obtained accurately ($k=20$, $\sigma_0=20$, $i=7613$, $\sigma=0.625$); (i) initial status, the sampling area is very scattered ($k=10$, $\sigma_0=2$, $i=1$, $\sigma=2$); (j) sampling points gather gradually ($k=10$, $\sigma_0=2$, $i=250$, $\sigma=2$); (k) three competitive sampling areas appear ($k=10$, $\sigma_0=2$, $i=2500$, $\sigma=2$); (l) two competitive sampling areas compete with each other, and the global optimal solution is not obtained accurately ($k=10$, $\sigma_0=2$, $i=20\,000$, $\sigma=2$)

Conclusions

1. The Griewank function first becomes more difficult and then easier to optimize with the increase of dimension.
2. Structural, mathematical, and quantum analyses on the Griewank function explain the reason for this phenomenon.
3. MQHOA is used to optimize the generalized Griewank function and it performs well.